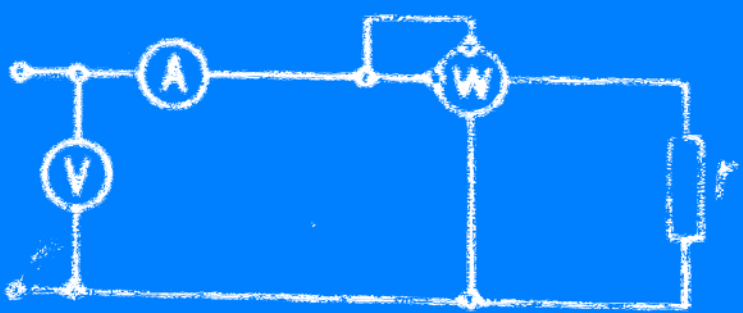
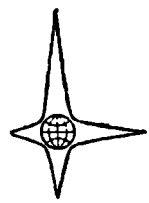
M. Kuznetsov

Fundamentals of Electrical Engineering





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ОСНОВЫ ЭЛЕКТРОТЕХНИКИ

ИЗДАТЕЛЬСТВО «ВЫСШАЯ ШКОЛА»

МОСКВА

На английской языке

M. Kuznetsov

FUNDAMENTALS OF ELECTRICAL ENGINEERING

Translated from the Russian

by

BORIS KUZNETSOV

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Chapter One

ELECTROSTATICS

1. Atoms and Molecules

After centuries of research, scientists have concluded that all things around us, despite their diversity, consist of a limited number of simple elements. To date, a total of 104 elements have been discovered.

Each element is made up of exceedingly small particles called atoms. The atoms of different elements differ from one another; those of one element all have the same specific properties. The atoms of an element which have the same charge on the nucleus (or the same atomic number) but differ in mass (or mass number) are called *isotopes*. The isotopes of an element are chemically identical but have somewhat different physical properties.

Two or more atoms are capable of uniting with one another to form a higher order of particles called *molecules*. If the atoms which compose the molecules of a substance are all of the same kind, the substance is called an *element*. If the atoms are not all of the same kind, the material is a chemical compound.

The number of atoms in molecules varies within broad limits. A molecule of water consists of two hydrogen atoms and one oxygen atom; a molecule of sulphuric acid is composed of two hydrogen atoms, one sulphur atom and four oxygen atoms. Some acids have molecules consisting of several hundred atoms. A protein molecule has thousands of atoms of hydrogen, carbon, oxygen, nitrogen, phosphorus and sulphur.

In 1869, Mendeleyev noticed that by arranging the elements in order of increasing atomic weight (from hydrogen to lead and bismuth) he obtained a sequence of recurring physical and chemical properties of groups of elements. Mendeleyev's original table, known since then as the Periodic Table (or System), had seventeen columns with 92 places in which the sixty-four then known elements were entered. His confidence in the new

classification was so profound that he predicted the existence of a number of elements and even described their basic properties. The subsequent discovery of gallium, scandium and germanium confirmed the remarkable accuracy of Mendeleyev's predictions.

2. Fundamentals of Electricity and the Electron Theory

Electricity has been known to man since the early days of his history. The ancient Greeks, 2500 years ago, noticed that when rubbed with cloth, a piece of amber would acquire the ability to attract light objects. The phenomenon was called "electricity" from the Greek "electron" for amber. Glass, resin, ebonite, sealing-wax and many other substances exhibit the same property. They become "electrified". The process of producing this condition in an object is called *electrification.* An electrified object can transmit some of its charge to another (unelectrified) object if brought into contact or connected with it by a metal wire. If the connection is by means of a porcelain, glass or ebonite rod, no transfer of electricity (charge) will occur. This difference serves as a basis for a practical classification of materials into *conductors* and *insulators* (or *dielectrics*). Conductors embrace all metals, carbon, and solutions of salts, acids and bases. Insulators include all gases under ordinary conditions, many liquids and almost all solids except metals and carbon. Thus, ebonite, glass, rubber, mica, silk, paraffin wax, marble, transformer oil, etc., are insulators. It has been found that electrified bodies either repel or attract each other. If two bodies electrified by a glass rod brushed against a piece of leather are brought together, they will repel each other. The same will occur if both are charged by an ebonite rod stroked with a piece of cloth. But if one is charged from a glass rod and the other from an ebonite rod, they will attract. The charge on the glass rod is evidently unlike that on the rubber. These facts suggest that objects that are similarly charged repel each other; those with unlike charges attract one another. Thus we have two types of electricity: positive and negative.

Atoms were once thought to be primary, indivisible and immutable building blocks of matter, whence the name "atom", the Greek for "indivisible".

At the end of last century, experimenting with high-voltage currents passed through a tube containing a highly rarefied gas, physicists noticed a greenish glow at the cathode (the electrode connected to the negative terminal of an electric energy source). The glow was found to be caused by some invisible rays which were called *cathode rays.* They were attracted by a positive charge and repelled by a negative charge. This suggested that the cathode rays were beams of negatively charged particles.

In 1895 the German physicist Roentgen discovered what he called X-rays—an invisible radiation capable of passing through many materials opaque to light. Today, X-rays have found many applications in medicine and industry.

In 1896 it was found that salts and minerals of uranium emitted rays that gave an impression on a photographic plate covered with black paper. Soon Marie Sklodowska-Curie and her husband Pierre Curie disclosed that salts and minerals of thorium, like those of uranium, emitted invisible rays capable of passing through thin sheets of metal and other opaque materials. In 1898, the Curies discovered two more elements—radium and polonium—with the same property. This spontaneous emission of radiations was called *radioactivity*. In her investigations of the element radium, Mme. Curie found this soft silvery metal was luminous in the dark, could decompose water into oxygen and hydrogen, affect a photographic plate, and liberate heat continuously. On disintegrating, radium emits three types of radiation (particles or rays): alpha, beta and gamma, and ultimately changes to the stable element lead.

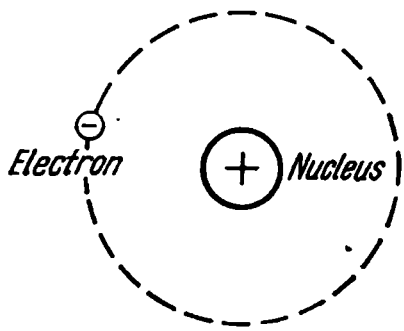


Fig. 1. The nucleus of the hydrogen atom

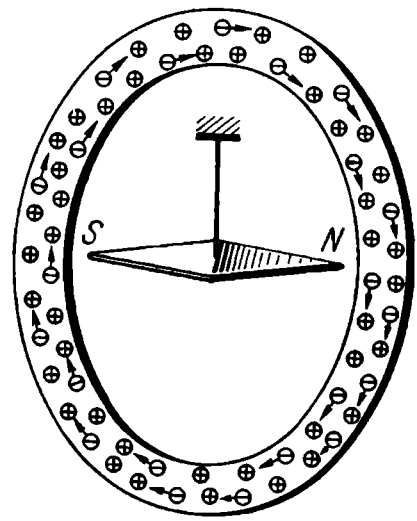


Fig. 2. Illustrating the experiment by Mandelstamm and Papalexi

Cathode rays, X-rays, radioactivity and other physical, chemical and magnetic phenomena suggested that the atom was not indivisible, but had a complex structure. Research has shown that atoms are composed of a positively charged central core, the *nucleus*, which also contains neutral particles, and a cloud of negatively charged *electrons*.

Ordinarily, atoms are electrically neutral. This, however, is not a sign that they are uncharged. Simply, they have equal amounts of positive and negative electricity.

The atoms of different elements differ as to weight (atomic weight), the magnitude of the positive charge on the nucleus, and the number of electrons around the nucleus. For example, in the simplest case, the hydrogen atom has only one electron (Fig. 1) rotating around the nucleus. The atom of copper has 29 electrons, that of gold 79 electrons, and that of uranium 92 electrons, all moving about the nucleus in various orbits. As can be seen,

the number of electrons always corresponds to the place an element occupies in the Periodic Table.

Although modern physics doubts that electrons can be accurately considered in orbital motion about the nucleus, the concept of orbits is helpful for an elementary discussion. The electrons in the outermost orbits are more loosely bound to the nucleus than those in the inner orbits. In various ways, the outer electrons can be knocked out of their orbits to be made *free electrons*. The number and freedom of motion of free electrons determine the properties of a material as a conductor of electricity. Metals have large numbers of free electrons and are good conductors. Nonmetals lack free electrons and are poor conductors.

Ordinarily, in a metal the atoms, ions and free electrons are in an irregular and random thermal motion. However, if caused to flow in one direction, they produce an electric current.

The existence of free electrons in metals was shown in a clever experiment by the Russian scientists Mandelstamm and Papalexi in 1912. They took a metal ring (Fig. 2) and placed a compass needle near the ring and opposite its centre. The ring was rotated at a high speed and then brought sharply to a standstill. At that moment, the compass needle would turn to align itself with the axis of the ring instead of pointing to the North. After some time, it would again seek North. The following explanation can be offered: when the ring is rotated, the free electrons are set into motion together with the atoms of the metal. When the ring is suddenly stopped, the atoms stop too, but the free electrons keep on moving by inertia, giving rise to an electric current in the ring. The current sets up a magnetic field around the ring, and the compass needle deflects (for the reason to be explained later on).

We have already learned that the positive and negative charges in an atom are equal. If, however, an atom loses some of its electrons, it becomes a *positive ion*. On the other hand, it may pick up one or more electrons in addition to its normal quota to become a *negative ion*. In both cases, the atom becomes electrically charged. The process by which a neutral (uncharged) atom or molecule acquires a charge, thus becoming an ion, is known as *ionization*. It can occur by a number of mechanisms. One of these mechanisms is electrification by rubbing. If a glass rod, for example, is rubbed with a piece of leather, it loses some of its electrons to become positively charged; the leather becomes negatively charged as it gains the electrons lost by the glass.

Another mechanism of ionization is the *photoelectric* (or more specifically *photoemissive*) effect which is utilized in photocells, discovered by the Russian physicist Stoletov. In this effect, radiation impinging on certain metals (sodium or potassium) causes bound electrons to go free.

When we heat a metal to a high temperature, some of the outermost electrons still held back by atomic forces escape from the surface of the

metal. This is known as *thermionic emission*—the basis of radio valves and other electronic devices.

Gases can be ionized by the action of high temperatures, X-rays, ultraviolet rays, nuclear radiations, high voltages, and also by collision. Liquids are usually ionized by electrolysis: the dielectric properties of the solvent weaken the electrostatic bonds within the molecule of the solute and cause it to break up into positive and negative ions.

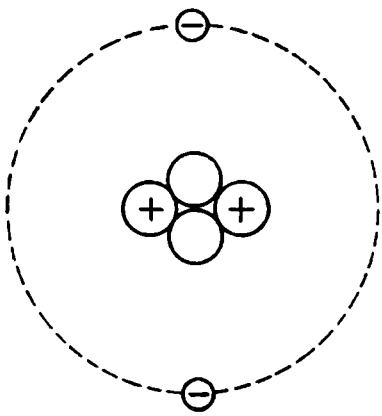


Fig. 3. The structure of the helium atom

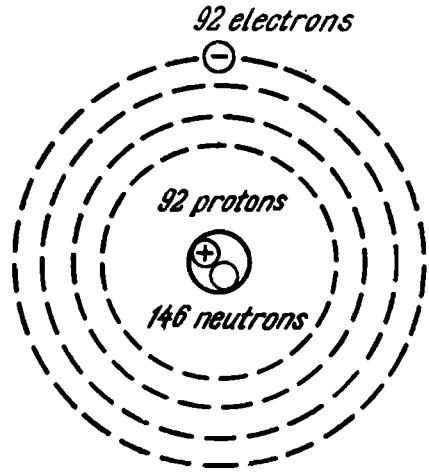


Fig. 4. The structure of the uranium atom

The rest mass (weight) of an electron is exceedingly small—about an eighteen hundredth (1/1839th) of that of the nucleus of a hydrogen atom, called the *proton*. It is therefore customary to assume that the whole mass of the hydrogen atom is concentrated in its nucleus.

The next element in the Periodic Table is helium. According to the electron theory, the helium atom has two protons in its nucleus and two electrons in motion about the nucleus. The positive charge on the nucleus of the helium atom is twice that of the hydrogen atom. Yet, the atomic weight of helium is 4 and not 2. Everything would dovetail, if we assumed that there are four protons in the nucleus of the helium atom. Then, however, the positive charge of the helium atom would exceed its negative charge—a condition which cannot exist. To explain the seeming discrepancy, the Soviet physicist Ivanenko suggested that in addition to protons the nucleus of every atom contains one or more neutrons, i.e., particles which have no charge but have mass equal to that of the proton. The nucleus of the helium atom would thus contain two protons and two neutrons, as shown in Fig. 3.

In the Periodic Table iron has atomic number 26 and atomic weight 56. This indicates that the nucleus of the iron atom has 26 protons and $56-26 = 30$ neutrons, with 26 electrons in orbital motion around the nucleus. Fig. 4 shows diagrammatically the structure of the uranium atom (atomic number 92, atomic weight 238).

In addition to protons, neutrons and electrons, the atom also contains other particles. One of them is the *positron*, a particle of positive charge and electron mass. It is produced by certain transformations inside the nucleus and was first recognized in cosmic radiation.

Another particle is the *meson*. Its existence was postulated by Yukawa in 1935 to account for the strong short-range nuclear forces between protons and neutrons. There are several species of mesons—neutral, positive and negative. They were first identified in cosmic rays. They are heavier than electrons but lighter than protons.

Still another fundamental particle is the *neutrino*, which has zero charge and mass much less than that of the electron. It is produced in the beta decay process. (As will be recalled, radioactive materials disintegrate to produce alpha particles, or the nuclei of helium atoms, which are positively charged, and beta particles which are negatively charged).

In 1955 still another fundamental particle was identified (the antiproton) and there was every reason to believe that ever more "fundamental" particles would be found. Since then many other so-called antiparticles have been discovered.

So far we have treated the difference between conductors and nonconductors from the view-point of classical physics which deals with microscopic bodies, i.e., bodies consisting of large assemblies of electrons and atoms. In the classical theory it was assumed that in nonconductors all the electrons in an atom are firmly attached to the nucleus, while in conductors the outermost (or valency) electrons are free to wander from atom to atom; moving in a regular way, they produce an electric current. Furthermore, the classical theory assumed that the atom can have any amount of energy (within a definite energy range) and that this energy changes continuously in any conceivable small portions. However, studies of optical spectra of elements and atomic interactions have shown that the energy changes within atoms are of discrete nature. Atomic and molecular physics states that the atom may only have a definite energy characteristic of this atom. The possible values of the internal energy of atoms are called *energy* or *quantum levels*. The energy levels which a given atom cannot have are called *forbidden levels*.

Quantum physics which investigates microscopic bodies and the laws governing their motion explains the difference between conductors and nonconductors thus. Free electrons exist in both conductors and nonconductors, the two groups only differing in the arrangement and occupancy of their energy levels—the basis of the band theory of electrical conduction. The total energy of the electrons revolving round the nucleus increases with increasing radius of the orbit. An electron may only occupy a definite quantum state or level which cannot be shared by any other electron. If an amount of extraneous energy is imparted to an electron, it may move to a new, higher quantum state or level. In this condition, both the electron and the atom to which it belongs are said to be excited. A trans-

fer from a higher to a lower energy level causes the electron to move into an orbit of a smaller radius. On returning to the lower level, the electron will give up the energy which had been spent to raise it to the higher level either as a quantum of light of definite wavelength characteristic of the atom, or to another electron. In a solid which consists of a multitude of atoms, the energy levels of the individual atoms fall into groups, or *bands*.

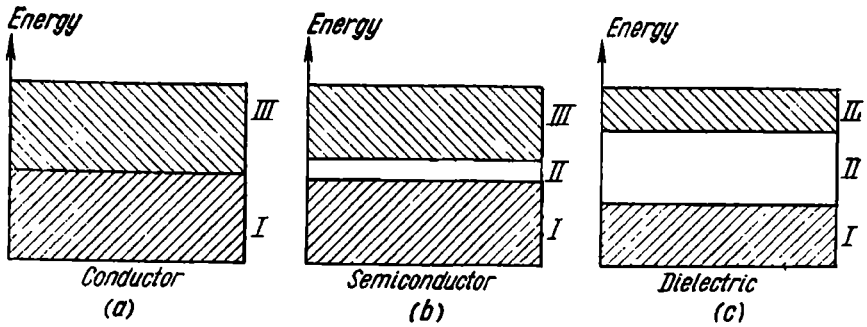


Fig. 5. Energy bands in a solid:

a—conductor; *b*—semiconductor; *c*—dielectric; *I*—filled (normal) band; *II*—forbidden band; *III*—empty (excitation) band

There may be a *filled band* occupied by normal, or unexcited electrons, and an *empty band* where electrons, when excited, can go; both bands are called *permitted*. The two bands are separated by an *energy gap*, or *forbidden band*. It should be noted in parenthesis that the word "band " must not be taken as meaning an area or a strip of any geometrical dimensions. When we speak of an energy band, we have in mind the amount of energy possessed by the electrons in this band. The width of the energy gap or forbidden band determines the properties of a material as a conductor of electricity.

Fig. 5 gives band diagrams for (a) a conductor; (b) a semiconductor; (c) an insulator (or dielectric). In case (a), the filled and empty bands overlap, there is no energy gap, and the electrons from the filled band are free to move into the empty band, and thus there is conduction.

In case (b) the energy gap between the permitted bands is narrow, and external factors (such as heat, light, electric fields, etc.) can easily move the electrons from the filled band across the energy gap into the empty band. Conduction then takes place by these excited electrons.

In case (c), there is a broad gap between the filled and empty bands. Electric fields of moderate strength cannot give enough energy to raise the electrons from the filled into the empty band, and there is practically no electric conduction.

Now we are in a position to define the term "electricity". *It is a property of matter* (a special form of motion of matter) *of a dual nature which manifests itself in elementary particles* (positive electricity in protons, positrons and mesons, and negative electricity in electrons, antiprotons, and mesons).

3. Systems of Electrical and Magnetic Units

In our study of electrical engineering we shall have to measure various electric and magnetic quantities. Several different systems of electrical and magnetic units are in common use for the purpose: the absolute c.g.s. electrostatic system, the absolute c.g.s. electromagnetic system, the practical system, the International System, and others.

In the absolute c.g.s. electrostatic system, as well as the absolute c.g.s. electromagnetic system, the fundamental units are the centimetre (cm), gram (g) and second (sec).

In the c.g.s. system the unit of velocity is 1 cm/sec; the unit of acceleration is 1 cm/sec^2; the unit of force is the dyne, which is 1 cm-g/sec^2; the unit of work is the erg (1 erg = 1 dyne$\times$1 cm).

Electrical and magnetic quantities expressed in the c.g.s. units, however, have magnitudes inconvenient for use in laboratory work and engineering. To meet the needs of the practical worker, a modified so-called International System, was introduced in which the fundamental units are the international ohm and the international ampere. They are defined as follows:

International Ohm: the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of uniform cross-sectional area, and 106.300 centimetres in length.

International Ampere: the unvarying electric current which, when passed through a standard specified solution of nitrate of silver in water, deposits silver at the rate of 0.001118 gram per second.

The other electrical units were based on the International Ohm and Ampere.

In the Soviet Union, use is currently made of the absolute rationalized m.k.s.a. system. This system is equally convenient for the measurement of electrical, magnetic and mechanical quantities.

The fundamental units in the m.k.s.a. system are the metre (m), kilogram (kg), second (sec) and absolute ampere (A).

The other units in the m.k.s.a. system are as follows: the unit of velocity is $v=1$ m/sec; the unit of acceleration is $a=1$ m/sec^2; the unit of force is the newton (1 N = 1 kg-m/sec^2) which gives the mass of one kilogram the acceleration of one metre per second; the unit of work is the joule (1 joule = = 1 N$\times$1 m).

In this system the absolute ampere is determined by means of a current (or ampere) balance, which measures the force of interaction between a moving and a stationary coil. The absolute ohm is defined as the resistance offered to the passage of a current of one absolute ampere so that one joule or 0.239 calorie of heat is liberated every second. The other electrical units are defined from the absolute ampere and ohm. The m.k.s.a. system is called "absolute" for the reason that three of its four basic units—the metre, the kilogram (mass) and the ampere—are multiples or decimal submultiples of the respective c.g.s. units. By way of example, the absolute

ampere is defined as exactly one-tenth of an ampere which is the electromagnetic unit of current. The word "absolute" with respect to the m.k.s.a. system will be omitted from now on.

The m.k.s.a. system is termed "rationalized" because it eliminates the factor 4π from the majority of equations describing electrical phenomena. This is true, for example, of the Ampere Law and its derivative relations, although the factor 4π appears in the Coulomb Law and the equations derived from it.

The relations between some of the m.k.s.a. units are as follows:

1 newton = 1 kg × 1 m/sec^2 = 1000 g × 100 cm/1 sec^2 = 10^5 dynes, or
1 newton = 10^5/981000 kgf = 0.102 kgf (kgf = kilogram force),
1 joule = 1 newton × 1 m = 10^5 dynes × 100 cm = 10^7 ergs.

Since 1 kgf-metre = 1000/427 calories, 1 joule = 1 newton × 1 m = = 0.102 kgf × 1 m = 0.102 × 1000/427 = 0.239 calories.

4. Coulomb's Law

This refers to the law of interaction between two concentrated electric charges or two magnetic poles. It reads as follows: the force, F, between two point charges, q_1 and q_2, is a pure attraction or repulsion and is directly proportional to the product of the charges placed in a homogeneous medium of permittivity $\varepsilon = \varepsilon_r \varepsilon_0$, and inversely proportional to the square of the dist nce, r, between them. In the m.k.s.a. system

$$F = \frac{q_1 q_2}{4\pi \varepsilon_r \varepsilon_0 r^2},$$

where ε_0 is the absolute permittivity of a vacuum. In the m.k.s.a. system in which electric charge (quantity of electricity) is expressed in coulombs,

$$\varepsilon_0 = \frac{1}{4\pi \times 9 \times 10^9} \text{ (farad/metre).}$$

The dimensions of ε_0 are derived from the Coulomb Law:

$$[\varepsilon_0] = \left[\frac{q_1 q_2}{4\pi \varepsilon_r F r^2}\right] = \frac{\text{coulomb} \times \text{coulomb}}{\text{newton} \times \text{m}^2}.$$

Since newton = joule/metre, and joule = volt × coulomb, we have

$$[\varepsilon_0] = \frac{\text{coulomb} \times \text{coulomb} \times \text{m}}{\text{volt} \times \text{coulomb} \times \text{m}^2} = \frac{\text{coulomb}}{\text{volt} \times \text{metre}}.$$

As will be shown later, coulomb/volt = farad, the unit of capacitance We thus finally have

$$[\varepsilon_0] = \text{farad/metre.}$$

The relative permittivity, or dielectric constant, ε_r, of a medium gives the factor by which this medium decreases the force of interaction between two charges in comparison with a vacuum (other conditions being equal). The dielectric constant is a dimensionless quantity. If the dielectric constant of a vacuum is taken as a reference standard and is unity, the constants for some of the materials used in electrical engineering are:

Petrol	2.3	Paraffin wax	2-2.4
Paraffin-impregnated paper	3.2	Rubber	2.6-3.5
		Mica	6-8
Oil-impregnated paper	3	Glass	5-10
Distilled water	81	Transformer oil	2.2
Air	1.0006	Ebonite (hard rubber)	3.1
Kerosene	2		
Marble	8.4		

Example. Find the force of interaction between two charges spaced a distance of 5 cm in a vacuum. The charges are 2×10^{-8} coulomb and 3×10^{-5} coulomb, respectively.

$$F = q_1q_2/4\pi\varepsilon_r\varepsilon_0r^2 = \frac{2\times10^{-8}\times3\times10^{-5}\times4\pi\times9\times10^9}{4\pi\times1\times0.05^2} = 2.16 \text{ newtons.}$$

Since 1 newton = 102 gf, $F = 2.16\times102 = 220$ gf.

The same charges separated by the same distance in kerosene will interact with a force

$$F = \frac{2\times10^{-8}\times3\times10^{-5}\times4\pi\times9\times10^9}{4\pi\times2\times0.05^2} = 110 \text{ gf,}$$

i.e., the force of interaction is half as great.

5. The Electric Field

Each electric particle projects into space a field of electric force, and as the particles move along a wire, the lines of force move with them. It is the motion of these lines of electric force that sets up a magnetic field transverse to them. A variable electric field is always accompanied by a magnetic field; and conversely, a variable magnetic field is accompanied by an electric field. The joint interplay of electric and magnetic forces is what is called an electromagnetic field and is considered as having its own objective existence apart from any electric charges or magnets with which it may be associated. Examples are the photon, or quantum of light, and the electromagnetic field radiated by an aerial.

Modern physics defines the electromagnetic field as a distinct form of matter possessing definite properties: it is distributed continuously in space; in a vacuum it propagates at the speed of light (300,000 km/sec); it interacts with charges and currents to convert itself into other forms of energy (thermal, mechanical, etc.).

The theory of the electromagnetic field was first stated by the Scotch physicist James Clerk Maxwell in his "Electricity and Magnetism" published in 1873.

In the case of a stationary charged body the magnetic fields built up by the elementary charges constantly moving inside it cancel each other, and there is practically no magnetic field. The same is true of a stationary permanent magnet which only displays a magnetic field and has no electric field. This condition enables us to investigate electric and magnetic fields separately.

We shall regard the electric field as one of the aspects of the electromagnetic field.

A measure of the strength of an electric field is given by the mechanical force per unit charge experienced by a very small body placed in this field and is denoted by the letter E.

By definition

$$E = \frac{F}{q}.$$

Taking one of the charges in the Coulomb Law equal to unity, the electric field strength E at a point distant r from the charge will be

$$E = \frac{q}{4\pi\varepsilon_r\varepsilon_0 r^2},$$

and *in vacuo*, where the relative permittivity is unity,

$$E = \frac{q}{4\pi\varepsilon_0 r^2}.$$

The dimensions of the electric field strength can be derived from its equation:

$$[E] = \left[\frac{F}{q}\right] = \frac{\text{newton}}{\text{coulomb}} = \frac{\text{joule}}{\text{metre} \times \text{coulomb}} =$$

$$= \frac{\text{coulomb} \times \text{volt}}{\text{metre} \times \text{coulomb}} = \frac{\text{volt}}{\text{metre}}.$$

Thus, in the rationalized m.k.s.a. system the electric field strength is expressed in volts per metre.

If the strength of an electric field is the same both in magnitude and direction at any point in space, the field is called *uniform.*

It is relevant to note that quantities which have both magnitude and direction are called *vectors*, as distinct from quantities which only have direction and called *scalars*. Typical vectors are force, speed, acceleration, while typical scalars are temperature, quantity of matter, energy, power. Vectors are shown graphically as arrows with their lengths giving magnitude on a chosen scale and the arrows themselves, direction.

An inertialess charge placed in an electric field would follow a path called a *line of force.* The total number of lines of electric force through

a surface placed in an electric field is called the *electric flux* and is denoted by the letter N. For a surface S normal to the vector of a uniform field of strength E, the flux is

$$N = ES.$$

The dimensions of the flux are

$$[N] = [ES] = \frac{\text{volt} \times \text{metre}^2}{\text{metre}} = \text{volt-metre}.$$

If the surface makes an angle α with the electric field vector, the flux will be

$$N = ES \cos \alpha = E_n S$$

where E_n is the component of electric field strength normal to the surface.

For a nonuniform field the flux is determined in a different way. The total surface is divided into surface elements, dS, which may be considered as placed in a uniform field. Then the flux through the entire surface will be

$$N = \Sigma E_n dS.$$

For a charge q placed in the centre of a sphere of radius R the electric field strength on the surface of the sphere is

$$E = \frac{q}{4\pi\varepsilon_r\varepsilon_0 R^2}.$$

The flux through the entire surface of the sphere is

$$N = \Sigma E_n dS.$$

Radiating from the central charge, the lines of electric force are normal to the surface elements of the sphere. Since the surface area of a sphere is $S = 4\pi R^2$, we have

$$N = \Sigma E_n dS = E\Sigma dS = E4\pi R^2.$$

Substituting the value of E in this equation gives us

$$N = \frac{q}{4\pi\varepsilon_r\varepsilon_0 R^2} 4\pi R^2 = \frac{q}{\varepsilon_r\varepsilon_0}.$$

The above equation of the electric field flux derived for a sphere is applicable to any other closed surface and expresses Gauss' theorem.

Fig. 6*a* gives a vector diagram of the strength of an electric field at points A and B distant r_1 and r_2, respectively, from a positive charge q immersed in a medium. The electric field strength is directed radially *away from* the charge decreasing with the square of the distance r_1 or r_2, and is different at points A and B. Fig. 6*b* gives a vector diagram of the strength of an electric field at points A and B, which are at the respective distances r_1 and r_2 from a negative point charge q immersed in a medium. The strength of the field is directed radially *towards* the charge.

Now let us examine the strength of an electric field due to two unlike charges, $+q_1$ and $-q_2$, at some point A (Fig. 7). In the absence of $-q_2$, the electric field strength at A would be E_1. Conversely, in the absence of $+q_1$, the electric field strength at A would be E_2. As E_1 and E_2 make an

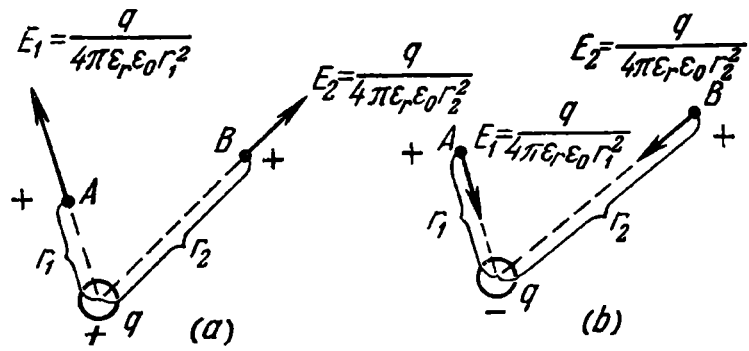


Fig. 6. Electric-field strength at different points in space

angle with each other, the resultant electric field strength E due to the joint action of $+q_1$ and $-q_2$ can be obtained by adding together the field vectors according to the parallelogram law. Using this method, it is possible to determine the electric field strength at any point for any number of electric charges.

Earlier we defined a line of electric force as the path that would be followed by an intertialess test charge. Placing a positive charge at different points in the field set up by a positively charged spherical body, we obtain a set of such paths, or lines of electric force. Obviously, any number of lines of electric force can be imagined in an electric field. In order to represent its strength, there is a well-established convention to draw as many lines of electric force through every square centimetre of area normal to the lines at a field point as will be equal to the field strength at that point. Consequently, the density of lines of force will give a graphic idea of the field strength. Fig. 8*a* shows the electric field of a positively charged sphere distant from any other charges, while Fig. 8*b* shows the electric field of a negatively charged sphere.

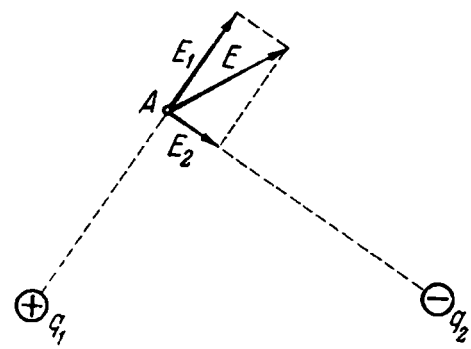


Fig. 7. Strength of a field due to two point charges

The electric field set up by two unlike charges presents a more complex picture (Fig. 9*a*). Its direction can be found by adding in succession the individual vectors for points *A*, *B*, *C*, *D* and *E* by the parallelogram law.

The broken line *ABCDE* thus obtained will give the direction of the electric field. With a greater number of points, such as in Fig. 9*b*, the broken line will give a more accurate indication. Obviously, the best approximation to

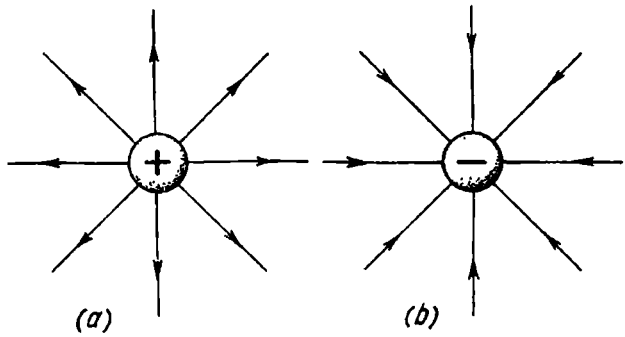


Fig. 8. Lines of force of a charged sphere

the direction of the field will be given by a line with an infinitely great number of points (Fig. 9*c*). It then becomes a smooth curve. The direction of the field at a given point will coincide with the field vector at this point and can be indicated by the direction of a line tangential to the line of electric force at the same point. Fig. 10*a* shows an electric field due to two unlike point charges, while Fig. 10*b* shows a field produced by two like point charges.

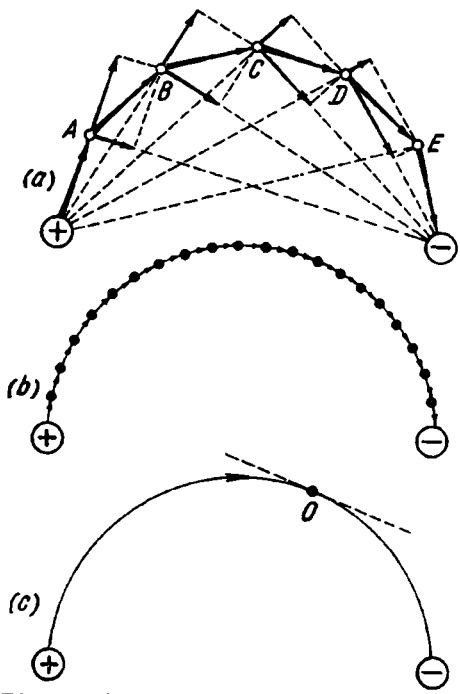


Fig. 9. Direction of a field at different points in space

In the case of a *uniform field*, the field vectors will be equal and parallel to one another, and the lines of electric force will be shown diagrammatically as parallel lines spaced at equal distances from each other.

As has been noted, like charges repel one another. Therefore, on any conductor the electric charge will concentrate only on its surface. The quantity of electricity per unit area is called the *surface charge density*. It depends on the quantity of electric charge on a given body and on the shape of the latter. On bodies of regular shape (a sphere, very long conductors of circular cross-section, and the like), the electric charge is distributed uniformly, and the surface charge density is the same at any point on the surface of such bodies.

On odd-shaped conductors the charge is not uniformly distributed. The charge density will be greater on projections and lower in hollows and depressions. It will be especially great at sharp points. This phenomenon is graphically explained in Fig. 11, where the diagram on the left illustrates a regular-shaped body (a sphere) and the diagram on the right, an irregular

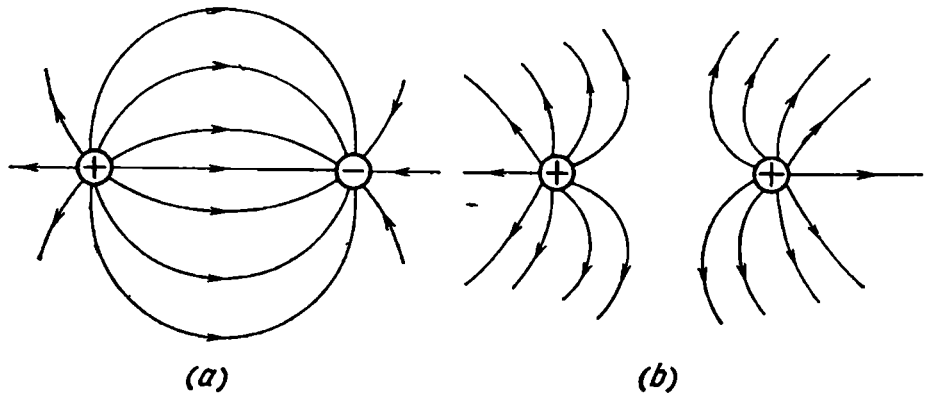


Fig. 10. Electric field due to two (a) unlike charges and (b) like charges

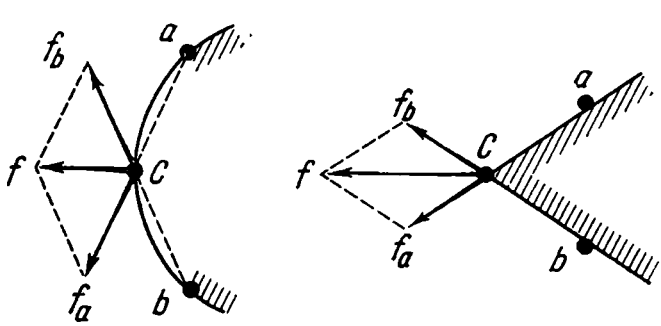


Fig. 11. Interaction between parts of a charge

body. Point charges *a* and *b* act on charge *C*. The forces f_a and f_b add together according to the parallelogram law to give their resultant force, *f*. With f_a and f_b respectively equal in both cases, the force *f* will be greater in the second case, since the angle between the component forces f_a and f_b is smaller. As a result, the charges at the sharp point will experience repulsion which will tend to remove these charges from the point. If the charge accumulating at the point is sufficiently large, it will set up an electric field around the conductor, and the air (or any other medium) will become ionized and conduct current. Then the charge will "leak" off the point. That is why in high-voltage engineering every effort is made to avoid sharp edges, ends or projections on conducting surfaces.

In the m.k.s.a. system the surface charge density is measured in coulombs per square metre.

6. A Conductor in an Electric Field

If an uncharged and insulated conductor is placed in an electrostatic field, the forces of the field will cause the charges of the conductor to separate. Fig. 12 shows a positively charged sphere A and a conductor B placed in the field of a sphere. The free electrons of the conductor are caused to drift in a direction opposite to that of the field. As a result, there will be an excess of electrons at the end near the sphere, giving this end a negative charge, while the opposite end will lack electrons, and its charge will be positive.

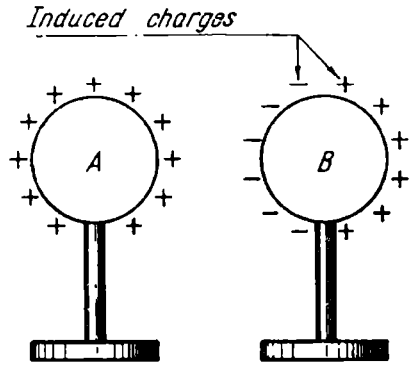


Fig. 12. Electrostatic induction

The separation of charges on a conductor due to the action of a charged body is called *electrostatic induction*, and the charges thus formed are termed *induced charges*. The number of induced charges on the conductor B will increase as it moves nearer to the sphere A. On the other hand, the electric field of A will change as soon as the conductor B has been placed within it. Instead of radiating uniformly, the lines of electric force of the field will curve towards the conductor. Furthermore, the conductor will cause the charges on the charged body to move to the side nearer to the conductor. The induced charges formed on the conductor can be separated by breaking the conductor in half.

When the conductor is removed from the field, the charges induced on it will disappear, and the conductor will again become uncharged.

7. Potential and Potential Difference

Picture to yourself an infinite and uniform electric field with a charge $+Q$ placed at point M. When left to itself, the charge will be caused (by the electric field) to move in the direction of the field through an infinitely great distance. In doing so, the electric field will expend some energy. If we relate this energy (or work) to a unit positive charge, then it will represent the potential energy (or simply potential) at a given point from which the field moves the charge to infinity. To move the charge $+Q$ from infinity back to point M, external forces have to perform work in order to overcome the force of the electric field. Then the potential φ at point M will be

$$\varphi_M = \frac{W}{q} = \frac{1 \text{ joule}}{1 \text{ coulomb}} = 1 \text{ volt.}$$

Since 1 joule $= 10^7$ ergs and 1 coulomb $= 3 \times 10^9$ statcoulombs (e.s. c.g.s. units of charge), in the m.k.s.a. system 1 V $= 1/300$ statvolts (e.s. c.g.s. unit of potential).

If a charge of 1 coulomb is moved from infinity to a point with a potential of 1 volt, the energy change, or work done, will be 1 joule. If a charge of 15 coulombs is moved from infinity to a point with a potential of 10 volts, the energy change, or work done, will be $10 \times 15 = 150$ joules.

Mathematically, this relation is expressed by the formula

$$W = q \cdot \varphi.$$

If a charge of 10 coulombs is to be moved from a point with a potential of 20 volts to another point with a potential of 15 volts, the energy change, or work done, is

$$W = q(\varphi_1 - \varphi_2) = 10(20-15) = 50 \text{ joules.}$$

The potential difference between two points of an electric field is also called *voltage*. It is designated by the letter V and is measured in volts.

Then the work done to move a charge from one point to another may be written as follows:

$$W = qV.$$

If a charge q is to be moved along a line of force from one point of a uniform field to another at a distance L, the following work will have to be done:

$$W = FL = EqL.$$

Since $W = qV$, $V = EL$, hence $E = \frac{V}{L}$.

The above expression relates the electric field strength and the voltage of a uniform field.

Around any electric field there exist points with equal potentials. Their actual location depends on the shape of the charged body. In the case of a charged metal sphere, points with equal potentials, or equipotential points, lie on spherical surfaces and the surfaces are called *equipotential surfaces*. Fig. 13 shows the equipotential surfaces of a positively charged sphere. By definition, the potential difference between any two points on the same surface is zero. Therefore, no work has to be done to move a charge between such points.

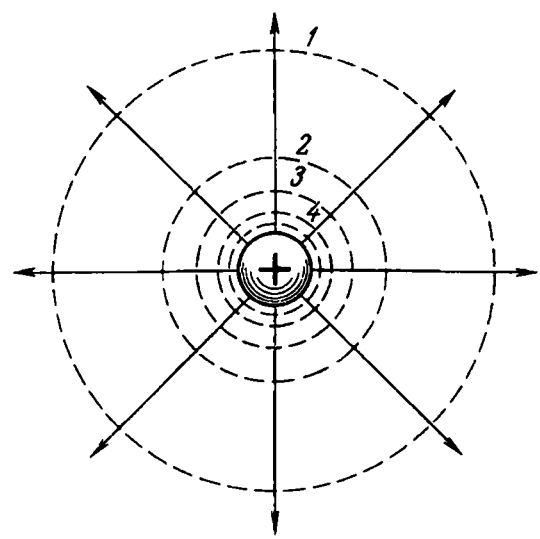


Fig. 13. Equipotential surfaces (shown by dotted lines) and lines of force (shown by solid lines) of a charged sphere distant from other objects

Equipotential surfaces are a very convenient method for illus-

trating electric fields. The convention in drawing them is that the potential difference between two adjacent equipotential surfaces should be the same, say, 1 volt. The surface of zero potential is drawn with an arbitrary radius; the radii of the other surfaces are chosen to maintain this difference of one volt between them. In Fig. 13 it will be seen that the spacing between equipotential surfaces decreases as we move nearer to the central charge. This is because the potential of the surfaces increases while the potential difference between them remains the same. If we move away from the central charge, the spacing increases. Lines of electric force are normal to an equipotential surface at any point, otherwise the work done to move a charge on an equipotential surface would be other than zero. The surface of the central charge is also an equipotential surface—all points on it are at the same potential, as are all the points inside the charged body.

If we connect two conductors at different potentials with a metal wire, the electric field that exists around the conductors due to the potential difference or voltage across them will cause the free electrons to flow towards the point of higher potential, and electric current will flow in the wire. The electrons will flow until the potentials of the conductors become equal, i.e., until there is no potential difference between them. A convenient analogy may be provided by two vessels filled with water to different levels. If they are connected at the bottom by a tube, the water from one vessel will flow into the other with less water until both are at the same level.

The potential of the earth is taken as zero for the reason that any conductor connected to it loses practically all of its electric charge.

8. Capacitance

The charge and potential of an isolated conductor bear a linear relation to each other and their ratio is constant for a given conductor:

$$\frac{q}{\varphi} = C.$$

The same is true of any other conductor, although its constant will be different, as it depends on a number of factors, one of which is the size of the conductor. Indeed, one and the same charge imparted to different conductors will produce different potentials on them.

This ratio of charge to potential is called the *capacitance* of a conductor and is designated by the letter C:

$$C = \frac{q}{\varphi}.$$

The unit of capacitance can be determined thus:

$$\text{Unit of capacitance} = \frac{\text{unit of charge}}{\text{unit of potential}}.$$

If the charge is in coulombs and the potential in volts, the capacitance will be in farads:

$$1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}}.$$

Thus, the farad is the capacitance of a conductor which acquires a charge of one coulomb when the potential difference across its terminals is raised by one volt.

The farad (F or f) is a very large quantity. In practice, use is made of the microfarad (μF) which is one-millionth of one farad:

$$1\ \mu\text{F} = 10^{-6}\ \text{F},$$

and of the picofarad (pF) which is one-millionth of one microfarad:

$$1\ \text{pF} = 10^{-6}\ \mu\text{F} = 10^{-12}\ \text{F}.$$

In c.g.s. electrostatic units, one farad is

$$\frac{1 \text{ coulomb}}{1 \text{ volt}} = \frac{3 \times 10^9}{1/300} = 9 \times 10^{11} \text{ statfarads}.$$

An elementary condenser or capacitor consists of two metal plates separated by a layer of dielectric. To charge the capacitor its plates are connected to the terminals of a current source. The opposite charges accumulating on the plates of the capacitor are linked by an electric field. As the plates of a capacitor are closely spaced, even a relatively low potential difference will produce a large electric charge. The capacitance of a capacitor is the ratio of the charge on its plates to the potential difference between them:

$$C = \frac{Q}{V} \quad \text{or} \quad Q = CV.$$

The capacitance of a capacitor increases directly with the area of the plates and inversely with the spacing between them. The material of the dielectric also affects the capacitance.

The conductors of long-distance power transmission lines can be regarded as the plates of a capacitor. The capacitance of such a conductor should be taken into account not only with respect to other conductors, but also relative to earth, walls and nearby structures. Underwater and underground cables have considerable capacitance since their conductors are spaced closely to one another.

Capacitors whose capacitance cannot be varied are called *fixed capacitors*. At present, fixed capacitors made from very thin foils separated by paraffin-treated paper or mica are in common use.

The capacitance of a capacitor is usually increased by connecting foils into two groups fitting into each other and separated by a dielectric, as

shown diagrammatically in Fig. 14. Alternatively, two long metal tapes interleaved with paraffin-treated paper are rolled into a compact package or cylinder. High-capacitance capacitors are in many cases placed in a metal box sealed with paraffin wax.

The capacitance of a parallel-plate capacitor can be calculated by Gauss' theorem which states: "The total number of lines of electric force emerging from a closed surface in an electric field is q_c/ε, where q_c is the total obvious charge enclosed by the surface and $\varepsilon=\varepsilon_r\varepsilon_0$ is the permittivity of the dielectric in which the surface lies," or

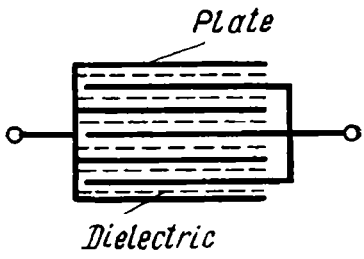


Fig. 14. A fixed capacitor (schematic)

$$N = ES = q/\varepsilon_r\varepsilon_0 . \qquad \text{(a)}$$

Assuming that the field of the capacitor is uniform (neglecting the distortion of the field at the edges of the plates), we obtain

$$E = \frac{V}{d} \qquad \text{(b)}$$

where d is the distance between the plates or the thickness of the dielectric. Substituting the value of E from Eq. (b) into Eq. (a) gives

$$\frac{V}{d} S = \frac{q}{\varepsilon_r\varepsilon_0},$$

whence

$$q = V\frac{S\varepsilon_r\varepsilon_0}{d}.$$

Since

$$C = \frac{q}{V},$$

the expression for the capacitance of a parallel-plate capacitor can be rewritten thus:

$$C = S\frac{\varepsilon_r\varepsilon_0}{d},$$

where

S = area of the plates, m^2;
d = thickness of the dielectric, m;
ε_r = relative permittivity (or dielectric constant) of the dielectric.

Thus, the capacitance of a parallel-plate capacitor can be increased by enlarging the area of its plates (S), by decreasing their spacing (d), and by using a dielectric of high permittivity (ε_r).

Capacitors whose capacitance can be varied are called *variable capacitors*. An elementary variable capacitor consists of two sets of copper or aluminium plates (which may have the shape of half-discs). Each set is mounted on a common shaft, one set being fixed and the other (which

interleaves with the former) movable. The degree of interleaving can be varied (thereby changing the capacitance of the capacitor) by rotating the shaft of the movable set of plates. The design and general view of a variable air capacitor are shown in Fig. 15*a* and Fig. 15*b*.

Radio engineering makes wide use of *electrolytic capacitors*, which may be either dry or wet. Both types employ oxidized aluminium. The oxide film on aluminium can be obtained by electrochemical means if the alu-

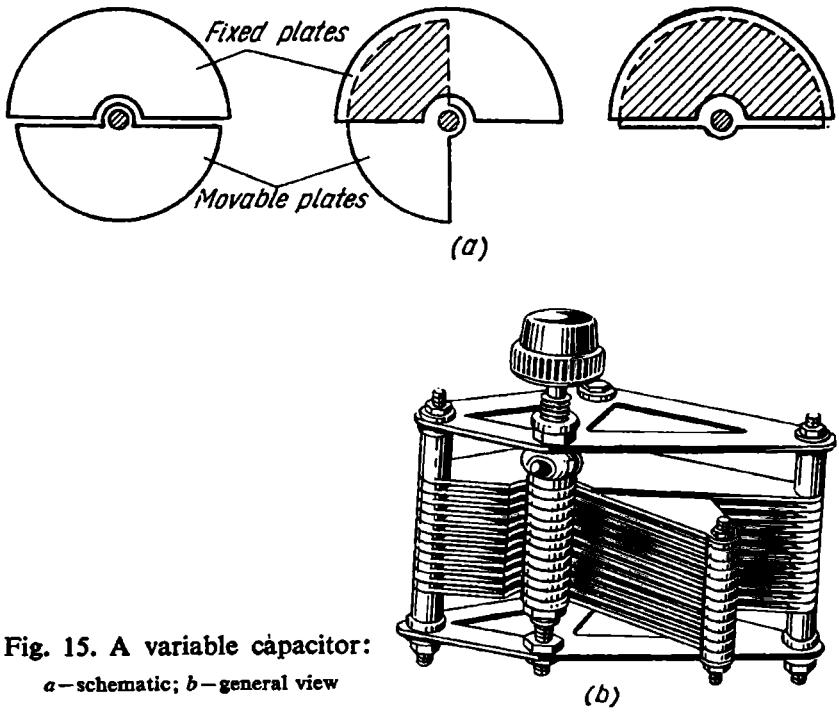


Fig. 15. A variable capacitor:
a—schematic; *b*—general view

minium is made an anode in a suitable electrolyte. The film of oxide (Al_2O_3) is very thin, of the order of several ten microns. The oxide film is a good insulator, strong mechanically and resistant to heat, but is moisture-absorbing.

In wet electrolytic capacitors the oxidized aluminium plate is placed in a metal case which serves as the other electrode. The space between the two is filled with an electrolyte, which is a mixture of boric acid and some additives.

A dry electrolytic capacitor consists of a thin aluminium foil (that serves as the anode member), layers of thin paper or cotton gauze, and another thin aluminium foil of the same area as the anode foil that serves as the cathode member. The electrolyte is contained in the layers of paper or gauze between the anode and cathode foils. Common practice is to concentrically wind the foils and separating materials into a compact roll, which is then placed in an aluminium case and sealed with bitumen. The

oxide film has a very high value of permittivity ($\varepsilon_r=9$), making it possible to obtain cheap capacitors of high specific capacitance.

When the capacitance of a single capacitor is too small, several capacitors may be connected in parallel (Fig. 16).

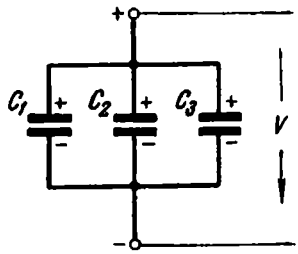


Fig. 16. Capacitors connected in parallel

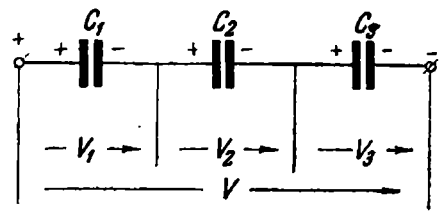


Fig. 17. Capacitors connected in series

In a parallel arrangement of capacitors, the voltage across the plates of each capacitor is the same. Therefore, we may write

$$V_1 = V_2 = V_3 = V.$$

The charge of each capacitor is

$$q_1 = C_1V; \qquad q_2 = C_2V; \qquad q_3 = C_3V.$$

The total charge of a bank of capacitors will then be

$$q = q_1+q_2+q_3,$$

or

$$q = C_1V+C_2V+C_3V = V(C_1+C_2+C_3).$$

Taking the capacitance of the entire bank of capacitors as C, we have

$$q = CV$$

and

$$CV = V(C_1+C_2+C_3),$$

or, finally,

$$C = C_1+C_2+C_3.$$

In other words, for several capacitors connected in parallel, the total capacitance is the sum of the capacitances of the individual capacitors. Each capacitor receives the full voltage of the line.

A different condition is observed when several capacitors are connected in series (Fig. 17).

Suppose that the left-hand plate of the first capacitor is charged positively, then (by electrostatic induction) the right-hand plate of the same capacitor will be charged negatively due to the negative charge from the left-hand plate of the second capacitor, and so on for the rest of the capaci-

tors. This means that each capacitor in a series bank will have the same charge, irrespective of their individual capacitances:

$$q_1 = q_2 = q_3 = q.$$

The voltage applied to the bank of capacitors will be the sum of the voltages across each capacitor:

$$V = V_1 + V_2 + V_3.$$

Since $$V_1 = \frac{q}{C_1}; \quad V_2 = \frac{q}{C_2}; \quad V_3 = \frac{q}{C_3},$$

for the entire bank we have

$$V = \frac{q}{C}.$$

Now we may write

$$\frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3};$$

eliminating q, we get the final form:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

In other words, for a series arrangement of capacitors, the reciprocal of their total capacitance is the sum of the reciprocal values of their individual capacitances. Each capacitor receives a voltage which is lower than the full voltage of the line.

9. Dielectrics

As is the case with a conductor, a dielectric placed in an electrostatic field will be subjected to electrostatic induction. There is a fundamental difference, however, between the electrostatic induction of a conductor and a dielectric. While in a conductor the application of an electric field causes the free electrons to move through the entire volume of the material, in a dielectric the charged elements are unable to move. Instead, the electric field will twist and strain the molecules to orient the positive charges in the direction of the field and the negative charges in the opposite direction. This condition is called *polarization.*

There exist two classes of dielectric materials. In the first class of dielectrics the positive and negative elements in the uncharged condition are located so closely to one another that their action is cancelled. The effect of the application of an electric field is to shift slightly the negative and positive charges within the molecules to form a *dipole.*

In the second class of dielectrics, the molecules form dipoles even in the absence of an electric field. Such dielectrics are called polar and embrace water, ammonia, ether, acetone, etc. In the absence of an electric field, however, the dipoles of such dielectrics are arranged at random, and

the resultant electric field of a polar dielectric is nil. The superposition of an electric field causes the molecules (and with them, the dipoles) to turn so that their axes align themselves with the direction of the external field. This shift gives rise to an instantaneous current, called *displacement current*, which ceases to flow in 10^{-13} to 10^{-15} second.

As distinct from the induced charges of a conductor, the polarization charges of a dielectric cannot be separated. The polarization of a dielectric vanishes as soon as the electric field is removed.

Generally speaking, the polarization of a dielectric is an elastic shift of electric elements in the dielectric material. If the electric field strength is too great, the dielectric will break down, i.e., it will cease to insulate.

There is another kind of current that can be detected in dielectrics. All real dielectrics have ions and free electrons, however small their number may be. The application of an electric field will cause them to flow inside the dielectric, producing what is known as *conduction current*. In some cases, the conduction current may be several times the displacement current. In passing through the material, conduction current dissipates Joule's heat, thereby impairing the performance of the piece of equipment in which it is used.

The properties of dielectrics important in electrical engineering are: (1) resistivity; (2) dielectric constant (or permittivity); (3) dielectric-loss angle; and (4) electric (or dielectric) strength.

10. Resistivity

As follows from the discussion above, technical dielectrics are not ideal insulators. Of course, the current that they conduct is very small in comparison with working currents carried by wires, cables, or busbars.

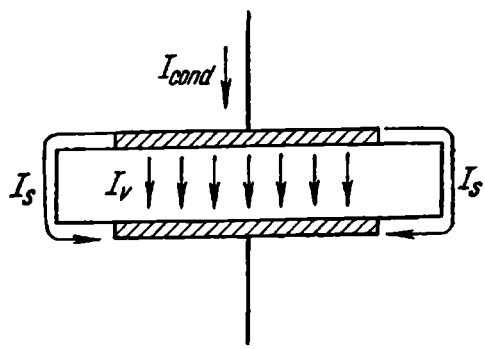


Fig. 18. Conduction current of a dielectric

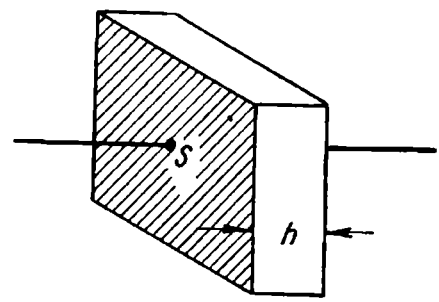


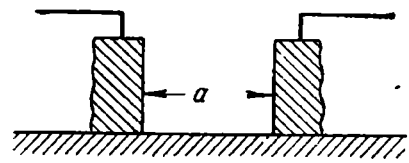
Fig. 19. Volume resistance of a dielectric

In general there are two possible leakage paths: (a) through the body of the material and (b) over its surface (Fig. 18). The total conduction current, I_{cond}, is the sum of the current passing through the body of the die-

lectric, I_v, and the current passing over its surface, I_s:

$$I_{cond} = I_v + I_s.$$

The former has to overcome volume resistance and the latter surface resistance. The resistance of a unit volume is called *volume resistivity* (or specific resistance), ϱ_v. The unit of volume resistivity is the resistance between opposite faces of a unit cube of a given material at a given temperature (Fig. 19). It is usually expressed in ohms per centimetre cube. Then the volume resistance of a material will be:



$$r_v = \varrho_v \frac{h}{S},$$

where h is the thickness of the dielectric in cm and S is the area of the cube face in cm^2.

The resistance of a unit surface is called *surface resistivity*, ϱ_s, and is expressed in ohms (per centimetre square). The unit of surface resistivity is the resistance between two opposite edges of a surface film which is 1 cm square (Fig. 20). The surface resistance of a material is

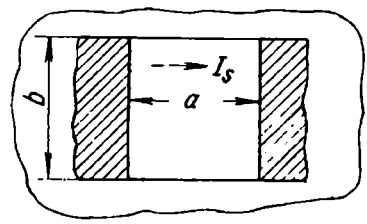


Fig. 20. Surface resistance of a dielectric

$$r_s = \varrho_s \frac{a}{b},$$

where a is the distance between the edges of the film and b is the width of the film.

The resistivity of a dielectric varies with its state (solid, liquid or gaseous), composition, ambient humidity and temperature.

11. Permittivity (Specific Inductive Capacity, Dielectric Constant)

Permittivity, also known as specific inductive capacity or dielectric constant, is that property of a dielectric which controls the capacitance of a system (say, a capacitor) in which this dielectric is used. It has been shown that the capacitance of a capacitor depends on the surface area of its plates (the larger the area, the greater the capacitance), the separation between the plates or the height of the intervening dielectric (the thicker the dielectric, the smaller the capacitance), and also on the material of the dielectric. It is the material of the dielectric that is described by its permittivity or dielectric constant.

Permittivity is numerically equal to the ratio of the capacitance of a capacitor in which the material is the dielectric to the capacitance of the

same capacitor with a vacuum as the dielectric. For practical purposes the permittivity of air at standard pressure and temperature can be assumed as the standard of reference.

For the most part, dielectrics have a permittivity of several units. Materials having high values of permittivity are preferred for capacitors where economy of space is important. Dielectrics of high and super-high permittivity are sometimes used.

12. Dielectric-loss Angle

When a dielectric is subjected to a changing electric field, some of the electric energy is transformed into heat and lost. This energy loss is termed *dielectric loss.* In Section 86 it will be shown that in an ideal dielectric subjected to an alternating voltage the displacement current is a pure capacitance current and leads the voltage by a phase angle of 90° (or is in perfect quadrature with the voltage). In a real dielectric the phase angle is less than 90° due to the conduction current. This difference angle due to the shift of the current vector relative to the voltage vector is called the *dielectric-loss angle* (also phase-difference or phase-defect angle) δ (the Greek letter "delta"). Since this angle is quite small, dielectric loss (or the power factor of a dielectric) is expressed in terms of the tangent of δ, tan δ.

It has been established that the power factor of a dielectric is directly proportional to the square of the voltage, the frequency, the capacitance, and tan δ.

Putting it another way, dielectric loss increases and the quality of a dielectric decreases with increasing tan δ. Materials with tan δ of the order of 0.08 or more are poor insulators. Conversely, materials having tan δ in the neighbourhood of 0.0001 are good insulators.

13. Electric Strength

Electric strength (or dielectric strength) is the property of an insulating material which enables it to withstand electric stress without injury. It is usually expressed in terms of the minimum *electric stress,* $E_{b\text{-}d}$ (i.e., potential difference per unit distance) which will cause failure or "breakdown" of the dielectric under certain conditions of service:

$$E_{b\text{-}d} = \frac{V}{h} \text{ (kV/cm or kV/mm),}$$

where h is the thickness of the dielectric, and V is the *breakdown voltage.*

Obviously, in actual service insulating materials should not be subjected to excessive voltages, or a breakdown will be inevitable.

The breakdown of a dielectric may take several forms. One of them is due to purely electric stress which destroys the structure of the material. While in a weak electric field the electric charges of molecules are elasti-

cally strained, the field at a breakdown voltage causes them to break away and the material is broken down.

Another form is due to the thermal effect of dielectric loss. In some materials, the resistance decreases with increasing temperature (they are said to have a negative temperature coefficient of resistance). As a result, more current will flow through a heated material, and more heat will be generated. This will continue until the material melts, burns or otherwise decomposes.

The breakdown of gaseous dielectrics (air) is caused by ionization due to excessive electric stress. The ions first produced collide with neutral molecules to give birth to more ions (ionization by collision), until the gas becomes conductive, and breakdown takes place.

When a high voltage stress is applied to conductors in a nonuniform electric field (between two points, or a point and a flat surface, or two wires of a high-voltage transmission line), breakdown of the intermediate air will occur when a certain, critical, stress is attained. It is accompanied by a discharge characterized by a humming or hissing sound and a violet-glow (corona discharge). As the stress increases, the corona discharge may grow into a spark discharge, and this into a brush discharge, which may also be followed by a continuous arc. The voltage at which this takes place is usually termed *spark-over* or *flash-over* values and the discharge itself is called *surface breakdown* or *flash-over*. To prevent this from happening, insulators are made corrugated to increase the surface area.

The electric strength of liquid dielectrics greatly depends on the amount of moisture, gases, and mechanical or chemical impurities present. Breakdown of liquid dielectrics occurs by overheating and the destruction of molecules.

14. Principal Insulating Materials

An insulating material is a material which offers relatively high resistance to the passage of an electric current. Thus, it may serve to separate a piece of electrical equipment from earth or from another equipment, so as to channel the current where required.

The materials used for insulation in electrical machines and apparatus are numerous and of the most diverse nature: solid, liquid and gaseous; organic and inorganic; natural and man-made. Some of the most important insulating materials are briefly described below.

Insulating varnishes are solutions of resinous materials (such as gums, asphalts, etc.) in volatile liquids (such as petrol, benzene, alcohols, ethers, acetone, turpentine, oil, etc.). According to use, they may be divided into impregnating, finishing or coating, and sticking or bonding. Impregnating varnishes serve to treat porous, fibrous or solid insulators (paper, pressboard, yarn, cloth) used in electrical machines and apparatus. Finishing or coating varnishes produce a hard lustrous coating on materials which is more or less resistant to air moisture. Sticking varnishes are used to cement

cloth, paper, mica, etc. It is usual to class varnishes as either air-drying or baking, depending on the method of drying.

Impregnating, *filling* and *sealing* compounds are used to fill voids and seal enclosures containing live parts against ingress of moisture and also to improve cooling. They are prepared from petroleum bitumens and mixtures of mineral oils and rosin. Sometimes, an addition of quartz as a filler is made in order to increase the thermal conductivity of the compound.

Natural resins or *gums* include rosin, shellac and copals. *Rosin* is a brittle material light yellow or brown in colour, obtained from the sap of pine-trees. Rosin dissolves in petroleum oils, liquid hydrocarbons, vegetable oils, alcohol and turpentine. Its softening point is 50 to 70°C and the electric strength is 10 to 15 kV/mm. It is ordinarily used as a spirit varnish for subsequent laminating and hot rolling, pressing or moulding. *Shellac* is a purified resin made from lac which consists of excretions of certain insects found in tropical countries. The crude lac is known as "stick lac". It is suitably treated for marketing in various shapes, such as flakes, buttons and cakes, known respectively as shellac, button lac and garnet lac. Shellac has a melting point of 100 to 200°C and is widely used in making spirit varnishes for a variety of applications. The dielectric constant is 2.7 to 3.7, approximately.

Mineral waxes include paraffin, ceresine and montan waxes and also ozokerite from which ceresine wax is obtained. *Paraffin wax* is a substance usually obtained by the distillation of petroleum. When thoroughly purified, paraffin wax is a white crystalline substance. It is used for impregnating wood, paper, fibre, etc., and also for filling radio coils and transformers, and as an ingredient of some compounds. It does not dissolve in water or alcohol, but is soluble in liquid hydrocarbons: petroleum oil, petrol, benzene. It has a specific gravity of 0.85 to 0.9, a melting point of 50 to 55°C, an electric strength of 16 to 30 kV/mm, and a permittivity of 2.1 to 2.2. *Ceresine wax* is a yellow to white material obtained by purifying or refining ozokerite, a native mineral wax, or petrolatum, the grease-like residue left after the distillation of paraffin-base petroleum. In comparison with paraffin wax, ceresine has a higher melting point (65 to 80°C) and a higher resistance to oxidation. It is employed for impregnating capacitors and as a constituent of insulating compounds. The electric strength is 15 kV/mm.

Synthetic waxes are principally chlorinated naphthalene, such as Halowax (a trade name). They melt at anywhere from 95 to 135°C. They have a high permittivity (about 5), so that smaller paper capacitors can be made. As distinct from natural waxes, synthetic waxes are not combustible.

Varnished cloth is made from cotton, silk or glass fibre impregnated with oil or oil-bitumen varnishes. Their uses include the insulation of windings in electrical machines and apparatus. Varnished cotton cloth, also known as varnished cambric and varnished muslin, is available in thick-

nesses from 0.15 to 0.25 mm and has an electric strength of 35 to 40 kV/mm. Varnished silk cloth is manufactured in thicknesses from 0.05 to 0.1 mm and has 1.5 to 2 times the permittivity of varnished cotton cloth.

Silk is the thread from a silkworm cocoon. The fibre is solid and from 0.01 to 0.015 mm in diameter. It is used as wire coverings and is woven into fabrics.

Rubber may be natural and synthetic. Natural rubber is obtained from the milky sap of tropical trees. Synthetic rubber is manufactured from alcohol or oil products. Heated to 50°C, rubber softens and becomes sticky; at low temperatures rubber grows brittle. It dissolves readily in hydrocarbons and carbon bisulphide. Its mechanical strength, heat and frost resistance and also stability towards solvents can be enhanced by adding 3 to 10 per cent sulphur—a process called vulcanization. Vulcanized rubber goes to insulate wires and cables, to make insulating sleeves, gloves, boots, matting, etc. Vulcanized rubber is a good insulator and has a high resistance to water and gases. Unfortunately, it is sensitive to temperature (it softens when heated to over 60 to 75°C), swells under the action of mineral oils, and ages when exposed to light. The electric strength is 24 kV/mm and the permittivity, 2.5 to 3.

Hard rubber (or *ebonite*) is a rubber compound containing 20 to 25 per cent sulphur and highly vulcanized. It is available in boards, rods and tubing. Though mechanically brittle, it can be worked, machined and polished. It is widely used in light-current engineering as conduits for concealed wiring and partition insulators.

Bakelite, although a trade name, covers a class of synthetic resins produced by condensation of phenolic materials with formaldehyde. In the formation of bakelite there are three stages: *A*, *B* and *C*. Stage *A* bakelite has a softening point of about 80°C and readily dissolves in alcohol and acetone. When heated to 110-140°C, it changes to stage *C* bakelite, which neither melts nor dissolves. It is used for coating paper, fabrics and wood veneers in the manufacture of laminated products. The electric strength is 10 to 20 kV/mm and the permittivity, 4.5 to 6.

Paper is manufactured from wood pulp, rags or plant fibre by a suitable chemical process. Electrical engineering uses several grades of insulating paper: for capacitors, cables, laminates, bakelite sleeves, and mica paper.

Pressboard differs from paper in thickness. Ordinarily, two grades of pressboard are used, one intended for operation in air and the other for immersion in oil. It is made into assorted small parts. The electric strength is 8 to 10 kV/mm and the permittivity, 2.5 to 4.

Vulcanized or *hard fibre* is produced by treating cotton-base paper with a chemical (usually zinc chloride). Fibre is easy to machine, but it readily dissolves in water on immersion, softening and swelling. Acids and alkalis attack it easily. Fibre is made into gaskets, coil formers and a variety of small parts. The grade 0.1 to 0.5 mm thick is termed leatheroid. The electric strength is 5 to 11 kV/mm and the permittivity, 2.5 to 5.

Built-up or *laminated insulation* can be based on either paper (synthetic resin-bonded paper) or fabrics (synthetic resin-bonded fabrics). *Synthetic resin-bonded paper* consists of sheets of paper bonded together with synthetic resins (chiefly phenolformaldehyde type) under heat (160-165°C) and pressure (usually in hydraulic presses). This material is available in several grades known under a variety of trade names (Pertinax, Paxolin, etc.) and in thicknesses varying from 0.5 to 50 mm. Paper laminates are easy to drill, turn, mill and saw. Built-up boards 2.5 to 3 mm thick can be pressed and stamped. An electric arc will char the surface of a paper laminate, thereby making it conducting. Paper laminates are made into panels, terminal boards, coil flanges, packings, cleats, etc. The electric strength is 20 to 25 kV/mm and the permittivity, 5 to 6. *Synthetic resin-bonded fabric* consists of several plies of fabric bonded together with a phenolic resin and pressed under heat (150°C). In comparison with laminated paper, it is less brittle, more durable, and has a great resistance to attrition. Unfortunately, it has lower electric properties, reduced resistance to moisture and is more expensive (five to six times that of laminated paper). Laminates based on glass fabric possess high electric properties, heat resistance and high mechanical strength and are moisture-proof. Fabric-base laminates are easy to machine. They are made into rollers, silent gears, bearing liners, etc. The electric strength is 27 to 45 kV/mm.

Moulded materials, or plastics, are mixtures consisting essentially of binders (resins, bitumens, etc.) and fillers (stone flour, wood flour, cotton, asbestos or glass fibre). They may also include plasticizers, which are added to reduce their brittleness, and pigments which give the desired colour. The mixture of a binder and filler is put into a mould or a forming press and is shaped under pressure and heat (or, sometimes, only pressure). Moulded products may be of a structural and an insulating variety.

Wood is useful where long pieces are needed with high mechanical strength in bending; hard woods such as birch, oak, beach and maple being preferred for switchgear operating rods, slot wedges, packing and cleats in machines and transformers, line supports, etc. The timber is very thoroughly dried and impregnated with paraffin wax, linseed oil or varnished with several coats of good insulating varnish.

Mica is a term which is applied to a class of minerals of finely laminated structure and very easy cleavage, the flakes of which (known as splittings) are very flexible, tough and highly resistant to heat. The electric strength is very high (80 to 200 kV/mm). The two classes most commonly used in electrical engineering are *muscovite* and *phlogopite*, of which the former is a better insulator. Mica splittings can be pressed into square plates for capacitors, washers for electrical appliances, etc. Most often, however, mica splittings are bonded together by means of a sticking varnish (shellac, Glyptal, or oil-bitumen varnish) and built up into sheets, plates and various commercial products known under many trade names (such as Micanite, Micabond, etc). Built-up mica plates are classed into commutator

segment plates, cold-moulding flexible mica plates (used for slot insulation, coil wrappers and coil sheaths), hot-moulding mica plates (mouldable to desired shapes under heat and pressure), and heater mica plates (for heating appliances). Sometimes, built-up products employ paper or fabric backing or facing, or both. They are available in sheet and tape forms and known as *foliums*. The fabrics may be silk, cotton, fibre glass, etc.

Asbestos is a fibrous crystalline mineral. Asbestos fibres are from a few tenths of a millimetre to several centimetres long. It is made into yarn, tape, cloth, paper, board, etc. The most valuable property of asbestos is its heat resistance—heating to 300 or 400°C will not change its properties. Owing to its low thermal conductivity, asbestos is an efficient thermal insulator at elevated temperatures. The high moisture absorption of asbestos can be reduced by adding to it resins, bitumen, etc. Asbestos fibre impregnated with bitumen and bonded to wire by varnish makes Deltabestos (a trade name). Asbestos is also used as a filler in moulded products with synthetic resins as binders. The properties of asbestos as an electrical insulator are not very good. The electric strength is 0.6 to 1.2 kV/mm, for which reason it is not recommended for operation at high voltages.

Asbestos board is a monolithic material composed of asbestos fibre and cement combined under heavy pressure. It is good for shields, panels, apparatus compartments, conduits, etc. Asbestos board is strong mechanically, resistant to arching and heat, and noncombustible, but its properties as an electrical insulator are moderate. As a method of reducing its moisture absorption, it can be impregnated with paraffin wax, linseed oil, bitumen and the like (impregnated asbestos board).

Glass is made by fusing together silica, SiO_2, as sand, and oxides of metals, such as sodium, potassium, lead or calcium (as carbonates, nitrates, borates, etc.). Glass is an amorphous material and as such it has no definite melting point. When heated it softens and finally melts. It can then be blown, drawn, pressed or cast. The physical and mechanical properties of glass depend on its composition and treatment. While ordinary glass is brittle, "quenching" can make it resistant to impact. Glass is practically impervious to water and resists the attack of acids (except hydrofluoric acid) and bases. Glass containing only alkaline oxides (Na_2O and K_2O) will readily dissolve in water (water glass). Glass is a very efficient electrical insulator, but it loses its insulating properties on heating. In electrical engineering, glass is manufactured into envelopes for lamps and radio valves, insulator fibre down to 0.005-0.006 mm in diameter, cloth and tape. The electric strength is 10 to 40 kV/mm and the permittivity is 5.5 to 10.

Electrical porcelain is the most commonly used ceramic insulating material. It is made from kaolin (china clay), fireclay, quartz and feldspar. The raw materials are finely ground and intimately mixed in a liquid state. The mixed material is made plastic by filtering under pressure and then moulded or worked into the desired shape, dried to remove excess water, dipped in glaze, and finally fired at a slowly rising heat (1320-1450°C)

which fuses the body of the material and converts the glaze into a smooth glass coating, which, among other things, reduces the moisture absorption of porcelain.

Porcelain is highly resistant to heat and electric arcs, and has a low moisture absorption. It is made into pin-type and suspension line insulators; pedestal, bushing and stand-off insulators; cleats; fuse holders; lampholders; socket bases; etc. The electric strength is 6 to 10 kV/mm and the permittivity, 5 to 6.5.

Steatites make up another group of ceramic insulating materials. They are formed by firing rock talc (soapstone). Steatite bodies are better insulators than porcelain and have higher physico-mechanical properties.

Marble is a limestone of the crystalline variety which can be sawn into slabs and given a high polish. Marble has a number of shortcomings: it readily absorbs moisture, is brittle and will crack on heavy heating; it is easily attacked by acids. Its moisture absorption can be practically eliminated by treatment with paraffin wax, bitumen, or rosin. The electric strength is 2.5 to 3.5 kV/mm and the permittivity is 8.

Slate is a microgranular crystalline stone characterized by an easy cleavage. When quarried, slate is wet, but after drying it will not re-absorb moisture to any appreciable degree. Electrical slate is easy to machine and is manufactured into panels, switchboards, etc. The electric strength is 1.5 to 3 kV/mm and the permittivity is 6 to 7.5.

Insulating oils are obtained from petroleum by fractional distillation. The principal one is transformer oil. In electrical apparatus it serves as an electric insulator. In power transformers it also doubles as a cooling medium. In oil circuit-breakers it additionally quenches the arc. Furthermore, it may be used as a filler for high-voltage cable boxes and terminals and as a constituent of many filling compounds. When suitably purified, it may also be used in the manufacture of capacitors and oil-filled cables. The electric strength is 5 to 18 kV/mm and the permittivity is 2.2.

Exercises

1. Two electric charges of 5×10^{-5} coulomb, and 3×10^{-4} coulomb are spaced 10 cm apart in a vacuum. Determine the force of interaction between them.
2. Two electric charges of 40 statcoulombs and 25 statcoulombs are spaced 5 cm apart in transformer oil. Determine the force of interaction between them.
3. A charge of 2×10^{-7} coulomb is acted upon by a force of 0.1 newton. Determine the distance to the other charge of 4.5×10^{-7} coulomb. Both charges are in a vacuum.
4. Determine the strength of an electric field 20 cm from a charge of 2×10^{-6} coulomb in a vacuum.
5. Determine the charge that produces an electric field strength of 40 V/cm at a distance of 30 cm in a vacuum.
6. Determine the potential at a point in an electric field if it takes 0.05 J of work to move a charge of 5×10^{-7} coulomb to this point.
7. The potential at point A is 50 V and at point B 80 V. Calculate the work that has to be done in order to move a charge of 5 coulombs from point A to point B.

8. Calculate the capacitance of a conductor, if it is imparted a charge of 2×10^{-3} coulomb and the potential of the conductor has increased by 500 V.

9. Determine the capacitance of a parallel-plate capacitor if the area of its plates is 40 cm^2. The dielectric is paraffin-treated paper 0.1 mm thick.

Review Questions

1. How does the electron theory explain atomic structure?
2. What is a conductor and an insulator? Give examples.
3. Formulate Coulomb's Law. Where is Coulomb's Law used?
4. What is an electric field?
5. What is the strength of an electric field?
6. How can the strength of an electric field produced by several point charges be determined graphically?
7. How is an electric charge distributed over the surface of various conductors?
8. What will happen to a current-carrying wire if it is placed in an electric field?
9. What will happen to a dielectric if it is placed in an electric field?
10. What is electric potential?
11. What are the units of potential? How is potential determined?
12. What is capacitance? What are the units of capacitance?
13. How is a capacitor constructed? How is it used?

Chapter Two

THE PRINCIPAL LAWS OF DIRECT CURRENT

15. Electric Current Defined

If two metal spheres on insulating supports (Fig. 21) are charged with unlike electricity (say, sphere *A* positively and sphere *B* negatively) and connected by a metal conductor, electrons will flow from *B*, where they are in excess, to *A* where they are lacking. *The flow of electrons in a conductor is called an electric current.** Instead of travelling the whole length of the conductor, electrons move through what is known as the *free mean path*, i.e., until they collide with other electrons, atoms or molecules.

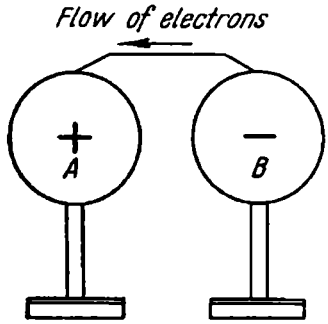


Fig. 21. Flow of electrons in a conductor

If sphere *A* is continuously charged with positive electricity and sphere *B* with negative electricity, there will be a continuous flow of electric current in the conductor.

Electric current cannot be observed directly. An indication that it is flowing is evident from its effects. The effects of electric current will be taken up in detail elsewhere. Here, it will suffice to mention the following signs of current flow:

(1) in passing through solutions of salts, bases, and acids as well as through molten salts, electric current decomposes them into their constituents;

* In engineering practice, it is assumed that the flow of electric current coincides with the movement of positive charges.

(2) every conductor carrying an electric current becomes heated;
(3) current flowing through a conductor sets up a magnetic field around the latter.

16. The Electric Circuit and Its Components

An elementary electric circuit consists of a source of electric energy (which may be a galvanic cell, a storage battery, a generator, etc.), various loads or devices in which the energy received from the source is converted into other forms (such as incandescent lamps, heaters, electric motors, etc.), and wires which connect the source and loads (Fig. 22).

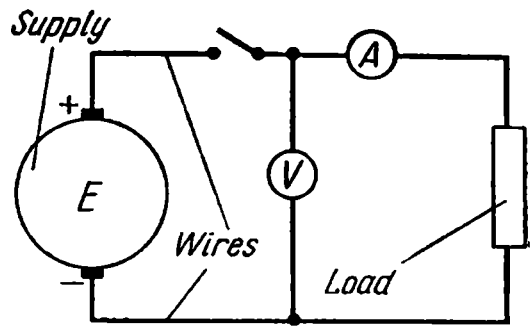


Fig. 22. An elementary electric circuit

It is customary to divide an electric circuit into an internal and an external circuit. The internal circuit consists of the energy source, while the external source includes the wires, loads, switches, instruments, etc.

Current will flow around a circuit only when it is completed or closed. No current can flow around an interrupted, or open circuit.

17. The Magnitude and Density of an Electric Current

If Q coulombs of electricity flow in a conductor during t seconds, then the quantity of electricity flowing in the conductor during one second will give us unit current (designated I):

$$I = \frac{Q}{t}.$$

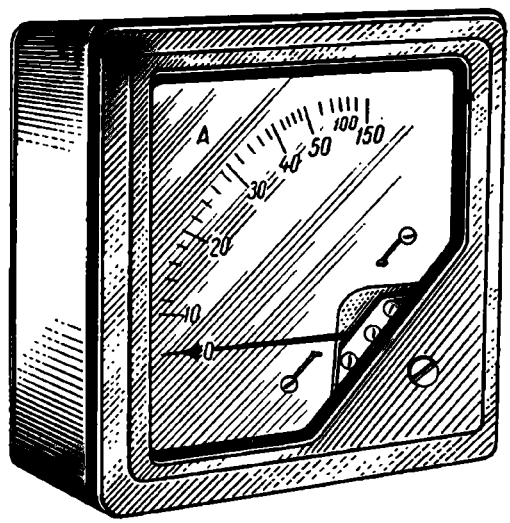


Fig. 23. General view of an ammeter

If one coulomb flows in a conductor during one second, the unit current will be the ampere:

$$1 \text{ ampere} = \frac{1 \text{ coulomb}}{1 \text{ second}}.$$

Current flowing in a circuit is measured with an ammeter (Fig. 23).

Sometimes it is more convenient to measure current in milliamperes, or thousandths of an ampere, which is measured by a milliammeter.

Should the quantity of electricity flowing around a circuit change, the current will also change. The mean value of current during a given span of time will be

$$i = dQ/dt$$

where i = instantaneous value of current;
dQ = change in quantity of electricity;
dt = time increment.

The smaller the time increment dt, the less the difference between the mean value and the instantaneous value of current.

Current that does not change in either magnitude or direction is termed direct current. Sources of direct current are galvanic cells, storage batteries, and d-c generators, if their load remains unchanged.

The ratio of a current I to the cross-sectional area, S, of a conductor is known as *current density* and is designated by the Greek letter δ (delta):

$$\delta = \frac{I}{S}.$$

Ordinarily, the cross-sectional area of conductors is expressed in square millimetres. Therefore, the dimensions of current density will be amperes per square millimetre, or A/mm^2.

18. Resistance and Conductance

When a circuit with a potential difference at its terminals is closed, an electric current begins to flow. Owing to the electric forces of the field, free electrons move along the conductor, encounter atoms of the conductor material, impart to them some of their kinetic energy, and slow down their flow. Then the field speeds them up again and the cycle is repeated. Putting it another way, every conductor always offers some resistance to the flow of free electrons, and therefore heats up.

That property of a body by virtue of which it resists the flow of electricity, causing a dissipation of electrical energy as heat is termed *resistance* and is designated by the letter R or r.

The device which introduces resistance into a circuit is known as a *resistor*. Fig. 24*a* gives two conventional signs for a fixed resistor in circuit diagrams. A resistor may be made variable (then it is called a *rheostat*) and used to vary the current in a circuit. Conventional signs for a rheostat are given in Figs. 24*b* and *c*. Ordinarily, a rheostat is a length of wire wound on an insulating core. Moving across the winding is a slider or contact arm. Its position on the winding determines the amount of resistance brought into the circuit incorporating the rheostat.

There are several factors that affect resistance.

A long conductor of small cross-section will offer a higher resistance to the flow of electricity than will a short conductor of large cross-section.

Ammeters connected to two conductors of the same size and shape but of different materials will indicate different currents, which is proof that the resistance of a conductor depends on its material.

The temperature of a conductor also affects its resistance. The resistance of metals increases with increasing temperature, while that of liquids and carbon decreases. There are several metals, however, such as manganin, constantan, nickeline, etc., whose resistance remains practically unaffected by temperature rise.

To sum up, the factors that affect the resistance of a conductor are:

(1) length,
(2) cross-sectional area,
(3) material,
(4) temperature.

The unit of resistance is the *ohm*, often designated by the Greek letter Ω (omega). Then, instead of a verbal description such as "The resistance of the conductor is 15 ohms", we write $r = 15\ \Omega$.

A thousand ohms equal one kiloohm (1 Kohm or 1 KΩ).

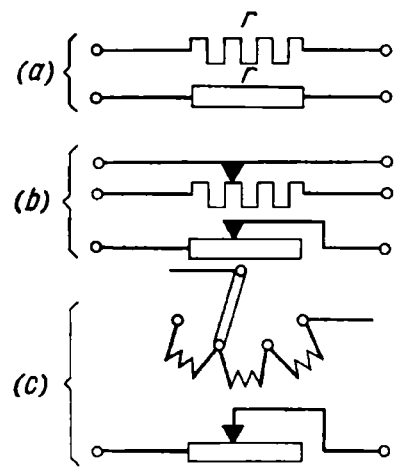


Fig. 24. Symbols for resistance in circuit diagrams

A million ohms make one megohm (1 Mohm or 1 MΩ).

In comparing the resistance of different conductors, their specimens should be of equal length and of equal cross-section. Then we can judge which conductor is better.

The resistance, in ohms, of a conductor 1 metre long and 1 mm^2 in cross-section is called *resistivity* and is designated by the Greek letter ϱ (rho). Table 1 summarizes the resistivity of some materials.

Referring to the table, 1 metre of iron wire 1 mm^2 in cross-section has a resistance of 0.13 ohm. To obtain a resistance of 1 ohm, 7.7 metres of this wire are needed. The lowest resistivity is displayed by silver: one would have to use 62.5 metres of silver wire in order to obtain a resistance of 1 ohm. Although silver is an excellent conductor, it is too expensive to be used extensively for this purpose. Silver is followed by copper: 1 metre of copper wire with a cross-section of 1 mm^2 has a resistance of 0.0175 ohm. For a resistance of 1 ohm, 57 metres of copper wire would be required.

Electrical engineering makes use of chemically pure, refined copper which goes to make wires, cables, and windings for machines and apparatus. Aluminium and iron conductors are also employed.

The resistance of a conductor can be found from the equation

$$R = \frac{\varrho l}{S}$$

where

R = resistance of the conductor in ohms;
ϱ = resistivity of the conductor, ohm-mm²/m;
l = length of the conductor in metres;
S = cross-sectional area of the conductor in mm².

Table 1

Resistivity of Some Materials*

Material	Resistivity, ϱ = ohms × × mm²/m
Silver	0.016
Copper	0.0175
Aluminium	0.03
Tungsten	0.05
Iron	0.13
Lead	0.2
Nickeline (copper-nickel-zinc alloy)	0.42
Manganin (copper-nickel-manganese alloy)	0.43
Constantan (copper-nickel-aluminium alloy)	0.5
Mercury	0.94
Nichrome (nickel-chromium-iron-manganese alloy)	1.1

* Metals and alloys are characterized in detail in Tables 8 and 9.

Example 1. Find the resistance of 200 metres of iron wire 5 mm² in cross-section:

$$R = \frac{\varrho l}{S} = \frac{0.13 \times 200}{5} = 5.2 \text{ ohms.}$$

Example 2. Find the resistance of 2 metres of aluminium wire 2.5 mm² in cross-section:

$$R = \frac{\varrho l}{S} = \frac{0.03 \times 2000}{2.5} = 24 \text{ ohms.}$$

Conversely, if the resistance of a conductor is known, its length, resistivity and cross-section can be found by the above equation.

Example 3. A resistor of 30 ohms should be wound with nickeline wire 0.21 mm² in cross-section for a radio receiver. Find the length of wire required:

$$l = \frac{RS}{\varrho} = \frac{30 \times 0.21}{0.42} = 15 \text{ metres.}$$

Example 4. Determine the cross-sectional area of nichrome wire if it takes 20 metres to wind a resistor of 25 ohms:

$$S = \frac{\varrho l}{R} = \frac{1.1 \times 20}{25} = 0.88 \text{ mm}^2.$$

Example 5. A resistor of 16 ohms is wound with 40 metres of wire 0.5 mm² in cross-section. Identify the material of the wire.

Any material can be identified on the basis of its resistivity. Then:

$$\varrho = \frac{RS}{l} = \frac{16 \times 0.5}{40} = 0.2.$$

The table of resistivities indicates that the material is lead.

As has been noted, the resistance of conductors varies with temperature. This can be proved by the following experiment. Wind a coil of a few metres of fine wire and connect its ends to a storage battery in series with an ammeter for current measurement. Heat the coil over the flame of an alcohol lamp and watch the ammeter: as the wire grows hotter, the readings of the ammeter decrease, thereby showing that the resistance of the wire increases with rising temperature.

The change of resistance of a wire with temperature is utilized in resistance thermometers. A resistance thermometer is in most cases a length of platinum wire wound on a mica core. By measuring the resistance of the platinum coil initially and again when the coil is placed in, say, a furnace, the temperature of the furnace may be obtained from the increase in resistance.

The change in the resistance of a conductor per ohm of the initial resistance and per degree change of temperature is termed the temperature coefficient of resistance and is designated by the letter α:

$$\alpha = \frac{R_t - R_0}{R_0(t - t_0)}$$

where

R_0 = initial resistance of the conductor;
R_t = final resistance of the conductor;
t_0 = initial temperature of the conductor;
t = final temperature of the conductor.

The resistance-temperature coefficients of some metals are given in Table 2.

The effect of temperature can be assessed from the expression

$$R_t = R_0[1 \pm \alpha(t - t_0)].$$

Example 6. Find the resistance of iron wire heated to 200°C if its resistance at 0° is 100 ohms.

$$R_t = R_0[1 + \alpha(t - t_0)] = 100(1 + 0.0066 \times 200) = 232 \text{ ohms}.$$

Table 2

Resistance-Temperature Coefficients of Some Metals

Metal	α	Metal	α
Silver	0.0035	Mercury	0.0090
Copper	0.0040	Nickeline	0.0003
Iron	0.0066	Constantan ...	0.000005
Tungsten	0.0045	Nichrome	0.00016
Platinum	0.0032	Manganin	0.00005

Example 7. A platinum resistance thermometer has a resistance of 20 ohms at 15°C. Placed in a furnace, it showed a resistance of 52 ohms. Find the temperature of the furnace.

$$t = \frac{R_t - R_0}{R_0 \alpha} + t_0 = \frac{52-20}{20 \times 0.0032} + 15 = 515°\text{C}.$$

The property a conductor has of allowing the passage of electric current is called *conductance*. A circuit with large conductance has low resistance, and vice versa. Therefore, resistance and conductance are reciprocal quantities. Conductance is represented by the letter G or g.

The reciprocal of 5 is 1/5, that of 1/7 is 7. Similarly, $R = 1/G$ and $G = 1/R$. The unit of conductance is the reciprocal ohm, or *mho* (which is "ohm" spelled backwards).

Example 8. A conductor has a resistance of 20 ohms. Find its conductance.

If $R = 20$ ohms, then $G = 1/R = 1/20 = 0.05$ mho.

Example 9. A conductor has a conductance of 0.1 mho. Find its resistance.

If $G = 0.1$ mho, then $R = 1/0.1 = 10$ ohms.

19. Electromotive Force. Voltage

For an electric current to flow continuously around a circuit, the terminals of the energy source must be maintained at different potentials. A very close analogy is provided by two vessels filled with water to different levels and connected by a tube: water will flow from one vessel into the other as long as the difference in their levels is maintained. This can be done by drawing water from one vessel and pouring it into the other.

The factor that establishes and maintains the potential difference and causes a current to flow around a circuit against its internal and external resistance is called *electromotive force* (abbreviated *emf* and designated by the symbol E).

Electromotive force can be produced in several ways. In chemical sources of electric energy (galvanic cells and storage batteries) an emf is set up by the action of certain chemical solutions on dissimilar metals; in electric generators it is developed by a combination of magnetic and mechanical means; in thermocells it is set up by thermal energy, etc.

The difference of potentials at two points that causes a movement of electricity from one point to the other is called *voltage*.

Both emf and voltage are measured in volts by means of voltmeters (Fig. 25). The volt has several multiples and submultiples: the kilovolt, or a thousand volts, and the millivolt, or one-thousandth of a volt, being the most common ones. Voltages in the millivolt range are measured with a millivoltmeter and in the kilovolt range by a kilovoltmeter.

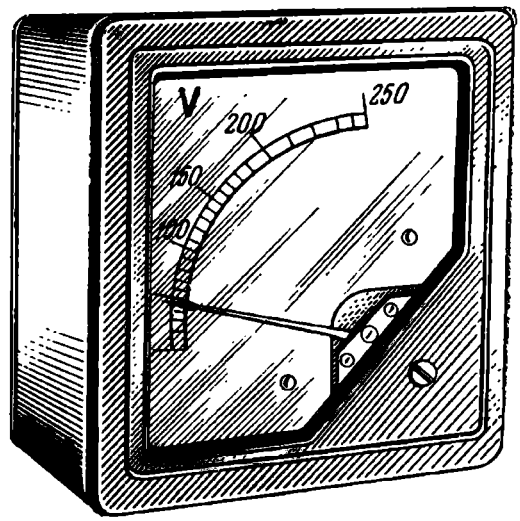


Fig. 25. General view of a voltmeter

In measuring the emf of a source, a voltmeter should be connected to its terminals with the external circuit open (Fig. 26). For a measurement of the voltage of a partial circuit, a voltmeter should be connected across that portion of the circuit (Fig. 27).

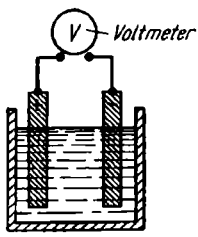


Fig. 26. Measuring the emf of a cell with a voltmeter

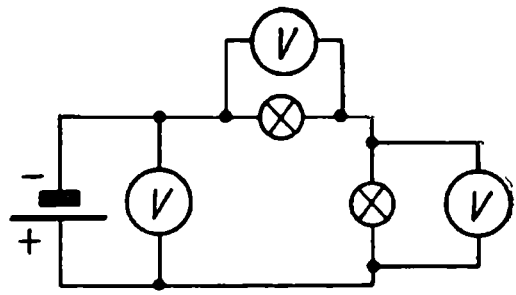


Fig. 27. Measuring voltage with a voltmeter

20. Ohm's Law

Fig. 28*a* shows an electric circuit consisting of a 2-V battery (1), a dial-type rheostat (2), a voltmeter (3), an ammeter (4) and connecting leads (5). With the rheostat set to obtain a circuit resistance of 2 ohms, the voltmeter connected across the battery terminals will read 2 volts and the ammeter connected in series with the circuit will read 1 ampere. Adding an-

other 2-V battery to the circuit will raise the voltage across it to 4 volts (Fig. 28*b*). For the same circuit resistance of 2 ohms, the ammeter will now read a current of 2 amperes. Further addition of a 2-V battery (Fig. 28*c*) will raise the voltage to 6 volts, and the reading of the ammeter will be 3 amperes. These changes are tabulated in Table 3.

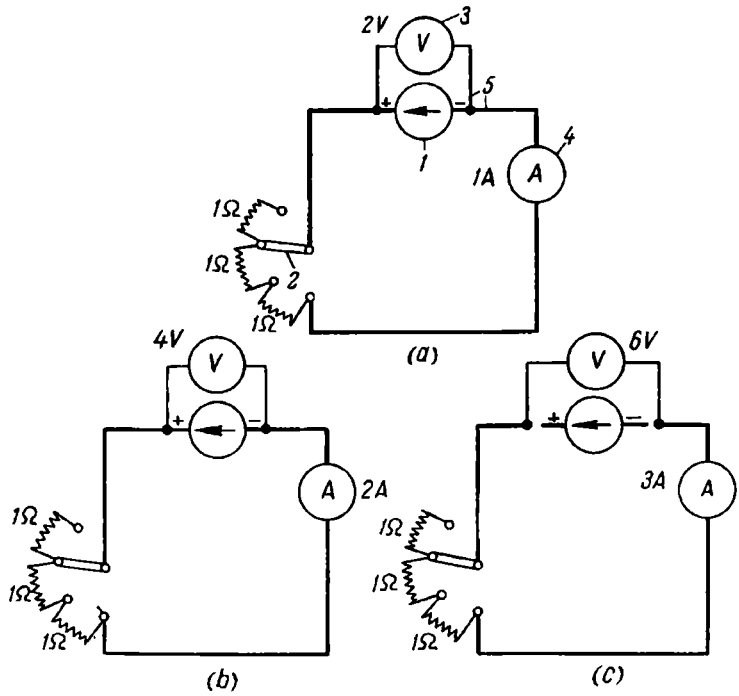


Fig. 28. Current in a circuit at varying voltage and constant resistance

Table 3

Current Versus Voltage at Constant Resistance

Voltage across circuit, volts	Current in circuit, amperes	Circuit resistance, ohms
2	1	2
4	2	2
6	3	2

The conclusion that can be drawn from the above observation is that the current in a circuit of constant resistance increases with increasing voltage across the circuit as many times as the voltage does.

Fig. 29*a* shows an electric circuit incorporating a 2-V battery and having a resistance of 1 ohm. The ammeter will read a current of 2 amperes. If the circuit resistance is increased to 2 ohms (Fig. 29*b*) by means of the rheostat, the ammeter will read 1 ampere for the same voltage across the circuit. With a circuit resistance of 3 ohms (Fig. 29*c*), the current flowing in the

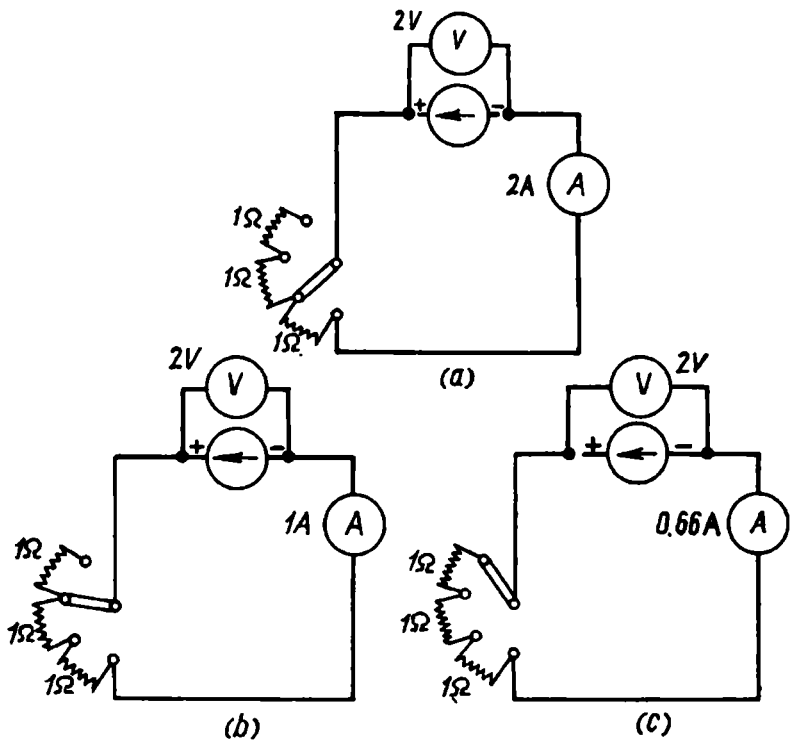


Fig. 29. Current in a circuit at varying resistance and constant voltage

circuit as read from the ammeter will be two thirds of an ampere. These observations are tabulated in Table 4.

Table 4

Current Versus Resistance at Constant Voltage

Voltage across circuit, volts	Circuit resistance, ohms	Current in circuit, amperes
2	1	2
2	2	1
2	3	2/3

The conclusion that may be drawn from the above observations is that when the voltage across a circuit is constant the current flowing in it will increase with decreasing resistance as many times as the circuit resistance does.

Both conclusions may be combined as follows: *The current flowing in a circuit is directly proportional to the applied voltage and inversely proportional to th eresistance.* This relationship is known as *Ohm's Law:*

$$I = \frac{V}{R},$$

where I is the current in amperes, V is the applied voltage in volts, and R is the resistance of the circuit in ohms. Thus, the current flowing in a portion of a circuit is equal to the voltage across that portion divided by its resistance.

Example 10. Find the current flowing in the filament of a lamp having a constant resistance of 240 ohms and connected in a 120-V mains.

$$I = \frac{V}{R} = \frac{120}{240} = 0.5 \text{ ampere.}$$

The equation of Ohm's Law may be transposed so that each of the three quantities may be found when the other two are known.

$$V = IR,$$

i.e., the voltage acting is equal to the current in amperes multiplied by the resistance in ohms, and

$$R = \frac{V}{I},$$

or the resistance of a circuit is equal to the applied voltage divided by the current.

Example 11. Find the voltage required to cause a current of 20 amperes to flow in a circuit of 6 ohms resistance.

$$V = IR = 20 \times 6 = 120 \text{ V.}$$

Example 12. A current of 5 amperes is flowing in the resistor of an electric range connected in a 220-V mains. Find the resistance of the range.

$$R = \frac{V}{I} = \frac{220}{5} = 44 \text{ ohms.}$$

Keeping the current in a circuit to 1 ampere and the resistance to 1 ohm, the voltage across the circuit will be one volt:

$$1 \text{ volt} = 1 \text{ ampere} \times 1 \text{ ohm.}$$

Hence, we may say that a voltage of 1 volt acts across a circuit having a resistance of one ohm and carrying a current of one ampere.

Fig. 30 shows an electric circuit consisting of a storage battery, a resistor r and long connecting leads having a definite resistance of their own.

Referring to the figure, the voltmeter connected across the battery reads 2 volts. Connected somewhere in the middle of the line, the voltmeter reads 1.9 volts, while near the resistor r its reading is 1.8 volts. This reduction in voltage across various points down a circuit is called the *voltage drop.*

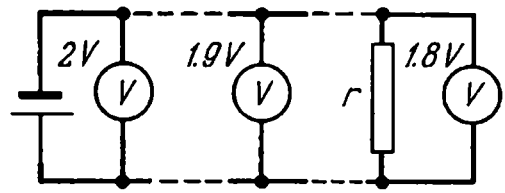


Fig. 30. Voltage drop across a circuit

The voltage drop is due to the fact that some of the applied voltage is spent to overcome the resistance of the circuit. From Ohm's Law, the voltage drop in volts across a part of a circuit is equal to the current in amperes flowing in that part multiplied by the resistance in ohms of the same part of the circuit, or

$$V\,(\text{drop}) = IR.$$

Example 13. A generator with a terminal voltage of 115 V powers an electric motor through wires having a resistance of 0.1 ohm. What is the voltage across the moto terminals if the motor takes a current of 50 amperes?

Obviously, the voltage across the motor terminals will be lower than across the generator terminals due to the voltage drop across the connecting line. From Ohm's Law,

$$V\,(\text{drop}) = IR = 50 \times 0.1 = 5 \text{ volts.}$$

If the voltage drop across the line is 5 volts, the terminal voltage of the motor will be $115 - 5 = 110$ volts.

Example 14. A generator produces a voltage of 240 V which is applied to a motor taking 15 amperes over a line of two copper wires each 350 m long and 10 mm^2 in cross-section. Find the voltage across the motor terminals.

The terminal voltage of the motor will be equal to that of the generator less the voltage drop across the line. The voltage drop across the line is $V = IR$.

As the resistance of the wires is unknown, we determine by the equation

$$R = \frac{\varrho l}{S} = \frac{0.0175 \times 700}{10} = 1.22 \text{ ohms}$$

(the length l is taken equal to 700 metres, as the current will have to flow from generator to motor and back).

Substituting R in the equation, we obtain

$$V\,(\text{drop}) = IR = 15 \times 1.22 = 18.3 \text{ volts.}$$

Thus, the terminal voltage of the motor is $240 - 18.3 = 221.7$ V.

Example 15. What should be the cross-section of aluminium conductors to apply a voltage of 120 V and a current of 20 amperes to an electric motor. Power will be drawn from a 127-V generator over a line 150 m long.

The maximum permissible voltage drop will be

$$127 - 120 = 7 \text{ volts.}$$

Then the resistance of the line conductors should be

$$R = \frac{V}{I} = \frac{7}{20} = 0.35 \text{ ohm.}$$

From

$$R = \frac{\varrho l}{S},$$

the cross-sectional area of the conductor will be:

$$S = \frac{\varrho l}{R} = \frac{0.03 \times 300}{0.35} = 25.7 \text{ mm}^2$$

For a copper conductor

$$S = \frac{\varrho l}{R} = \frac{0.0175 \times 300}{0.35} = 15 \text{ mm}^2.$$

It may be added that in some cases a voltage drop is sought for deliberately so as to swamp an excess voltage.

Example 16. For an electric arc to be stable, a current of 10 amperes is required at 40 volts. Find the required resistor to be connected in series with the arc so that it may be operated from a 120-V mains.

The voltage drop across the series resistor will be

$$120 - 40 = 80 \text{ volts.}$$

From Ohm's Law the resistance of the dropping resistor will be

$$R = \frac{V}{I} = \frac{80}{10} = 8 \text{ ohms.}$$

So far we have ignored both the resistance within, and the resulting voltage drop across, the energy source itself. Yet, a proportion of its emf is taken up by both. To take care of this, Ohm's Law has to be slightly modified:

$$E = V_0 + V = IR_0 + IR = I(R_0 + R),$$

or

$$I = \frac{E}{R_0 + R},$$

where

E = emf in volts,
I = current in amperes,
R = resistance of external circuit in ohms,
R_0 = resistance of internal circuit in ohms,
V_0 = internal voltage drop in volts,
V = external voltage drop in volts.

Ohm's Law then reads: *The electric current flowing in a circuit is equal to the emf divided by the sum of the internal and external resistances.*

Example 17. The emf of a cell is $E = 1.5$ V and its internal resistance, $R_0 = 0.3$ ohm. The cell is connected across a resistor of 2.7 ohms resistance. Find the current in the circuit.

$$I = \frac{E}{R_0+R} = \frac{1.5}{0.3+2.7} = 0.5 \text{ A.}$$

Example 18. Find the emf E of a cell connected across a resistor, $R=2$ ohms, if the current in the circuit is $I = 0.6$ A. The internal resistance of the cell is $R_0 = 0.5$ ohm.

With the circuit closed, a voltmeter connected across the cell terminals will read the voltage or voltage drop in the circuit:

$$V = IR = 0.6 \times 2 = 1.2 \text{ V.}$$

Obviously, some of its emf goes to overcome the internal resistance, and so the 1.2 V is only what the cell delivers into the circuit.

The internal voltage drop is

$$V_0 = IR_0 = 0.6 \times 0.5 = 0.3 \text{ V.}$$

As $E = V_0+V$, we have

$$E = 0.3+1.2 = 1.5 \text{ V.}$$

The same answer can be obtained from Ohm's Law for the whole circuit:

$$I = \frac{E}{R_0+R},$$

or

$$E = I(R_0+R) = 0.6(0.5+2) = 1.5 \text{ V.}$$

A voltmeter connected across an energy source when the external circuit is closed will read its closed-circuit voltage. When the circuit is open, no current will flow either in it or in the source, and there will be no internal voltage drop. Then the voltmeter will read the emf or open-circuit voltage of the source.

To sum up, a voltmeter connected across an energy source will read:

(a) the voltage across the circuit when the circuit is closed; and

(b) the emf of the source when the circuit is open.

Example 19. A cell has an emf of 1.8 V and is connected across a resistor of 2.7 ohms resistance. The current flowing around the circuit is 0.5 A. Find the internal resistance of the cell, R_0, and the internal voltage drop, V_0.

$$R_0+R = \frac{E}{I} = \frac{1.8}{0.5} = 3.6 \text{ ohms.}$$

Since $R = 2.7$ ohms,

$$R_0 = 3.6-2.7 = 0.9 \text{ ohm;}$$

$$V_0 = IR_0 = 0.5 \times 0.9 = 0.45 \text{ V.}$$

From the examples given above it can be seen that readings of a voltmeter connected across the terminals of an energy source vary with conditions in the circuit. When the current flowing in the circuit increases, the internal voltage drop also rises. Hence, if the emf remains constant,

the external circuit will account for a progressively decreasing proportion of applied voltage.

Table 5 relates the voltage, V, of an electric circuit to its external resistance, R, the emf, E, and internal resistance, R_0, of the source being constant.

Table 5

Circuit Voltage as a Function of Circuit Resistance at Constant EMF and Internal Resistance

E	R_0	R	$I=\frac{E}{R_0+R}$	$V_0=IR_0$	$V=IR$
2	0.5	2	0.8	0.4	1.6
2	0.5	1	1.33	0.67	1.33
2	0.5	0.5	2	1	1

21. Combinations of Resistances. Kirchhoff's First Law

Commonly, resistances are found connected in series, in parallel, or in series-parallel.

In a *series combination*, the finish of one resistance is connected to the start of another resistance, the finish of the latter to the start of a third resistance, and so on (Fig. 31).

When two or more resistances are connected in series, the equivalent resistance of the combination is equal to the sum of the resistances of the individual resistors:

$$R_{eq} = R_1+R_2+R_3+\ldots \text{ etc.}$$

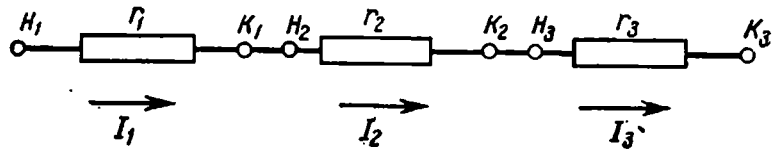


Fig. 31. Resistances in series

The current flowing in each of the resistances will be the same:

$$I_1 = I_2 = I_3 = I.$$

Example 20. Fig. 32 shows an electric circuit consisting of three resistors connected in series: $R_1 = 2$ ohms; $R_2 = 3$ ohms; and $R_3 = 5$ ohms. Find the readings of voltmeters V_1, V_2 and V_3, if the current in the circuit is 4 A.

The equivalent (or total) resistance of the circuit

$$R_{eq} = R_1+R_2+R_3 = 2+3+5 = 10 \text{ ohms}.$$

From Ohm's Law, the terminal voltage of a circuit is the current in the circuit multiplied by its resistance, or

$$V = IR = 4 \times 10 = 40 \text{ V}.$$

Thus, a voltmeter connected across the energy source will read 40 V.
The voltage appearing across the resistor R_1 (or its voltage drop) will be

$$V_1 = IR_1 = 4 \times 2 = 8 \text{ V}.$$

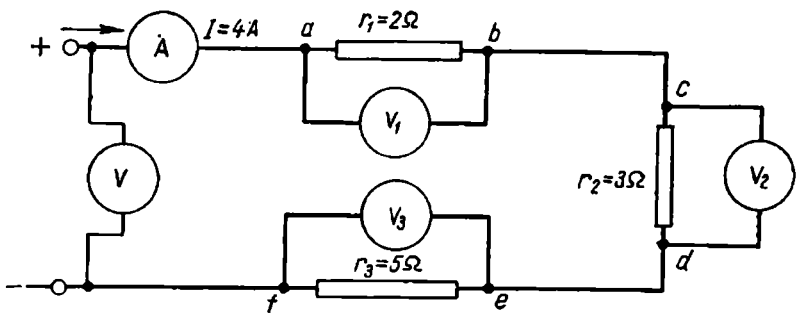


Fig. 32. Voltage measurements on partial circuits

The voltmeter V_1 connected between points a and b will read 8 V.
The voltage drop developed across R_2 will be

$$V_2 = IR_2 = 4 \times 3 = 12 \text{ V}.$$

The voltmeter V_2 connected between points c and d will read 12 V.
Finally, the voltage drop across R_3 will be

$$V_3 = IR_3 = 4 \times 5 = 20 \text{ V}.$$

The voltmeter V_3 connected between points e and f will read 20 V.
The voltmeter connected between points a and d will read the potential difference between these two points, which will be equal to the sum of the individual voltage drops across R_1 and R_2 (8+12 = 20 V).
Similarly, the voltmeter V connected between points a and f and reading the terminal voltage of the circuit will give the potential difference between these points equal to the sum of the individual voltage drops across R_1, R_2 and R_3.
Thus, the sum of the individual voltage drops across the component resistances of a circuit is equal to the closed-circuit voltage.

Since the current in a circuit with series-connected resistances is everywhere the same, the voltage drop across each resistance is directly proportional to its resistance.

Example 21. Three resistors of 10, 15 and 20 ohms, respectively, are connected in series as shown in Fig. 33. The current in the circuit is 5 A. Find the voltage drop across each of the resistors.

$$V_1 = IR_1 = 5 \times 10 = 50 \text{ V};$$
$$V_2 = IR_2 = 5 \times 15 = 75 \text{ V};$$
$$V_3 = IR_3 = 5 \times 20 = 100 \text{ V}.$$

The voltage applied to the circuit as a whole is equal to the sum of the individual voltage drops, or

$$V = V_1+V_2+V_3 = 50+75+100 = 225 \text{ V}.$$

In a *parallel combination*, resistances meet at common junction points at both ends, as shown in Fig. 34. Each of the junction points is connected to one of the terminals of an energy source.

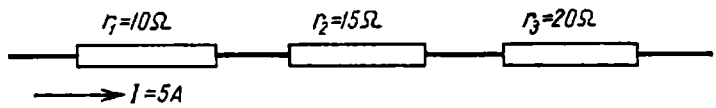


Fig. 33. Resistances in series (see Ex. 21)

Referring to the figure, there are several paths for current flow in a parallel combination of resistances. Arriving at junction point A, it divides between the three resistances. So the total current equals the sum of the currents in the three resistances:

$$I_{tot} = I_1+I_2+I_3.$$

Assigning the "+" sign to the currents arriving at a junction point and the "—" sign to the currents leaving the junction point, we may write

$$\sum_{k=1}^{k=n} I_k = 0,$$

This relationship, known as Kirchhoff's first Law, states that *the algebraic sum of the currents flowing into a circuit junction is zero.*

In practical circuit calculations, the directions of the currents in the branches are usually unknown. To make the application of Kirchhoff's first Law possible, an arbitrary positive direction for each branch is assumed by marking arrows along the branches. If the direction of the current is not in the direction of the arrow, the computation will show the value of the corresponding current to be negative.

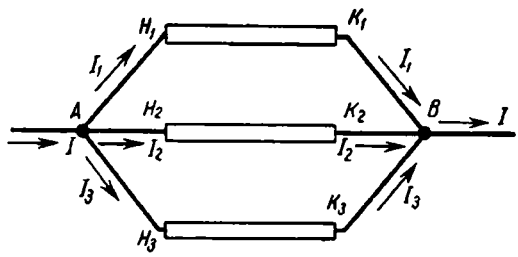


Fig. 34. Resistances in parallel

The formula for finding the total resistance of resistances in parallel may be derived from Ohm's Law.

The total current arriving at junction A is

$$I = \frac{V}{R}.$$

The currents in the individual branches will be

$$I_1 = \frac{V}{R_1}; \quad I_2 = \frac{V}{R_2}; \quad I_3 = \frac{V}{R_3}.$$

According to Kirchhoff's first Law, we have

$$I = I_1 + I_2 + I_3$$

or

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}.$$

Simplifying, we get

$$\frac{V}{R} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$

Dividing through by V, we obtain the following formula for the total conductance:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}, \quad \text{or} \quad g = g_1 + g_2 + g_3.$$

Thus, conductances are added when in parallel. As conductance increases, resistance decreases.

Example 22. Find the equivalent resistance of three resistors of 2 ohms, 3 ohms and 4 ohms, respectively, connected in parallel.

$$g_{tot} = g_1 + g_2 + g_3 = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{6+4+3}{12} = \frac{13}{12} \text{ mhos},$$

whence

$$R = \frac{12}{13} = 0.92 \text{ ohm}.$$

Example 23. Five resistors of 20, 30, 15, 40 and 60 ohms, respectively, are connected in parallel. Find their equivalent resistance.

$$g_{tot} = g_1 + g_2 + g_3 + g_4 + g_5 = \frac{1}{20} + \frac{1}{30} + \frac{1}{15} + \frac{1}{40} + \frac{1}{60} = \frac{6+4+8+3+2}{120} = \frac{23}{120} \text{ mho},$$

whence

$$R = \frac{120}{23} = 5.2 \text{ ohms}.$$

It should be noted that in a circuit with resistances in parallel, the equivalent or total resistance is *less* than the lowest of the resistances present.

If the resistances in parallel are of the same value, the total resistance will be equal to the resistance of one branch divided by the number of branches, or

$$R = R_1/n$$

where R_1 is the resistance of a branch and n is the number of branches.

Example 24. Find the equivalent resistance of four resistances of 20 ohms each connected in parallel:

$$R = R_1/n = 20/4 = 5 \text{ ohms.}$$

As a check, find the total resistance from the equation

$$g = g_1+g_2+g_3+g_4 = \frac{1}{20}+\frac{1}{20}+\frac{1}{20}+\frac{1}{20} = \frac{4}{20} \text{ mho,}$$

or

$$R = \frac{20}{4} = 5 \text{ ohms.}$$

The same answer is obtained in both cases.

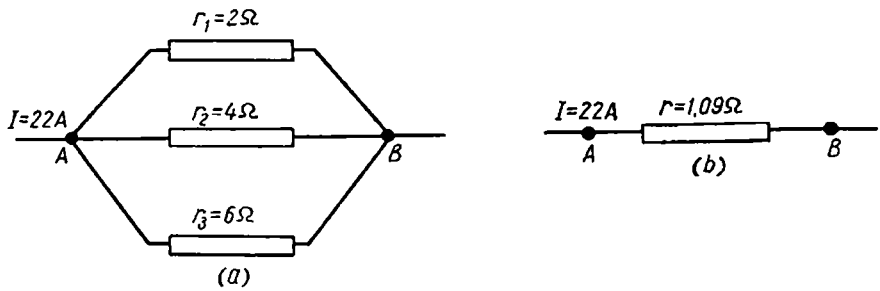


Fig. 35. Resistances in parallel (see Ex. 25)

Example 25. Find the currents in each branch of the parallel combination shown in Fig. 35*a*.

The total resistance of the combination is

$$g = g_1+g_2+g_3 = \frac{1}{2}+\frac{1}{4}+\frac{1}{6} = \frac{6+3+2}{12} = \frac{11}{12} \text{ mho}$$

or

$$R = \frac{12}{11} = 1.09 \text{ ohms.}$$

Now the combination may be represented by a single resistance (Fig. 35*b*).

The voltage drop between points A and B is

$$V = IR = 22 \times 1.09 = 24 \text{ V.}$$

Going back to Fig. 35*a*, the voltage across each of the three resistors will be 24 V, as they are connected between points A and B.

For R_1, with a voltage of 24 V and a resistance of 2 ohms, the current will be

$$I_1 = V/R_1 = 24/2 = 12 \text{ A.}$$

For R_2:

$$I_2 = V/R_2 = 24/4 = 6 \text{ A.}$$

For R_3:

$$I_3 = V/R_3 = 24/6 = 4 \text{ A.}$$

Kirchhoff's first Law provides a check on the answer:

$$I = I_1+I_2+I_3 = 12+6+4 = 22 \text{ A.}$$

Thus, the problem has been solved correctly.

Arranging the resistances and currents of the three branches in table form, we have.

1st branch $R_1 = 2$ ohms $I_1 = 12$ A,

2nd branch $R_2 = 4$ ohms $I_2 = 6$ A,

3rd branch $R_3 = 6$ ohms $I_3 = 4$ A.

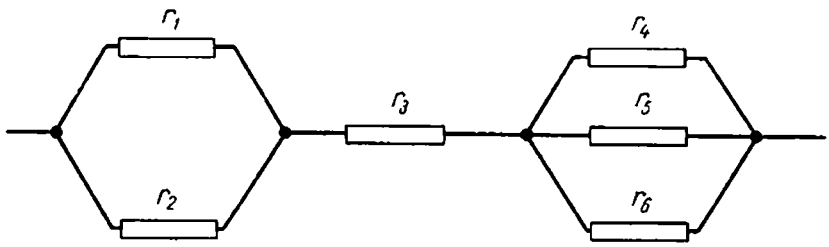


Fig. 36. Resistances in series-parallel

It will be noted that the resistance of the 1st branch is half the resistance of the 2nd branch, while the current through the former is twice as great as through the latter. The resistance of the 3rd branch is three times that of the 1st branch, but the current through it is one third of the currens through the first branch. The conclusion that can be drawn from thie observation is that in a parallel combination the currents through the branches are inversely proportional to their resistances.

For only two resistances in parallel the formula becomes

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}.$$

In a *series-parallel combination*, like the one shown in Fig. 36, some of the resistances are in series and some in parallel. The method of solving such circuits is self-explanatory from the example supplied.

Example 26. Find the equivalent resistance of the series-parallel combination show in Fig. 36 if

$$R_1 = 2 \text{ ohms}, \quad R_2 = 3 \text{ ohms}; \quad R_3 = 5 \text{ ohms}; \quad R_4 = 4 \text{ ohms};$$
$$R_5 = 8 \text{ ohms}; \quad R_6 = 6 \text{ ohms}.$$

First, find the equivalent resistance of R_1 and R_2:

$$R_{1,2} = 1/g_{1,2}$$

$$g_{1,2} = g_1 + g_2 = \frac{1}{2} + \frac{1}{3} = \frac{3+2}{6} = \frac{5}{6} \text{ mho}$$

or

$$R_{1,2} = \frac{6}{5} = 1.2 \text{ ohms}.$$

Second, find the equivalent resistance of R_4, R_5 and R_6:

$$g_{4,5,6} = g_4+g_5+g_6 = \frac{1}{4}+\frac{1}{8}+\frac{1}{6} = \frac{6+3+4}{24} = \frac{13}{24} \text{ mho}$$

or

$$R_{4,5,6} = \frac{24}{13} = 1.85 \text{ ohms.}$$

Now the problem has been reduced to a simple series combination in which the resistances have to be added to give their equivalent resistance:

$$R_{eq} = R_{1,2}+R_3+R_{4,5,6} = 1.2+5+1.85 = 8.05 \text{ ohms.}$$

22. Energy and Power of an Electric Current

Let a constant voltage V be applied across the circuit shown in Fig. 37:

$$V = \varphi_A-\varphi_B.$$

Let Q be the quantity of electricity that will pass through the circuit during time t. Then we may say that the electric field acting along the conductor will move a charge Q from point A to point B. The work done by the field or, which is the same, by the current in moving this charge can be determined by the equation

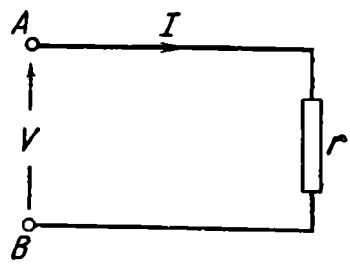


Fig. 37. Energy and power of electric current

$$W = Q(\varphi_A-\varphi_B) = QV.$$

Since $Q = It$, we finally have $W = VIt$,
where W = work in joules;
I = current in amperes;
t = time in seconds;
V = voltage in volts.

From Ohm's Law $V=IR$. Therefore we may rewrite the expression for work as follows:

$$W = I^2Rt.$$

Since energy is the ability to do work, the above expression also gives energy.

The rate of doing work, or energy per unit time, is termed *power* and is designated by the letter P:

$$P=W/t.$$

From this expression we have

$$W = Pt.$$

Substituting the expression for the work done by an electric field (or the energy of an electric field) in the equation of power will give

$$P = \frac{VIt}{t} = VI;$$

or

$$P = \frac{I^2Rt}{t} = I^2R.$$

The unit of power, called the *watt* (W), is equal to one joule divided by one second or to one volt multiplied by one ampere.

Common submultiples and multiples for power are the *milliwatt* (mW), one-thousandth of a watt, the *kilowatt* (kW), or one thousand watts, and the *megawatt* (MW), or one million watts.

Power is measured with a wattmeter. A wattmeter generally has two coils: the potential (or shunt) coil and the current (or series) coil. Their definitions indicate both what they measure and how they are connected in the circuit under measurement. (Fig. 38).

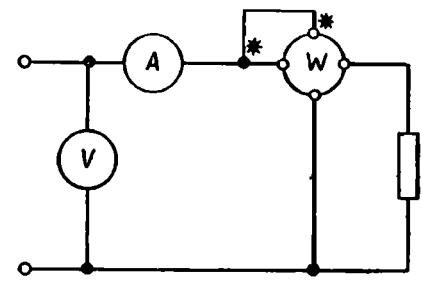


Fig. 38. Connection of a wattmeter in the circuit

Alternatively, power can be measured by a voltmeter and an ammeter. Then, according to $P = VI$, their readings have to be multiplied. For example, with an ammeter reading 3 A and a voltmeter reading 120 V, the power used up in the circuit will be

$$P = VI = 120 \times 3 = 360 \text{ watts.}$$

For practical purposes, the joule is too small a unit. Large units of energy can be obtained by measuring time in hours instead of in seconds; the most common units are the *watt-hour* (Wh) and the *kilowatt-hour* (kWh).

Electric energy is measured by watthour-meters.

Example 27. How much power will be used up by an electric motor if the current is 8 A and the voltage applied is 220 V?

$$P = VI = 220 \times 8 = 1760 \text{ W} = 1.76 \text{ kW.}$$

Example 28. How much power will be used up in a 24-ohm electric range taking a current of 5 A?

$$P = I^2R = 25 \times 24 = 600 \text{ W} = 0.6 \text{ kW.}$$

In converting mechanical power into electrical power it should be remembered that

$$1 \text{ hp} = 736 \text{ W}$$

$$1 \text{ kW} = 1.36 \text{ hp}$$

Example 29. How much energy will be consumed by a 600-W electric range during 5 hours?

$$W = Pt = 600 \times 5 = 3000 \text{ Wh} = 3 \text{ kWh}$$

Example 30. What will be the power bill for 12 lamps burning a month (30 days), if four of them are 60-W lamps and are on six hours a day, and the remaining eight are 25-W lamps and are on four hours a day. The charge is 0.4 kopek per 100 Wh.

First, find the power of the four 60-W lamps:

$$P = 60 \times 4 = 240 \text{ W}.$$

Their on-time during a month will be

$$t = 6 \times 30 = 180 \text{ hours}.$$

Their energy consumption will be

$$W = Pt = 240 \times 180 = 43{,}200 \text{ Wh}$$

Second, find the power of the remaining eight 25-W lamps

$$P = 25 \times 8 = 200 \text{ W}.$$

Their on-time during a month will be

$$t = 4 \times 30 = 120 \text{ hours}.$$

Their energy consumption will be

$$W = Pt = 200 \times 120 = 24{,}000 \text{ Wh}.$$

The total energy consumption is

$$43{,}200 + 24{,}000 = 67{,}200 \text{ Wh}.$$

The bill will be

$$67{,}200 \times 0.4 = 2 \text{ roubles } 69 \text{ kopeks}.$$

23. A Short-circuit

Fig. 39 shows an incandescent lamp connected in an electric circuit. If the lamp has a resistance of 240 ohms and the voltage applied is 120 V, the current flowing through the lamp will be

$$I = V/R = 120/240 = 0.5 \text{ A}.$$

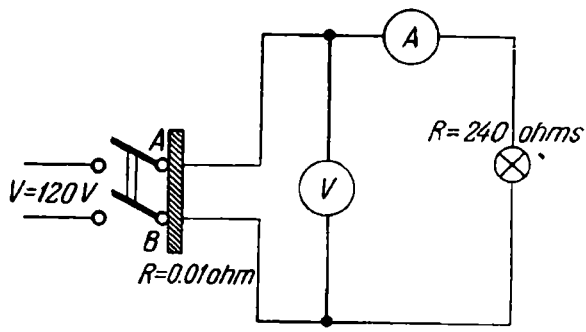


Fig. 39. Short-circuit across a knife-blade switch

Let us examine what will happen if the wires to the lamp turn out to be connected by a low resistance (short-circuited), say, by a heavy metal bar placed inadvertently on the wires. The current arriving at the point *A* will divide between two paths, the larger portion flowing through the low-resis-

tance bar and the smaller portion through the lamp which has a higher resistance.

Assigning the bar a resistance of 0.01 ohm, the current through it will be

$$I = V/R_b = 120/0.01 = 12{,}000 \text{ A}.$$

In an actual short-circuit the current will be smaller, since the mains voltage will be lower than 120 V due to an increased voltage drop across the bar. Still, the current will be many times the value ordinarily flowing through the lamp.

Flowing through a conductor, an electric current heats it. In our example, the wires are designed to withstand the heat generated by a current of 0.5 A. As the short-circuit current is many times the normal current, heat generation will also increase. As a result, the insulation of the wires will char or burn completely, the wires melt the instruments connected in the circuit damaged, the contacts of the switches fused, etc. Indeed, the energy source may also be damaged, and a fire may break out.

The condition under which the current through a circuit dangerously rises due to reduced resistance is called a *short-circuit.* To avoid it, the following rules should be observed at all times:

(1) the insulation of the wires in a circuit should be adequate for the voltage applied and the load carried;

(2) the cross-section of the wires should be such that they will not grow dangerously hot under normal conditions of voltage and current;

(3) the wires should be reliably protected against mechanical injury;

(4) the wires should be laid on insulators of sufficient mechanical and electrical strength;

(5) all joints and tap-offs should be insulated as reliably as the wires themselves;

(6) at crossings, the wires should not touch one another;

(7) where passed through walls, ceilings or floors, the wires should be so laid as to be protected from moisture, mechanical or chemical injuries, and well insulated.

As a protection against short-circuits and the accompanying damage, circuits incorporate fuses. A fuse is a piece of fine low-melting wire series-connected in the circuit to be protected. Should the current rise dangerously, the fuse link will melt (blow), the circuit will be automatically interrupted, and no current will flow through it. Fuses are described in greater detail in Sec. 52.

24. Kirchhoff's Second Law. Application of Kirchhoff's Laws to Circuit Calculations

Practical electric circuits are often connected into closed networks. In addition to resistances, such networks may include sources of emf. Fig. 40 shows part of a network. The polarity of all the emfs is known. The posi-

tive directions of the currens may be chosen arbitrarily. Now let us move around the network clockwise from, say, the point A. There will be a voltage drop across each of the individual legs of the network, and the respective potentials will be as follows.

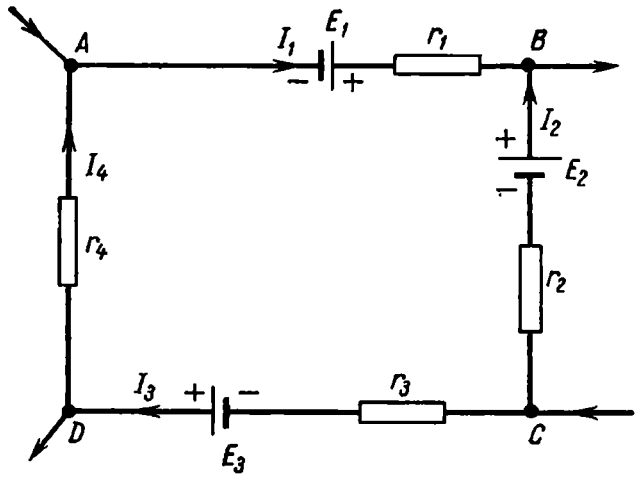


Fig. 40. A portion of an electric network

Section AB:

$$\varphi_A + E_1 - I_1R_1 = \varphi_B.$$

Section BC:

$$\varphi_B - E_2 - I_2R_2 = \varphi_C.$$

Section CD:

$$\varphi_C - I_3R_3 + E_3 = \varphi_D.$$

Section DE:

$$\varphi_D - I_4R_4 = \varphi_A.$$

Adding together the four equations termwise, we obtain

$$\varphi_A + E_1 - I_1R_1 + \varphi_B - E_2 - I_2R_2 + \varphi_C - I_3R_3 + E_3 + \varphi_D - I_4R_4 = \\ = \varphi_B + \varphi_C + \varphi_D + \varphi_A$$

or

$$E_1 - I_1R_1 - E_2 - I_2R_2 - I_3R_3 + E_3 - I_4R_4 = 0.$$

Transposing the products IR to the right side gives

$$E_1 - E_2 + E_3 = I_1R_1 + I_2R_2 + I_3R_3 + I_4R_4.$$

Generally,

$$\Sigma E = \Sigma IR.$$

This is Kirchhoff's second Law which reads as follows: *The algebraic sum of the emfs round a closed circuit is equal to the sum of all the IR (voltage) drops round it.*

Let us examine an elementary closed network (see Fig. 41). According to Kirchhoff's second Law,

$$\Sigma E = \Sigma IR.$$

Therefore,

$$E = IR_0 + IR = I(R_0 + R).$$

Whence,

$$I = \frac{E}{R_0 + R}.$$

The above expression is Ohm's Law for the whole circuit.

Here are a few examples which can be solved by using Ohm's Law and Kirchhoff's two Laws.

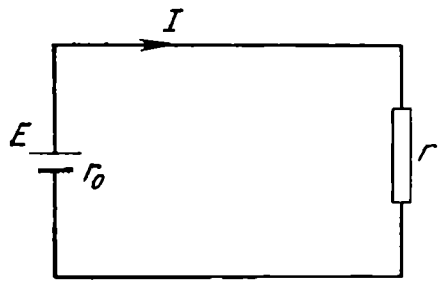


Fig. 41. A simple closed circuit

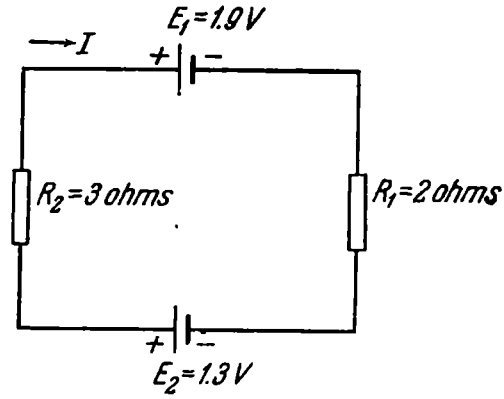


Fig. 42. An electric circuit (see Ex. 31)

Example 31. Find the current flowing around the network shown in Fig. 42. Indicate the positive direction of the current along the branch by an arrow. Moving around the network clockwise, we write the equation of Kirchhoff's second Law:

$$-E_1 + E_2 = IR_1 + IR_2; \quad -1.9 + 1.3 = I(2+3); \quad -0.6 = 5I; \quad I = -0.12 \text{ A}.$$

The computation shows that the actual direction of the current is not in the direction of the arrow.

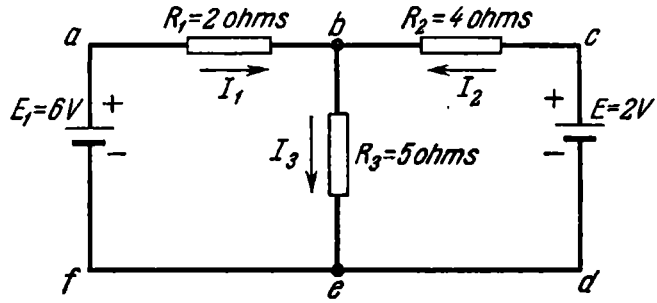


Fig. 43. An electric circuit (see Ex. 32)

Example 32. Find the currents of the sections of the network shown in Fig. 43. Indicate the positive directions of the individual currents in the branches by arrows. For the network *abef*,

$$6 = 2I_1 + 5I_3. \qquad (1)$$

For the network *acdf*,

$$6-2 = 2I_1-4I_2;\ 2 = I_1-2I_2. \tag{2}$$

By Kirchhoff's first Law, the current at the point *b* is

$$I_3 = I_1+I_2. \tag{3}$$

This leaves us with three equations in three unknowns. Their solution gives the magnitudes and directions of the individual currents. Substituting the value of I_3 from Eq. 3 in Eq. 1 gives

$$6 = 2I_1+5I_1+5I_2;$$

$$+$$

$$2 = I_1-2I_2$$

or

$$12 = 14I_1+10I_2$$

$$+$$

$$10 = 5I_1-10I_2.$$

Adding the last two equations, we obtain

$$22 = 19I_1, \text{ whence } I_1 = 1.156 \text{ A}.$$

Substitute the value of I_1 in Eq. 1:

$$6 = 2\times1.156+5I_3;$$

$$I_3 = \frac{6-2\times1.156}{5} = 0.74 \text{ A}.$$

Substitute the value of I_1 in Eq. 2:

$$2 = 1.156-2I_2$$

whence

$$I_2 = \frac{-2+1.156}{2} = -0.422 \text{ A}.$$

The computation shows that the actual direction of the current is not in the direction of the arrow.

25. The Superposition Theorem

Electric circuits with several sources of emf can be solved on the basis of the superposition theorem which states: *The current in any branch of a network is the sum of the currents due to each source of emf separately, all other emfs being taken equal to zero.*

Example 33. Find the currents in each of the branches of the network shown in Fig. 44*a*, if $E_1 = 27$ V; $E_2 = 24$ V; $R_1 = 3$ ohms; $R_2 = 4$ ohms, and $R_3 = 6$ ohms.

Taking $E_2 = 0$, find the current produced by E_1 (Fig. 44*b*). Assume the positive drections of the currents and determine resistances and currents of the branches. Since R_2 and R_3 are connected in parallel, the equivalent resistance will be

$$R_{2,3} = \frac{R_2R_3}{R_2+R_3} = \frac{4\times6}{4+6} = 2.4 \text{ ohms}.$$

The total resistance of the network

$$R_{tot} = R_1+R_{2,3} = 3+2.4 = 5.4 \text{ ohms}.$$

The total current

$$I_1 = \frac{E_1}{R_{tot}} = 27 \div 5.4 = 5 \text{ A}.$$

The voltage across AB

$$V_{AB} = E_1 - I_1 R_1 = 27 - 5 \times 3 = 12 \text{ V}.$$

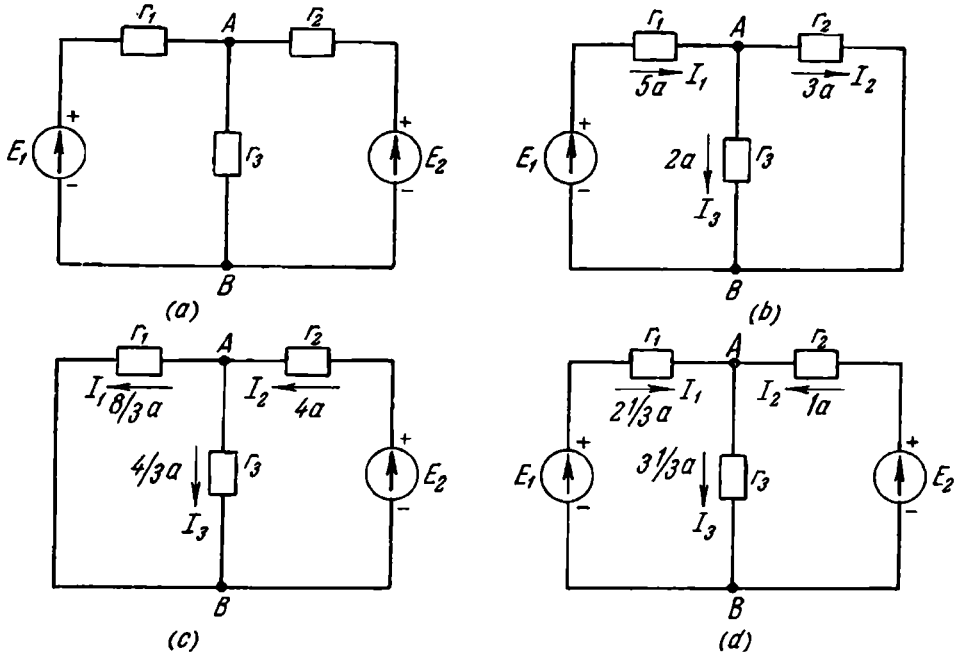


Fig. 44. The superposition theorem (see Ex. 33)

The current in the parallel branches

$$I_2 = V_{AB}/R_2 = 12 \div 4 = 3 \text{ A};$$
$$I_3 = V_{AB}/R_3 = 12 \div 6 = 2 \text{ A}.$$

The result may be verified as follows:

$$I_2 + I_3 = 3 + 2 = 5 \text{ A} = I_1.$$

Taking $E_1 = 0$, determine the currents produced by E_2. Assume the positive directions of the currents and determine the resistances and currents of the branches.

$$R_{1,3} = \frac{R_1 R_3}{R_1 + R_3} = \frac{3 \times 6}{3+6} = 2 \text{ ohms};$$
$$R_{tot} = R_2 + R_{1,3} = 4 + 2 = 6 \text{ ohms};$$
$$I_2 = V_{AB}/R_{tot} = 24/6 = 4 \text{ A};$$
$$V_{AB} = E_2 - I_2 R_2 = 24 - 4 \times 4 = 8 \text{ V};$$
$$I_1 = V_{AB}/R_1 = 8/3 \text{ A};$$
$$I_3 = V_{AB}/R_3 = 4/3 \text{ A}.$$

Verify the result as follows:

$$I_1 + I_3 = 8/3 + 4/3 = 12/3 = 4 \text{ A} = I_2.$$

Comparing the two latter diagrams (Figs. 44*b* and *c*), we see that two currents flow in each branch. Adding the two currents algebraically (Table 6), we obtain the actual current of a given branch.

Table 6

Branch current	Fig. 44*b*	Fig. 44*c*	Actual value and direction of current
I_1	clockwise, 5 A	counterclockwise, 8/3 A	clockwise, 7/3 A
I_2	clockwise, 3 A	counterclockwise, 4 A	counterclockwise, 1 A
I_3	down, 2 A	down, 4/3 A	down, 10/3 A

The results may be verified as follows.
By Kirchhoff's first Law, for the point A we have

$$I_1+I_2+I_3 = 7/3+1-10/3 = 0.$$

The distribution of the currents flowing in the network shown in Fig. 44*a* is illustrated in Fig. 44*d*.

26. The Parallel-generator Node-voltage Theorem

In practice, cases frequently arise where a network has only two terminal points between which any number of parallel branches may be connected. Their calculation can be greatly simplified by the use of the parallel-generator (or node-voltage) theorem.

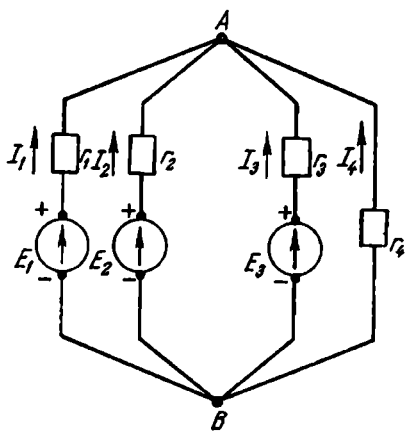


Fig. 45. The parallel-generator theorem

Fig. 45 shows a network with two common terminals A and B between which four parallel branches are connected. The first three branches contain generators of emf E_1, E_2 and E_3, respectively. Connected in series with the generators are resistances R_1, R_2 and R_3 (which also include the internal resistances of the generators themselves). It is assumed that the current in each branch flows from B to A. The current in the first three branches is in the direction of the respective emf. Designating the voltage between A and B by V we obtain the current in the first branch

$$I_1 = \frac{E_1-V}{R_1} = (E_1-V)g_1;$$

i.e.,

$$I_1 = (E_1-V)g_1.$$

Similarly for the other branches:

$$I_2 = (E_2 - V)g_2;$$
$$I_3 = (E_3 - V)g_3;$$
$$I_4 = (0 - V)g_4 = -Vg_4.$$

By Kirchhoff's first Law, for the junction A we have

$$I_1 + I_2 + I_3 + I_4 = 0.$$

Substituting the respective expressions for the currents, we can rewrite the above equation as follows:

$$(E_1 - V)g_1 + (E_2 - V)g_2 + (E_3 - V)g_3 - Vg_4 = 0,$$

whence

$$V = \frac{E_1g_1 + E_2g_2 + E_3g_3}{g_1 + g_2 + g_3 + g_4}.$$

The above equation gives the common terminal voltage. The numerator of the expression gives the algebraic sum of the products of the respective emfs and resistances. The denominator is the sum of the branch conductances. Should the emf of any branch in the network be in a direction opposite to the one shown in Fig. 44, it will have a minus sign in the common terminal voltage equation. In general form, the common terminal voltage may be written

$$V = \Sigma Eg / \Sigma g.$$

Example 34. The network shown in Fig. 45 contains generators of emf $E_1 = 110$ V $E_2 = 115$ V and $E_3 = 120$ V. The respective internal resistances of the generators are $R_{01} = 0.2$ ohm; $R_{02} = 0.1$ ohm; and $R_{03} = 0.3$ ohm. The branches have resistances $R_1 = 2.3$ ohms; $R_2 = 4.9$ ohms; and $R_3 = 4.7$ ohms; $R_4 = 5$ ohms. Find the currents in the branches.

Solution.

Determine the conductance of each branch.

$$g_1 = \frac{1}{R_{01} + R_1} = \frac{1}{0.2 + 2.3} = \frac{1}{2.5} = 0.4 \text{ mho};$$

$$g_2 = \frac{1}{R_{02} + R_2} = \frac{1}{0.1 + 4.9} = \frac{1}{5} = 0.2 \text{ mho};$$

$$g_3 = \frac{1}{R_{03} + R_3} = \frac{1}{0.3 + 4.7} = \frac{1}{5} = 0.2 \text{ mho};$$

$$g_4 = \frac{1}{R_4} = \frac{1}{5} = 0.2 \text{ mho}.$$

Find the common terminal voltage.

$$V = \frac{E_1g_1 + E_2g_2 + E_3g_3}{g_1 + g_2 + g_3 + g_4} = \frac{110 \times 0.4 + 115 \times 0.2 + 120 \times 0.2}{0.4 + 0.2 + 0.2 + 0.2} = 91 \text{ V}.$$

Now determine the currents of the branches.

$$I_1 = (E_1 - V)g_1 = (110-91)\times 0.4 = 7.6 \text{ A}$$
$$I_2 = (E_2 - V)g_2 = (115-91)\times 0.2 = 4.8 \text{ A}$$
$$I_3 = (E_3 - V)g_3 = (120-91)\times 0.2 = 5.8 \text{ A}$$
$$I_4 = Vg_4 = -91\times 0.2 = -18.2 \text{ A}.$$

The "—" sign of I_4 shows that the actual current is not in the direction of the arrow in Fig. 45.

Let us examine operation of two shunt-wound generators of the same emf ($E_1 = E_2$) and of the same internal resistance ($R_{01} = R_{02}$). The generators are connected as shown in Fig. 45. Let $E_1 = E_2 = 110$ V and $R_{01} = R_{02} = 0.2$ ohm. The load has a resistance of $R_3 = 1$ ohm. Find the power developed by the generators.

Using the parallel-generator theorem we get

$$V = \frac{E_1 g_1 + E_2 g_2}{g_1 + g_2 + g_3} = \frac{2\times 110 \dfrac{1}{0.2}}{\dfrac{1}{0.2} + \dfrac{1}{0.2} + 1} = 100 \text{ V}$$

The current of the generators will be

$$I_1 = (E_1 - V)g_1 = (110-100)\times 1/0.2 = 50 \text{ A}.$$
$$I_2 = 50 \text{ A}$$

Their power will then be

$$P_1 = E_1 I_1 = 110\times 50 = 5500 \text{ W}$$

and

$$P_2 = P_1 = 5500 \text{ W}.$$

Hence, generators with the same emfs and internal resistances deliver an equal amount of power into the line.

Now let the emf of the second generator be $E_2 = 121$ V. Then the common terminal voltage will be

$$V = \frac{110\times \dfrac{1}{0.2} + 121 \times \dfrac{1}{0.2}}{\dfrac{1}{0.2} + \dfrac{1}{0.2} + 1} = 105 \text{ V}$$

and the currents of the generators will be

$$I_1 = (110-105)\times 1/0.2 = 25 \text{ A};$$
$$I_2 = (121-105)\times 1/0.2 = 80 \text{ A}.$$

The load current

$$I_3 = 105 + 1 = 105 \text{ A}.$$

The powers delivered by the generators

$$P_1 = 110\times 25 = 2750 \text{ W}$$
$$P_2 = 121\times 80 = 9680 \text{ W}.$$

Thus, in the case of two d-c generators with equal internal resistance operating in parallel, the one with the greater emf will carry a greater load.

Finally, take two parallel generators with equal emf but different internal resistances.

Example 35. Find the currents of generators of emf $E_1=E_2=110$ V and of interna resistances $R_{01}=0.2$ ohm and $R_{02}=0.25$ ohm. The external resistance is $R=1$ ohm.

(a) The common terminal voltage is

$$V=\frac{110\times\frac{1}{0.2}+110\times\frac{1}{0.25}}{\frac{1}{0.2}+\frac{1}{0.25}+1}=99\text{ V}.$$

(b) The currents of the generators

$$I_1=(110-99)\times 1/0.2=55\text{ A};$$
$$I_2=(110-99)\times 1/0.25=44\text{ A}.$$

(c) The load current is

$$I_3=99+1=99\text{ A}.$$

In the case of d-c generators of an equal emf but different interna resistances, operating in parallel, the one with the lower internal resistanc will carry the greater load.

27. The Maxwell Circulating-current Theorem

The Maxwell circulating-current theorem is applicable to networks having more than two terminals. Fig. 46*a* shows such a network. In fact, it consists of three closed circuits or meshes. The middle mesh has sections which are also common to the other two meshes. Also, there are sections which belong to only one mesh.

By the Maxwell theorem, in each mesh there is a circulating current. Then the current flowing through the sections common to two meshes will be equal to the algebraic sum of the currents in the latter two meshes.

Let us assign to each mesh a clockwise circulating current as shown by arrows in Fig. 46. Kirchhoff's second Law can then be applied to each mesh and the equations written down for solution.

For mesh *I*:

$$E_1=I_1R_1+(I_1-I_2)R_2. \qquad \text{(a)}$$

For mesh *II*:

$$0=I_2R_3+(I_2-I_1)R_2+(I_2-I_3)R_4. \qquad \text{(b)}$$

For mesh *III*:

$$E_2=I_3R_5+(I_3-I_2)R_4. \qquad \text{(c)}$$

The total number of independent equations is equal to the number of meshes (i.e. there, are fewer equations than in a purely Kirchhoffian solution). Therefore they can be solved as simultaneous equations to obtain the circulating currents and then the mesh currents.

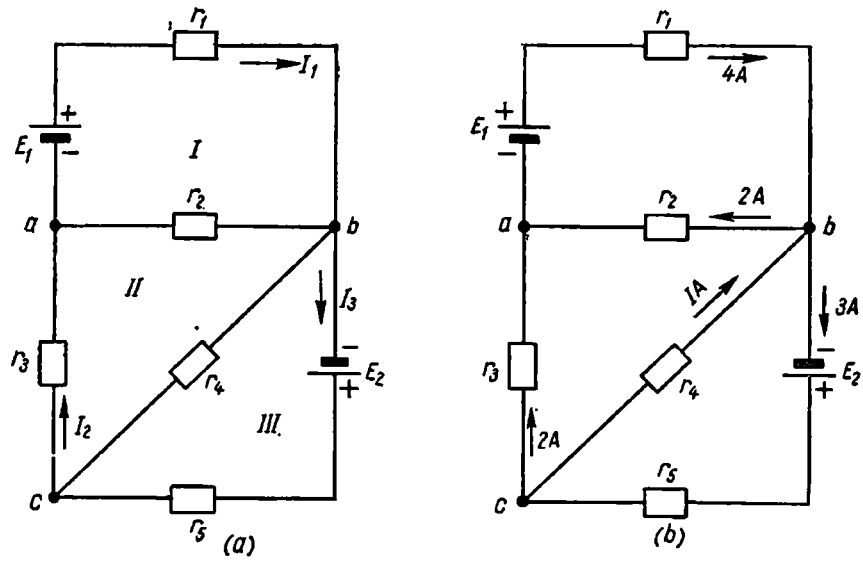


Fig. 46. The Maxwell circulating-current theorem

Example 36. Find the current distribution in the network shown in Fig. 46*a* if $E_1=14$ V, $E_2=20$ V, $R_1=2$ ohms, $R_2=3$ ohms, $R_3=4$ ohms, $R_4=2$ ohms, and $R_5=6$ ohms.

Substituting the values given in the respective formulas (a), (b) and (c) gives a set of simultaneous equations for meshes *I*, *II* and *III*, respectively:

(a)
$$14 = I_1 2+(I_1-I_2)3,$$
$$14 = 2I_1+3I_1-3I_2,$$
$$14 = 5I_1-3I_2. \qquad (a')$$

(b)
$$0 = I_2 4+(I_2-I_1)3+(I_2-I_3)2,$$
$$0 = 4I_2+3I_2-3I_1+2I_2-2I_3,$$
$$0 = 9I_2-3I_1-2I_3. \qquad (b')$$

(c)
$$20 = 6I_3+(I_3-I_2)2,$$
$$20 = 6I_3+2I_3-2I_2,$$
$$20 = 8I_3-2I_2,$$
$$10 = 4I_3-I_2. \qquad (c')$$

Adding (a') and (b') gives

$$\begin{aligned} 14 &= 5I_1-3I_2 \\ + \\ 0 &= 9I_2-3I_1-2I_3 \end{aligned}$$

or

$$\begin{array}{r} 42 = 15I_1 - 9I_2 \\ + \\ 0 = 45I_2 - 15I_1 - 10I_3 \\ \hline 42 = 36I_2 - 10I_3. \end{array} \qquad (d)$$

Adding together (c′) and (d) gives

$$\begin{array}{r} 10 = 4I_3 - I_2 \\ + \\ 42 = 36I_2 - 10I_3 \end{array}$$

or

$$\begin{array}{r} 50 = 20I_3 - 5I_2 \\ + \\ 84 = 72I_2 - 20I_3 \\ \hline 134 = 67I_2, \end{array}$$

whence $I_2 = 134 \div 64 = 2$ A.

Substituting the value of I_2 in Eq. (c′), we get

$$10 = 4I_3 - 2,$$

$$10 + 2 = 4I_3,$$

whence $I_3 = 12 \div 4 = 3$ A.

Substituting the value of I_3 in Eq. (a′), we get

$$14 = 5I_1 - 6,$$

$$20 = 5I_1,$$

whence $I_1 = 20 \div 5 = 4$ A.

The currents in the individual meshes of a network are found by adding the circulating currents algebraically. The current distribution in a network is shown in Fig. 46*b*.

28. Basic Properties of Conductors

The properties of conductors of importance to the electrical engineer are:

1. Resistivity.
2. Resistance-temperature coefficient.
3. Thermal conductivity.
4. Contact potential-difference and thermal emf.
5. Ultimate tensile strength and percentage elongation.

1. Resistivity (ϱ), or specific resistance, of a conductor gives a quantitative idea of the ability of a material to resist the passage of an electric current. The formula for resistivity is

$$\varrho = \frac{RS}{l}.$$

For long conductors (wires, cords, cable cores, busbars) the length l is measured in metres, the cross-sectional area S in square millimetres, and the resistance R in ohms. Then the dimensions of resistivity are

$$[\varrho] = \text{ohm-mm}^2/\text{m}.$$

The resistivities of some of the metal conductors are given on page 84.

2. The resistance-temperature coefficient, α, describes the rate of change of resistivity with temperature.

The average value of the resistance-temperature coefficient in the temperature range t_2-t_1 can be found from the formula

$$\alpha = \frac{R_2-R_1}{R_1(t_2-t_1)} \text{ deg}^{-1}.$$

The resistance-temperature coefficients of some of the metal conductors are given on page 84.

3. Thermal conductivity, λ, describes the quantity of heat passing through a layer of a material per unit time. The dimensions are kcal/m-hr-deg It is essential to take full account of thermal conductivity in calculating electrical machines, apparatus, cables, etc.

Thermal Conductivity, λ, of Some Materials (kcal/m-hr-deg)

Silver	350-360
Copper	340
Aluminium	180-200
Brass	90-100
Iron and steel	40-50
Bronze	30-40
Concrete	0.7-1.2
Brick	0.5-1.2
Glass	0.6-0.9
Asbestos	0.13-0.18
Wood	0.1-0.15
Cork	0.04-0.08

From the table it can be seen that thermal conductivity is highest in metals and lowest in nonmetals, especially in porous materials employed as thermal insulation. According to the free electron theory, thermal conductivity is accounted for by the same conduction electrons which are responsible for electrical conductivity.

4. Contact potential difference and thermo-emf are of importance where two junctions of dissimilar metals may be formed in the same circuit. In the case of metals, positive ions make up the framework of the crystal lattice. The space between them is filled by free electrons, much as a gas would do, for which reason the term "electron gas" is sometimes used. The pressure of the electron gas in a metal is proportional to the absolute temperature and the number of free electrons per unit volume, which varies from metal to metal. At a junction of two dissimilar metals, the pressure of electron gas tends to an equilibrium. Due to the diffusion of electrons, one metal loses some and becomes positively charged, the other metal acquires some and becomes negatively charged, and a potential

difference is set up at the junction. This potential difference depends on the properties of the metals making up the junction and is proportional to the difference in their temperatures. If the circuit containing such a junction is closed, a thermo-current will flow around it. The emf causing this flow of current is termed the *thermo-emf.*

Since thermo-emfs may sometimes run high, they can distort measurements of low currents or voltages. To avoid inaccuracies, it is essential to match contact materials as close as possible.

5. Ultimate tensile strength and percentage of elongation describe a material from a mechanical point of view. Mechanical strength is especially vital in overhead power transmission lines. This is why all conductors are tested for both tensile strength and elongation prior to use.

The ultimate tensile strength of a material is obtained by dividing the load that causes the material to break in a test by the cross-sectional area of the specimen at fracture, or U.T.S. $= P_{us}/A$ kg/mm^2. The ultimate tensile strength of some of the materials is tabulated on page 84.

The percentage of elongation (or per cent elongation) is found by dividing the increase in the length measured across the fracture by the original length and multiplying the ratio by 100, or P.E. $= \frac{l_1 - l}{l} \times 100$ per cent.

Sometimes, it will suffice to find *absolute elongation.* This is the difference between the original length and the length across the fracture, or $dl = l_2 - l_1$.

29. Current Conducting Materials

Conductors may be solid, liquid and, sometimes, gaseous. Metals are solid conductors. Electrolytes and molten metals are liquid conductors. Ionized air is a gaseous conductor.

Metals may be divided into two groups: high-conductance metals and high-resistance metals. The first group embraces chemically pure copper and aluminium which go to make wires, cables, machine and transformer windings. Some alloys, such as brass, bronze, aluminium alloys and steel, are also good conductors. These are preferred because of their low cost in comparison with pure metals and their high mechanical strength.

The second group of metals covers metals and alloys used in heaters, incandescent lamps, rheostats, and the like.

In addition to metals, electrical engineering makes wide use of carbon products (electrodes in flood-lights and electric furnaces, brushes in electric machines, inserts in microphones, etc.).

30. High-conductance Metals

Copper is the most commonly used metal in electrical engineering. Copper possesses the following useful properties: low resistivity (0.0172 to 0.0175 ohms mm^2/metre); good mechanical strength (an ultimate tensile

strength of 25 to 40 kg/mm^2); good resistance to corrosion; ready availability in almost any shape; and easy and efficient jointing by a variety of methods, including soldering and welding.

Where high mechanical strength, hardness and resistance to abrasion are vital, use is made of tough-pitch copper. Such applications include transmission-line conductors, overhead contact wires, switchgear buswork, commutator segments for electric machines. Heated to 400 to 700°C and cooled, tough-pitch copper becomes soft and is referred to as annealed copper. Annealed copper is used in the manufacture of round and square wire, cable cores, and magnet wire.

Bronze is an alloy of copper, tin, silicon, phosphorus, beryllium, cadmium and some other elements. It is stronger than copper (bronze wire has a tensile strength of 80 to 135 kg/mm^2), but it has a lower electrical conductivity (10 to 95 per cent that of copper).

Brass is an alloy of 50-70 per cent copper and 30-50 per cent zinc. It is employed as a structural material. Brass can be readily machined, drawn and pressed.

Aluminium ranks third in electrical conductivity after silver and copper (0.029 ohm mm^2/metre).

It is inferior to copper as far as mechanical strength is concerned. Annealed aluminium has a tensile strength of 8-9 kg/mm^2, while hard aluminium will stand up to tensile loads of 18 kg/mm^2. For higher mechanical strength aluminium is alloyed with silicon, iron, or manganese. One such alloy, known under the trade name of Aldrey, is used for overhead conductors. It consists of 0.3-0.5 per cent Mg, 0.4-0.7 per cent Si, 0.2-0.3 per cent Fe, remainder Al. Overhead lines also use stranded steel-cored conductors which consist of an envelope of aluminium wires around a core of one or more high-tensile galvanized steel wires. Other electrical uses for aluminium include wire, round or square busbars, capacitor foil, and cable sheaths.

Steel is a highly durable material. Steel grades used for conductors run 0.1 to 0.15 per cent carbon and have a tensile strength of 70 to 75 kg/mm^2. The resistivity is 0.1 ohm mm^2/metre, or one-sixth to one-seventh of that of copper. As distinct from copper or aluminium, steel is a more common material and is cheaper to make. A serious drawback of steel is its low resistance to corrosion. This shortcoming may be offset by applying a coat of a more corrosion-resistant metal (such as zinc). Being a ferromagnetic material, steel is subject to the skin effect when carrying an alternating current (see Sec. 85). Therefore its resistance to an alternating current is higher than to a direct current. To prevent overheating in a steel conductor alternating-current values for conductors and busbars should be less than direct-current values. Furthermore, hysteresis loss is inevitable in a steel conductor carrying an alternating current (see Sec. 61).

In the electrical industry steel is made into wires, busbars, earthing matts, tramway and railway rails.

In an attempt to economize on copper, copper-clad steel conductors have been developed. The core of such a conductor is of steel hot-dipped or galvanized with copper. The two metals make an intimate union. The copper content is 50 per cent. The tensile strength of a copper-clad steel conductor is 55 to 70 kg/mm^2.

31. Metals Used in the Electrical Industry

Lead is a soft-ductile metal grayish in colour. The tensile strength of lead is 1.6 kg/mm^2. It resists the attack of water, sulphuric and hydrochloric acids. In air a thin oxide film forms on lead, protecting it from further oxidation. Acetic and nitric acids, rotting organic substances, lime, and fresh concrete easily attack lead. It is made into cable sheaths and acid-battery plates. Lead and its compounds are poisonous.

Tin is a silver-white soft metal. It resists the action of water and air, while diluted acids attack it very slowly. Tin goes to plate sheet iron as a protection against corrosion (tinplate). Tin foil (thin rolled sheets of tin) is used in the manufacture of capacitors. Tin is also added to bronze, babbitt metals and solders.

Tungsten is a silver-white hard metal obtained by a suitable chemical process from ore concentrates. It is obtained in powder form. The powder is then pressed into cubes under a pressure of up to 2000 atm. When the cubes are heated to 700°C, a dense sintered metal is produced. Sintered ductile tungsten can be drawn into wire or rolled into sheets. Tungsten has the highest melting point of all metals: 3370 ± 50°C. Ductile tungsten is made into lamp filaments, valve electrodes, cathode-ray anticathodes, etc.

Mercury is a silver-white metal liquid under normal conditions. It is employed in mercury-arc rectifiers, mercury-vapour lamps, and mercury switches. Mercury is poisonous, especially its fumes.

Table 7 gives the principal characteristics of most of the metals and alloys used for electrical purposes.

32. Manufactured Carbon

Carbon for electrical uses is manufactured from carbon black, coal, coke, or naturally occurring graphite. The raw material is finely ground, sized, and mixed with a suitable binder (which may be coal-tar or water glass). The mix is then drawn through a die (in the manufacture of carbon rods) or pressed in moulds (when made into odd shapes). Then the green products are fired at 800 to 3000°C.

In the case of brushes for electric machines, additions are made of graphite, copper, and bronze for better wearability.

Table 7

Principal Characteristics of Metals and Alloys

Material	Density, g/cm³	Melting point, °C	U.T.S., kg/mm²	Resistivity, ohms mm²/m	Resist. temp. coeff. 1/deg. C	Uses
Aluminium	2.69-2.7	657-660	8-25	.026-0.028	.00403-.00429	Wire, cables, busbars, foil
Aldrey	2.7	1100	30-38	.029-.032	.0036-.0038	H.V. line conductors
Bronze	8.3-8.9	885-1050	31-135	.02-.05	.004	Contacts, springs, trolley conductors
Tungsten	19.3-20	3370±50	100-300	.053-.055	.004-.005	Lamp filaments, valve electrodes, contacts
Gold	19.3	1063	—	.022-.023	.0036	Silver-alloy contacts
Brass	8.4-8.7	900-960	30-70	.031-.079	.002	Contacts, terminals
Copper	8.7-8.9	1083	27-44.9	.0175-.0182	.004	Wire, cables, busbars
Molybdenum	10.2	2570-2630	80-250	.048-.054	.0047-.005	Valve anodes, supports and grids
Nickel	8.8-8.9	1452	40-70	.07-.079	.006	Valve cathodes, anodes, grids
Tin	7.3	232	2-5	.11-.12	.0043-.0044	Solders, foil
Platinum	21.4	1773	15-35	.09-.1	.0025-.0039	Thermocouples, furnace heaters, contacts
Mercury	13.54-13.55	−38.9	—	.958	.099	Thermostats, temperature regulators, rectifiers
Steel	7.8	1400-1530	70-75	.103-.137	.0057-.006	Wire, cables, busbars, rails
Silver	10.5	960.5	15-30	.016-.0162	.0034-.0036	Contacts
Lead	11.34	327.4	.95-2	.217-.222	.0038-.004	Cable sheaths, fuse links, battery plates
Zinc	7.1	419.4-430	14-29	.05-.06	.0039-.0041	Plating, contacts
Cast iron	7.2-7.6	1200	12-32	.5-1.4	.0009-.001	Rheostats

Note. The upper limits apply to the hard (as-cast, etc.) condition while the lower limits are for the annealed condition.

33. Electrical Resistance Alloys

This group of alloys may be further subdivided into (a) alloys for resistance boxes, standards, multipliers and shunts; (b) alloys for resistors and rheostats; and (c) alloys for heating appliances and furnaces.

The alloys in subdivision (a) should have high resistivity, a resistance temperature coefficient close to zero, low thermo-emf in junctions with other metals (especially copper), time-independent resistance, and high resistance to corrosion. This subdivision is represented by manganin and constantan, both copper-base alloys.

Manganin is an alloy of 86 per cent copper, 12 per cent manganese and 2 per cent nickel, of a reddish-brown colour. It has the following properties: resistivity, 0.42-0.43 ohm mm^2/m; density, 8.4 g/cm^3; tensile strength, 40-55 kg/mm^2, low resistance temperature coefficient and thermo-emf; maximum working temperature, not over 60°C. Manganin is best of all suited for the manufacture of resistance boxes, standard resistances and shunts.

Constantan is an alloy of 60 per cent copper and 40 per cent nickel. Its properties are: resistivity, 0.5 ohm mm^2/m; density, 8.9 g/cm^3; tensile strength, 40-50 kg/mm^2. It is used in the manufacture of rheostats and heater resistors, provided their working temperature does not exceed 400 to 450°C. When in a junction with copper, constantan develops a high thermo-emf and therefore cannot be used in the manufacture of standard resistances for high-precision instruments, since the thermo-emf would falsify the readings. This property of constantan is, on the other hand, utilized in thermocouples for measuring temperatures up to several hundred degrees.

The alloys in subdivision (b) should be cheap to make, have high resistivities and low resistance temperature coefficients. These requirements are met by copper-base alloys such as constantan, nickeline, etc. For cost reduction, the nickel in some of the alloys in this group has been replaced by zinc and iron.

The alloys in subdivision (c) should be easy to work, mechanically strong, cheap to make, of high resistivity, and resistant to oxidation at elevated temperatures. This subdivision is represented by nichromes, fecraloy and chromal.

Nichromes are alloys of nickel and chromium in various proportions. This group also includes ferronichrome, which, in addition to nickel and chromium, also carries iron (its composition is: nickel, 58-62 per cent; chromium, 15-17 per cent ; balance, iron). The properties of nichromes are as follows: density, 8.4 g/cm^3; tensile strength, 70 kg/mm^2; resistivity, about 1 ohm mm^2/m. Nichromes are available in wire and tape form for application in heating appliances and furnaces with working temperatures up to 1000°C.

Alloy	Density, g/cm³	Melting point, °C	Ultimate tensile strength, kg/mm²	Resistivity, ohm-mm²/m
Constantan	8.7-8.9	1200-1275	40-55 65-70	0.45-0.48 0.46-0.52
Manganin	8.14-8.4	920-960	45-55 60-70	0.42-0.48 0.43-0.5
German silver	8.3-8.5	1050	35-40 45-53 55-60	0.30-0.35 0.40-0.45
Nichrome X15H60 (15% Cr, 60% Ni)	8.2-8.25	1380-1390	55-65	1.02-1.18
Nichrome X20H80 (20% Cr, 80% Ni)	8.4	1400	60-70	1.02-1.27
Fecraloy (X13IO4)	7.2-7.4	1450-1480	58-65	1.1-1.25
Nichrome (X20H80T)	8.0	1510-1520	65-70	1.02-1.17
Nichrome (X20H80T3)	8.0	1520-1530	65-75	1.18-1.36

Note. The lower limits of ultimate tensile strength and resistivity are given for

Table 8

Resistance Alloys

Resistance temperature coefficient, 1/°C,	Thermo-emf in junction with copper, μV/°C	Maximum safe temperature, °C	Uses
5×10^{-6}	39-42	450-500	Rheostats, resistors for low-accuracy instruments. Heaters for 400-450°C. Thermocouples with copper and iron
$(3\text{-}6) \times 10^{-5}$	09-1.0	250-300	Standard resistors, resistance boxes, shunts and multipliers for high-accuracy instruments
$(28\text{-}30) \times 10^{-5}$	14-16	200-250	Rheostats
0.17×10^{-3}	—	1000	Laboratory and industrial furnaces for up to 900°C
0.15×10^{-3}	—	1050	Laboratory and industrial furnaces for up to 1000°C
0.05×10^{-3}	—	850	Household electrical appliances, industrial furnaces with working temperature of up to 650°C
0.14×10^{-3}	—	1200	Industrial furnaces with work ing temperature up to 1150° C
0.08×10^{-3}	—	1200	Same

soft alloys (wire, ribbon), while the upper limits are for hard alloys.

Fecraloy is an iron-base alloy of 12-15 per cent chromium, 3-5 per cent aluminium. The density is 7.5 g/cm^3, the tensile strength 70 kg/mm^2, and the resistivity about 1.2 ohms mm^2/metre. The working temperature is about 800°C.

Chromal is an iron-base alloy of 28-30 per cent aluminium. The tensile strength is 80 kg/mm^2, the resistivity 1.3-1.4 ohms mm^2/m, and the working temperature, 1250°C.

A summary of electrical resistance alloys is given in Table 8.

34. Semiconductors

As the name implies, semiconductors are materials which lie between conductors (such as metals, electrolytes and carbon) and insulators (such as porcelain, mica, rubber, etc.).

The volume resistivity of conductors, semiconductors and dielectrics, in the order given, is 10^{-6} to 10^{-3}; 10^{-3} to 10^{8}; and 10^{8} to 10^{20} ohm-cm.

Belonging to semiconductors are metallic oxides (Al_2O_3, Cu_2O, ZnO, TiO_2, VO_2, WO_2 and MoO_3); sulphides, or sulphur compounds (such as Cu_2S, Ag_2S, ZnS, CdS, and HgS); selenides, or selenium compounds; tellurides, or tellurium compounds; several alloys ($MgSb_2$, ZnSb, Mg_2Sn, CdSb, AlSb, InSb and GLSb); the elements germanium, silicon, tellurium, selenium, boron, carbon, sulphur, phosphorus, arsenic, and also a variety of compounds (galena, carborundum, etc.).

In Russia the field of semiconductors was pioneered by Stoletov and explored in detail by Yoffe and coworkers.

In Section 2 we stated that a semiconductor is a material in which the lower permitted band of electrons is filled and there is a small gap between the lower and upper permitted bands. What is meant is that the electrons are all bound to the atoms and require a definite amount of energy to be set free. Under the influence of thermal vibrations or some other factors, some of the electrons may be shaken free from their associated ions. These electrons can then move through the crystal structure as free electrons. The crystal becomes conductive since the electrons are free to move under the action of an applied electric field.

This can be proved by the following experiment. Heat a metal rod at one end, leaving the other end cold. An electroscope will show the presence of a negative charge at the cold end and the presence of a positive charge at the hot end. Apparently this is due to the fact that the hot end lacks electrons (hence it carries a positive charge), while the cold end has an excess of electrons (it has a negative charge). This migration of electrons was caused by heat, which produced a momentary flow of electricity.

Obviously, the condition of the ion from which the electron is shaken free is different from the normal state: there is a net positive charge in the neighbourhood due to the absence of the electron. We say that there is a positive hole, or simply a hole, at any ion which has a missing valency

electron. It requires very little energy to take an electron from a normal ion to a neighbouring ion which has one electron missing. The result is that a positive charge (i.e., a hole) moves from ion to ion due to the movement of a bound electron between these ions. Thus, the hole takes part in conduction very much like a free electron, but its movement is in an opposite direction. Fig. 47*a* gives a diagrammatic representation of a valency bond, while Fig. 47*b* shows a hole. At *c* a normal ion contributes an electron to a hole.

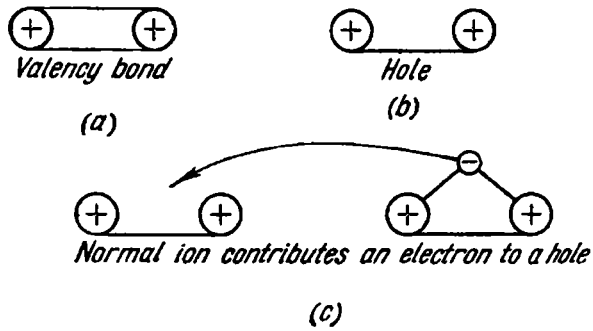


Fig. 47. Bonds between atoms

Under normal conditions, a free electron (or a hole) moves through the crystal in a random manner, changing its direction due to collisions. The application of an electric field superposes a steady drift upon the random motion, with the electrons moving against, and the holes with the field. It should be noted, however, that a flow of electrons against the field is taken as a current with the field, as is also a flow of holes. Therefore, both holes *and* electrons contribute additively to the current.

There are semiconductors which owe their conductivity to their own electrons shaken free. Hence their name—intrinsic semiconductors. Intrinsic semiconductors are usually of low conductivity, since equal numbers of free electrons and holes are produced. Germanium may be taken as an example. It has a valency of 4, i.e., it has four electrons in the outermost shell. Each atom is surrounded by four neighbours (at the corners of a tetrahedron) and shares its four electrons with these four neighbours. It is usual to consider the electrons as shared in four pairs, a pair being called a covalent bond. Whenever a covalent bond is broken up, an electron-hole pair is produced (Fig. 48).

On the other hand, electrons or holes can be produced in a semiconductor by adding to it (or doping it with) an impurity, thereby enhancing its conductivity.

The impurities that can be used for this purpose are phosphorus, arsenic and antimony, which all have five valency electrons. When, say, arsenic is present in a melt from which a single crystal of germanium has been produced, the arsenic atoms can take the place of some of the germanium atoms

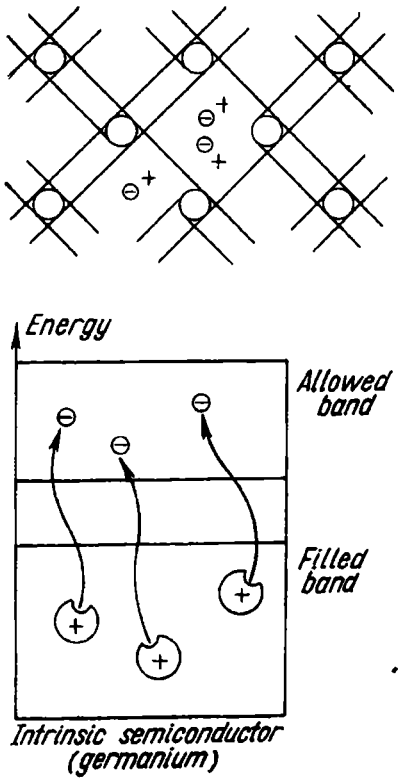


Fig. 48. Intrinsic semiconductor

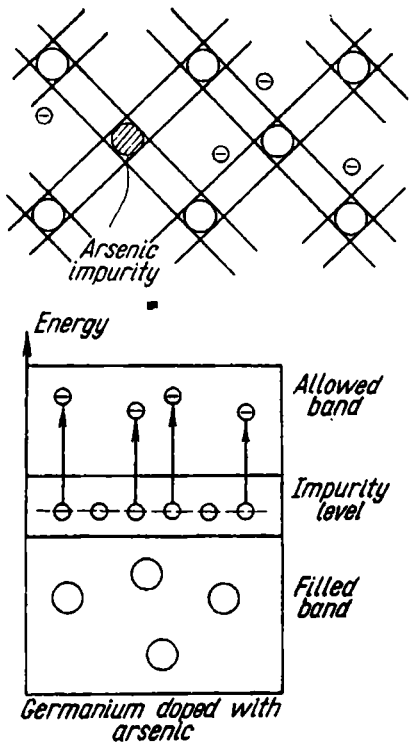


Fig. 49. *N*-type semiconductor

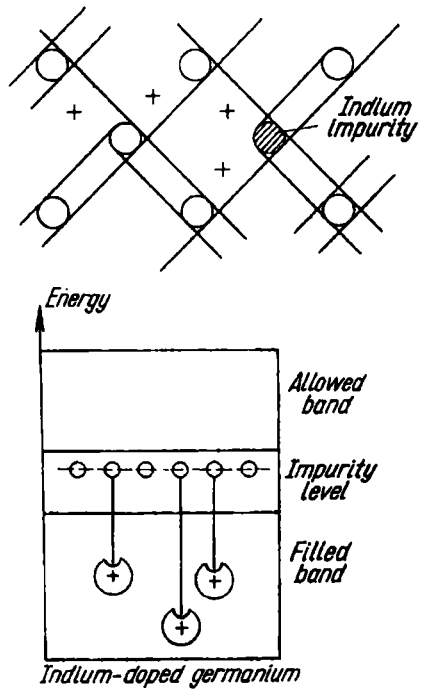


Fig. 50. *P*-type semiconductor

in the crystal structure. But only four of the five valency electrons of the arsenic atoms are held in the four covalent bonds, and the fifth can easily leave the arsenic atom to become a free electron available for conduction.

Arsenic is said to be a *donor*, as it donates an electron, and germanium (or any other semiconductor) doped with arsenic, antimony or phosphorus is said to be an *N*-type (negative) semiconductor, i.e., a semiconductor with electron conduction. These excess electrons have an energy level close to the conduction band and just below the gap voltage (Fig. 49). The donor atoms yield their electrons into the conduction band, and the semiconductor will be conducting even at normal temperatures when all the covalent bonds remain intact.

Boron, aluminium, gallium and indium have three valency electrons. If indium atoms replace germanium atoms in a crystal, there will be a missing electron in the four covalent bonds, and a neighbouring valency electron can be captured, thus forming a hole. Indium is called an *acceptor* impurity. The band diagram of Fig. 50 shows that the impurity electrons are between the allowed and filled bands, closer to the former. Accepting electrons from the filled band, they produce holes in the latter, which will be present even though there may be no free electrons in the material. The resulting material will thus be a *P*-type (positive) semiconductor, with conductivity due to its positive holes.

Hole conduction can be proved by an experiment similar to the one made to demonstrate electron conduction. In this case, however, the hot end of the bar (that has impurities which capture free electrons to produce negative ions) will be charged negatively and the cold end positively, due to the movement of holes towards the latter.

Exercises

1. Find the resistance of 100 metres of wire 1 mm in diameter.
2. What is the resistance of an aluminium wire 300 metres long and 2.5 mm^2 in cross-section?
3. What should be the cross-sectional area of a nickeline wire 5 metres long for a 20-ohm rheostat?
4. What should be the length of a nichrome wire 0.5 mm^2 in cross-section for a 24-ohm electric range?
5. Identify the material of a wire 200 metres long, 4 mm^2 in cross-section, and of 6.5 ohms resistance.
6. What is the cross-sectional area of a tungsten wire 25 cm long and of 0.05 ohm resistance?
7. What is the resistance of an iron conductor 150 km long and 4 mm in diameter?
8. What is the cross-sectional area of a nichrome wire 80 metres long and of 0.025 mho conductance?
9. What is the temperature of a copper motor winding if its resistance is 70 ohms at 15°C and 85.05 ohms when running?
10. What is the length of a nichrome wire 0.6 mm^2 in cross-section carrying a current of 5 A at 120 V?
11. What is the voltage that a current of 4 A can produce across a coil wound with copper wire 0.5 mm^2 in cross-section and 200 metres long?

12. For an arc lamp to burn steadily, a current of 10 A at 60 V is required. The lamp is powered by a generator operating at 120 V. Find the resistance of a series resistor to the lamp if the leads have a resistance of 0.2 ohm.

13. What should be the resistance of a multiplier resistor for a 20-V, 0.05-A voltmeter if it is to be connected to a 100-V supply?

14. What should be the resistance of a series resistor to obtain a current of 2 A in a circuit carrying 2.5 A at 60 V?

15. What will be the readings of voltmeters connected at points *A* and *B*, *C* and *D*, *E* and *F* in Fig. 51 if the voltage applied is 50 V, and the circuit contains three resistors connected in parallel (of 0.3, 2 and 4 ohms, respectively) by means of leads with a resistance of 1 ohm each?

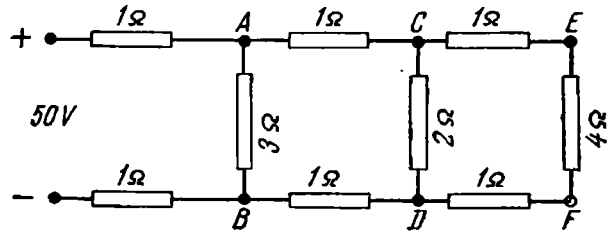


Fig. 51. See Ex. 15

16. What will be the resistance of a circuit containing 0.4-ohm wires, a 150-ohm lamp, and a 120-ohm rheostat, all connected in series?

17. Find the equivalent resistance of four resistances of 3, 4, 6 and 2 ohms connected in parallel?

18. What is the resistance of a copper conductor 800 metres long made of seven copper wires of 1.7 mm dia. each?

19. Find the equivalent resistance of an iron wire 20 m long and 1.5 mm^2 in cross-section, an aluminium wire 50 m long and 2 mm^2 in cross-section, and a nichrome wire 10 m long and 0.5 mm^2 in cross-section, all connected in series?

20. What is the equivalent resistance of a parallel combination consisting of an 18-ohm resistor in one branch and three series-connected resistors of 12 ohms each in the other branch?

21. What is the equivalent resistance of eight resistors 10 ohms each connected in four similar parallel combinations?

22. What is the equivalent resistance of a series-parallel combination if the series-connected resistor is 7 ohms, and those in the parallel combination are 2, 4, 6 and 8 ohms, respectively?

23. What is the equivalent resistance of a series combination consisting of two parallel combinations if the resistors in one parallel combination are 3, 8 and 6 ohms and those in the other parallel combination are 2, 7, 6 and 3 ohms?

24. What is the total current flowing through a parallel combination of three resistors of 3, 4 and 6 ohms, respectively, if the current through the first resistance is 2 A?

25. What is the resistance of each of the four lamps connected in parallel if their total current is 8 A at 12 V?

26. What is the current in each resistance of a combination consisting of three parallel-connected resistances of 2, 9 and 6 ohms connected in series with four parallel-connected resistors of 2, 4, 6 and 3 ohms, respectively, if the applied voltage is 30 V?

27. What is the emf of a 110-V generator supplying forty parallel-connected lamps 200 ohms each if the internal resistance of the generator is 0.2 ohm?

28. What is the current in each branch of a combination consisting of four parallel-connected resistances of 6, 4, 3 and 8 ohms, respectively, if the current flowing through the common terminal is 20 A?

29. What is the cross-section of the copper wires running from a 125-V source to a 120-V, 25-A motor over a distance of 200 metres?
30. A factory uses a current of 200 A supplied over a 300-metre-long power transmission line made of copper wire 150 mm^2 in cross-section from a power plant generating at 240 V. Find the voltage available at the factory terminals?
31. How much energy will be used up by an electric furnace operating at 120 V and 10 A during 30 minutes?
32. What is the resistance of the filament of a 200-W, 220-V lamp?
33. What is the current through the winding of a 3-kW motor connected to a 120-V supply?
34. Determine the power rating of a motor operating on 220 V at 8 A and the work done during 2 hours 30 min.
35. What will be the electricity bill for six lamps for one month if two are 40-W lamps and are on 5 hours a day, two are 60-W lamps and are on six hours a day, and the remaining two are 15-W lamps and are on 4 hours a day? The rate is 4 kopeks per kWh.
36. *Given*: a 5-kW, 110-V electric motor driving a mechanical saw; a 120-V generator 150 metres from the motor.
Find: (a) the cross-section of the copper wires between generator and motor; (b) generator kW rating; (c) power loss in the wires.
37. What is the cross-section of the copper wires connecting a 240-V generator and a 60-A load over a distance of 200 m? Power loss in the wires is 10 per cent of the power transmitted.
38. What is the hp of a petrol engine driving a 6-V, 900-A generator if the transmission efficiency is 95 per cent?
39. What is the power received by a load from a 115-V, 10-kW generator over aluminium wires 95 mm^2 in cross-section at a distance of 250 metres?

Review Questions

1. What is electrical resistance?
2. What factors affect electrical resistance?
3. What are the units of electrical resistance?
4. What is resistivity?
5. How can the resistance of a conductor be determined if its length, material and cross-section are known?
6. What is conductance?
7. Formulate Ohm's Law for a part of a circuit and give the formula.
8. Formulate Ohm's Law for the whole circuit and give the formula.
9. How can the voltage drop across a wire be determined if the resistance of the wire and the load current are known?
10. What is a short-circuit? What are its consequences? How can it be prevented?
11. What is a series combination of resistances?
12. What is a parallel combination of resistances?
13. Formulate Kirchhoff's first Law.
14. What is a series-parallel combination of resistances?
15. Formulate Kirchhoff's second Law.
16. How does the work done by an electric current manifest itself?
17. What are the units of work (energy) of an electric current?
18. What is electric power; what are the units of electric power? How is electric power measured?
19. How can electric power be measured without a wattmeter?

Chapter Three

ELECTRO-CHEMICAL EFFECTS AND CHEMICAL SOURCES OF ELECTRICITY

35. Electrolysis

As was stated earlier, there are two main types of conductors. In the first class, conduction is due to the movement of free electrons through the conductor, for which reason this class is usually designated as *electronic conductors.* They are also called metallic conductors, as they include metals, alloys and substances closely related to metals, such as carbon and graphite. Conduction in them is not accompanied by any chemical action or movement of material through the conductor.

In the second class, conduction is accompanied by chemical reaction and by the movement of matter through a liquid conductor; this class, known as *electrolytic* or *ionic conductors*, includes solutions of acids, bases and salts. The passage of a current through them is associated with the movement of the constituent particles (ions) through the solution, termed the *electrolyte.* The entire process of decomposition due to the passage of a current is called *electrolysis.*

Electrolysis is carried out in vessels called electrolytic cells. Current to the electrolyte in the cell is led by terminals which are termed *electrodes,* the positive pole being the *anode* and the negative pole the *cathode.* Electrolysis is accompanied by the deposition of metal and the evolution of hydrogen at the cathodes—an evidence that conduction in electrolysis involves the movement of matter.

Let us examine the current flow in electrolytes in greater detail.

According to modern theory, a neutral molecule, once it is in a solvent, decomposes (dissociates) into ions which have equal positive or negative electric charges. The reason for the dissociation is that in a medium with a permittivity ε_r, the force of interaction between charges is reduced ε_r times (see Coulomb's theorem). Consequently, the forces that hold a

molecule together in a solvent with a high permittivity (water has $\varepsilon_r=81$) are weakened, and thermal agitation proves sufficient to break up the molecule (i.e., cause it to dissociate) into ions.

Dissociation is accompanied by molisation, a process (opposite to dissociation) by which neutral molecules are formed.

Acids dissociate into positive hydrogen ions and negative acid radicals:

$$HCl \rightleftarrows H^+ + Cl^-$$
$$HNO_3 \rightleftarrows H^+ + NO_3^-$$
$$H_2SO_4 \rightleftarrows H^{++} + SO_4^{--}$$

Bases dissociate into metal ions and OH^- ions:

$$NaOH \rightleftarrows Na^+ + OH^-$$
$$KOH \rightleftarrows K^+ + OH^-$$
$$Ba(OH)_2 \rightleftarrows Ba^{++} + 2OH^{--}$$

Salts dissociate into metal ions and acid radicals:

$$NaCl \rightleftarrows Na^+ + Cl^-$$
$$KNO_3 \rightleftarrows K^+ + NO_3^-$$
$$CuSO_4 \rightleftarrows Cu^{++} + SO_4^{--}$$

The application of an unvarying voltage to the electrodes produces an electric field between them. Positive ions will move towards the cathode and negative ions towards the anode. Ions reaching the electrodes have their charges neutralized.

Electrolysis of Water. The electrolyte is water to which is added a little acid, base or salt for better conductivity. When a current is passed through the cell, gas bubbles are produced at the electrodes *A* and *K*. If the electrodes are covered by test tubes, the gas bubbles will rise, displacing the water. The test tube over the cathode *K* will be filled twice as fast as the one over the anode *A*. If the left-hand test tube (Fig. 52) is lifted and a burning match applied to its mouth, the gas in the tube will burn giving a bluish flame and a popping sound. This gas is hydrogen.

If a smouldering wooden stick is inserted into the test tube over the

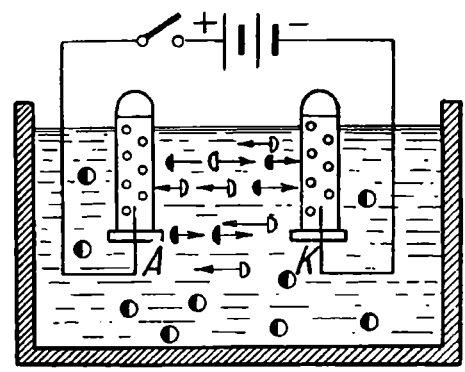


Fig. 52. Electrolysis of water

anode, the stick will flare up with a bright flame. This is an indication that the gas in the tube is oxygen.

The above experiment is used in school chemistry to prove that water consists of two gases—oxygen and hydrogen.

Electrolysis of Blue Vitriol. The electrolyte is a solution of blue vitriol, or Cu_2SO_4 (copper sulphate). The electrodes are two carbon plates. The molecules of blue vitriol decompose into the ions Cu^{++} and SO_4^{--}. When a d-c voltage is applied to the electrodes, the positive ions (or cations) move to the cathode and the negative ions (anions), to the anode. When in contact with the cathode, the cations receive two electrons from the cathode and are deposited on it as neutral molecules of copper. The anions in contact with the anode give up two electrons to the anode and become neutral, but unstable, acid radicals SO_4 which break up into SO_3 and O. The oxygen evolves at the anode while the SO_3 reacts with the water to form molecules of sulphuric acid:

$$SO_3 + H_2O = H_2SO_4.$$

With time, the deposit of copper on the cathode will increase while its content in the electrolyte will decrease. If the carbon anode is replaced by a copper plate, the SO_4 appearing at the anode will enter into a secondary reaction with the copper of the electrode:

$$SO_4 + Cu = CuSO_4$$

The concentration of the solution will then remain unchanged, but the deposition of copper at the cathode will be accompanied by the dissolution of the copper anode.

From the foregoing it is evident that an electric current in an electrolyte is due to the movement of ions—charged particles of matter. This is why electrolytes are said to possess ionic conduction as distinct from the electronic conduction of metal conductors.

As is the case with electronic conductors, the relationship between voltage (potential difference) and current in ionic conductors obeys Ohm's Law.

36. Faraday's First Law of Electrolysis

During 1883 and 1884 Michael Faraday published the results of his research into electrolysis. Among them were the now famous Faraday Laws. The first law can be expressed as follows: *The amount of chemical change (or of substance) produced at the electrodes in an electrolytic cell is directly proportional to the quantity of electricity passing through the electrolyte.*

One and the same current passing through different electrolytes during the same span of time will liberate different amounts of material at the electrodes. The weight of an element or group of elements set free by the

passage of one coulomb of electricity (one ampere × one second) is called the *electro-chemical equivalent* of that element or group. It is designated by the letter "α" (in the Soviet Union) or by the letter "z" (in Britain and elsewhere).

The electro-chemical equivalent of a chemical substance can be established experimentally as follows.

Pour a solution of blue vitriol into an electrolytic cell. Take a cathode plate 50 grams in weight and adjust the current through the electrolyte at 3 A. After 30 minutes turn off the current and weigh the cathode plate again. Its weight will be found to have increased to 51.7766 g. Thus, the passage of a current of 3 A during 30 min (1800 sec.) has liberated $51.7766 - 50 = 1.7766$ g or 1776.6 milligrams of copper. A current of 1 A during one second would liberate

$$1776.6 \div 1800 \times 3 = 0.329 \text{ mg of copper.}$$

This is the electro-chemical equivalent of copper found experimentally. The electro-chemical equivalents of other chemical substances may be found by a similar procedure.

Here are the electro-chemical equivalents of a number of substances:

Aluminium	0.0936	Lead	1.072	Nickel	0.304
Iron	0.289	Zinc	0.3387	Hydrogen	0.0104
Gold	0.681	Silver	1.118	SO_4	0.4975
Copper	0.329	Mercury	2.072		

Faraday's first Law may be written as follows:

$$w = \alpha It$$

or

$$w = \alpha Q,$$

where w = weight of the element or radical deposited at an electrode in milligrams;
α = electro-chemical equivalent;
I = current in amperes;
t = time in seconds;
Q = quantity of electricity in coulombs.

Example 1. How much metallic nickel will be liberated at the cathode by the passage of a current of 5 A during 20 minutes through a solution of nickel sulphate?

$I = 5$ A;
$t = 20$ min $= 1200$ sec;
$\alpha = 0.304$;
$w = ?$

$w = \alpha It = 0.304 \times 5 \times 1200 = 1824$ mg.

It should be noted that the weight of an element liberated during electrolysis is independent of the shape of the electrolytic cell, of the concentration of electrolyte, temperature, etc.

37. Faraday's Second Law

It is known from chemistry that the chemical equivalent weight of an element is the ratio of its atomic weight (A) to its valency, n:

$$m = A/n.$$

The ratio of electro-chemical equivalent to chemical equivalent is the same for all substances—0.01036 (see Table 9).

Table 9

Relationship Between Electro-chemical Equivalent and Chemical Equivalent

Substance	Electro-chemical equivalent, α	Atomic weight, A	Valency, n	Chemical equivalent, m	α/m
Hydrogen	0.0104	1.008	1	1.008	0.01036
Silver	1.118	107.88	1	107.88	0.01036
Nickel	0.304	58.68	2	29.34	0.01036
Aluminium	0.0936	27.1	3	9.03	0.01036
SO_4	0.4975	96.07	2	48.03	0.01036

Faraday's second Law can be stated as follows: *The amount of various substances liberated at the electrodes by the same quantity of electricity is directly proportional to their chemical equivalent weights.* In other words, Faraday's second Law establishes the relationship between the electro-chemical and chemical equivalents of substances. By the second law it is possible to determine the electro-chemical equivalent of any substance.

Example 2. Find the electro-chemical equivalent of zinc if its atomic weight is 65.37 and the valency is 2.

$$A = 65.37;$$
$$n = 2;$$
$$\alpha = ?$$

The chemical equivalent weight of zinc $m = A/n = 65.37 \div 2 = 32.68$,

$$\alpha = 0.01036 \times 32.68 = 0.3387.$$

Faraday's second Law shows what properties of a substance affect its electro-chemical equivalent:

$$\alpha = 0.01036\, A/n.$$

Referring to chemistry again, the gram equivalent of a substance is its weight in grams equal to its chemical equivalent weight. For silver, the chemical equivalent weight is 107.88, and the gram equivalent is 107.88 g.

If 1 coulomb of electricity liberates 1.118 mg of silver in passing through a solution of silver nitrate, the quantity of electricity required to liberate 107.88 grams of silver will be $107.88 \times 1000 \div 1.118 = 96{,}500$ coulombs. As much electricity will be required to liberate 1.008 grams of hydrogen, 29.34 grams of nickel, 9.03 grams of aluminium, etc. Thus, the liberation of one gram equivalent of any substance requires 96,500 coulombs. This quantity is called the faraday, F.

Experiments have shown that one gram equivalent of any substance has 6.06×10^{23} atoms. As many atoms will be contained in both 1.008 grams of hydrogen and 107.88 grams of silver. One gram equivalent of a bivalent substance will have half this number of atoms, and that of a trivalent substance one-third of the atoms contained in one gram equivalent of a univalent element.

If the liberation of one gram equivalent of a substance requires 96,500 coulombs and each gram equivalent contains 6.06×10^{23} atoms, each atom of a substance carries the following charge:

$$e = 96{,}500 \div 6.06 \times 10^{23} = 16 \times 10^{-20} \text{ coulomb.}$$

In a solution, the neutral molecule of common salt (NaCl) will decompose into a cation of sodium with a charge $+e = 16 \times 10^{-20}$ coulomb and an anion of chlorine with a charge $-e = 16 \times 10^{-20}$ coulomb. The molecule of zinc chloride, $ZnCl_2$, will dissociate into an acid-radical anion with a charge $-2e = 2 \times 16 \times 10^{-20}$ coulomb and a zinc cation with a charge $+2e = 2 \times 16 \times 10^{-20}$ coulomb. A hydrogen ion which has lost one electron will have a positive charge equal to $e = 16 \times 10^{-20}$ coulomb. Therefore, the number 16×10^{-20} coulomb is the elementary electric charge equal to the charge of an electron or proton.

Any n-valent element loses or acquires n electrons in becoming an ion. This is added proof in favour of the electron theory.

38. Industrial Applications of Electrolysis

The industrial applications of electrolysis are many.

(1) Electroplating, or the application of a coating of a metal to another metal.

One metal may be subjected to electroplating by another metal with one or more of the following aims in view: (1) protection against corrosion; (2) decoration; (3) as an intermediate manufacturing operation; (4) for reconstruction or repair. The metals used for the purposes may be gold, silver, chromium, nickel, or zinc, to mention but only a few.

The article to be electroplated is thoroughly cleaned, polished, degreased and dipped in an electroplating cell as its cathode. The electrolyte is a

solution of a salt of the coating metal. The anode is a plate of the same metal. Fig. 53 shows a nickel-plating cell. The electrolyte is an aqueous solution of nickel sulphate ($NiSO_4$), and the cathode is the article to be electroplated. The current is adjusted in proportion to the area to be treated. The current density in nickel-plating is usually 0.4 A per square decimetre. For a uniform coating of nickel to be obtained, the article is placed between two anodes. After electroplating, the workpiece is removed from the cell, dried and polished.

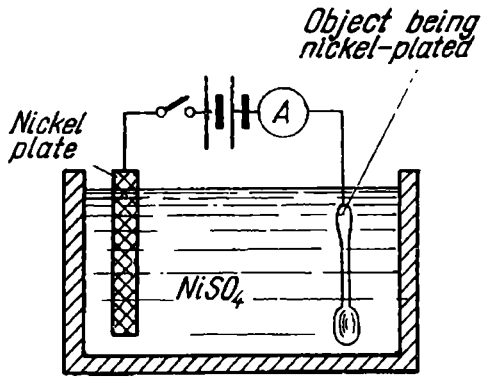


Fig. 53. Electroplating

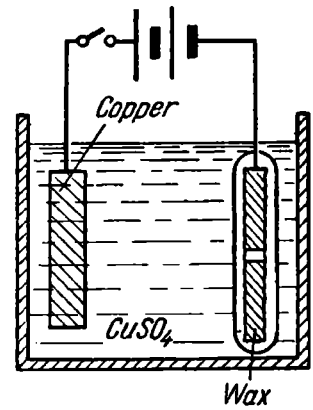


Fig. 54. Electroforming

(2) Electroforming, or galvanoplasty, is the reproduction of objects by electrodeposition on some sort of a mould or form.

In the reproduction of coins, medals, engravings and the like, a mould is first made by impressing the object in, say, wax. The surface of the wax mould is made conducting by giving it a coating of powdered graphite. The mould is then dipped in an electroforming cell where it is made a cathode. After the desired thickness of coating has been obtained, the article is removed, and the wax core is melted out of the metal shell.

In the manufacture of grammaphone records, the original wax record (Fig. 54) is coated with powdered gold to make it conducting and dipped into a blue vitriol electrolyte as the cathode. The solution is maintained saturated by using a copper anode. The plating of copper onto the wax record produces a negative master plate which goes to stamp a large number of shellac discs.

The oldest and most used variety of electroforming is *electrotyping*, the objective of which is to reproduce printing, set-up type, engravings, etc. In Russia, electroforming was pioneered by Yakobi in 1836.

(3) Electrolytic refining of metals has as its purpose to remove the last impurities from the crude metal. The crude metal is made the anode, and by electrolytic action the metal to be refined is dissolved from the anode and passes into solution. At the cathode, the prime metal is deposited on a

starting sheet. At regular intervals the deposit is stripped from the starting sheet and the anode is replaced.

Commercial copper is obtained from copper ores predominantly by smelting. The ore is ground and roasted in suitable furnaces to eliminate the iron, sulphur, arsenic, antimony, bismuth, lead, phosphorus and other impurities present in the ore. The roast is then smelted into what is known as matte, which is composed essentially of copper sulphide and iron sulphide. The matte is next blown in a converter, analogous to a Bessemer converter, to remove the iron and sulphur. The resulting metal is called blister copper and is 98 to 99 per cent pure. Specifications for copper for electrical applications (bare and insulated wire, cables, magnet wire for machines and transformers, busbars, tape, rods, commutator segments, etc.) call for a minimum purity of 99.92 per cent. This standard is achieved by electrolytic copper which is up to 99.95 per cent pure. The anodes of blister copper (Fig. 55) are suspended in an electrolyte of copper sulphate and sulphuric acid.

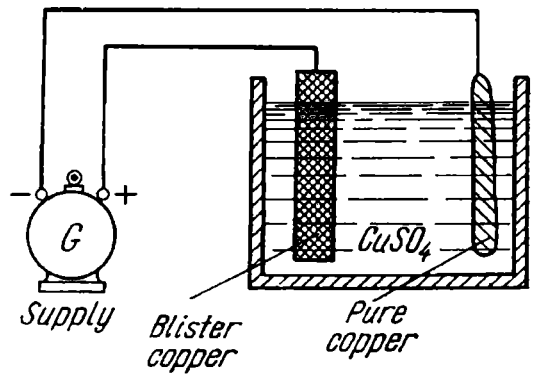


Fig. 55. Electrolytic refining of copper

(4) Electrolytic reduction of metals from their compounds is employed for several metals, aluminium being of primary interest to the electrical engineer.

Aluminium is produced from bauxite. High-grade bauxite contains up to 70 per cent aluminium oxide, or alumina, chemically combined with water forming a hydrated oxide. The ore also contains silica (silicon oxide) and iron oxide. After suitable treatment, which includes leaching, a virtually pure alumina is obtained, ready for reduction.

The reduction is accomplished electrolytically in large carbon-lined steel cells or pots (also known as electrolytic furnaces), using carbon blocks as anodes and the carbon lining and the accumulated metal as cathode. The charge is melted by the arc which strikes between the carbon anode and cathode, and is then maintained in a molten state by the heating action of the electric current. Certain fluorides are added to facilitate the action (as they bring down the melting point of the charge). The liquid metal collects at the bottom of the cell and is periodically siphoned out into large ladles from which it is poured into pig or ingot moulds. The resulting aluminium is up to 99.5 per cent pure. As the electrolytic process requires large blocks of electric power, it is customary to build aluminium factories near big hydro-electric stations. Electrical uses for aluminium include wires, cables, line hardware, etc.

39. Primary Cells

Chemical sources of an emf which are used for discharge only, thereby converting chemical energy into electrical energy, are called primary cells.

A simple primary cell (Fig. 56) consists of a containing jar filled with a solution of sulphuric acid (H_2SO_4) as the electrolyte, a zinc plate and a copper plate as electrodes. The reason for providing dissimilar electrodes is this. Immersed in diluted sulphuric acid, zinc easily passes into solution as positively charged ions, leaving the plate electronegative (i.e., with an excess of electrons). This will continue until a certain potential difference builds up at the interface between the plate and the electrolyte, preventing more zinc ions from passing into solution. The excess of electrons on the plate will determine its potential with respect to the electrolyte. If the other plate were also made of zinc, it would have the same potential; i.e., there would be no potential difference between the two plates, and no current would flow in the circuit. Copper, on the other hand, is less soluble in sulphuric acid, and its potential with respect to the electrolyte is lower, and so there will be a potential difference between the electrodes. In a simple zinc-acid-copper cell the potential difference (or emf) is 1.1 V.

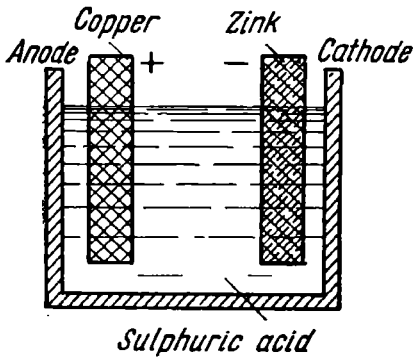


Fig. 56. An elementary primary cell

When the cell is connected in a closed electric circuit, the excess electrons of the zinc plate will move to the copper plate, reducing the potential difference between the electrolyte and the zinc plate; as a result, more zinc ions will pass into solution, leaving more electrons on the plate. In the electrolyte, the positive ions combine with radicals of sulphuric acid (negative ions) to form zinc sulphate ($ZnSO_4$).

On reaching the copper plate, the hydrogen ion H_2^{++} takes two electrons from it neutralizing its charge and becoming molecular hydrogen. The hydrogen is released at the cathode as bubbles covering the surface of the plate. On giving up its excess electrons, the copper plate becomes electropositive. From the foregoing it follows that the zinc plate is negative (it is called the cathode) and the copper plate is positive (it is called the anode). The respective terminals attached to them for external connections are called the negative pole and the positive pole.

Summing up, in the external circuit (from pole to pole) the electrons move from the zinc to the copper plate, while in the internal circuit (from electrode to electrode) negative ions move from copper to zinc and positive ions from zinc to copper.

Readings of an ammeter connected to a zinc-acid-copper cell will rapidly drop off. The reason for this is that the copper plate and the hydrogen bubbles on it form a cell within a cell, which generates an emf acting in opposition to the main emf. This is called *polarization.* Because of pronounced polarization, zinc-acid-copper cells have not found any appreciable application.

The most widely used primary cell of the nonpolarizing type is the Leclanché cell. It comprises a zinc and a carbon electrode in a solution of ammonium chloride (NH_4Cl), popularly called sal ammoniac. The

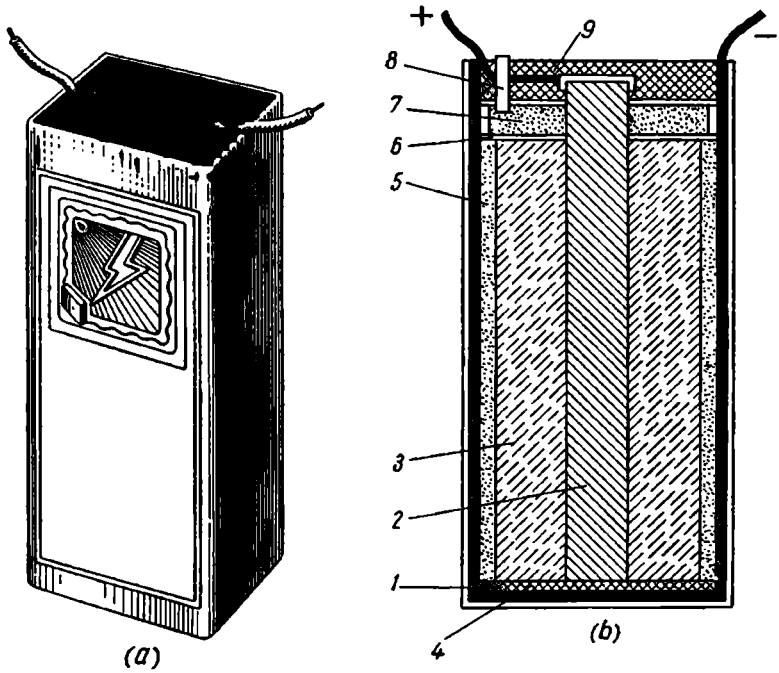


Fig. 57. Dry Leclanché cell

carbon (positive) electrode is packed in a mixture of manganese dioxide and carbon as *depolarizer.* The emf of the cell is 1.45 V. In operation, the NH_4 ions are split up into NH_3 and H, the latter being oxidized by the MnO_2 to become water:

$$MnO_2 + 2H = MnO + H_2O.$$

The removal of hydrogen prevents polarization.

The Leclanché cell is available in wet, inert and dry forms.

Fig. 57 shows a dry Leclanché cell. The cell is put into a zinc jar *1,* which also serves as the cathode. A carbon rod *2* placed centrally in the jar serves as the anode. The depolarizer *3* is a mixture of crushed manganese peroxide, graphite and carbon black impregnated with a solution of sal ammoniac. The depolarizer is wrapped in muslin and placed around

the carbon rod. The sides and bottom of the jar are lined with pasteboard *4* to separate the depolarizer from the zinc and with a layer of absorbing material *5* made of flour impregnated with sal ammoniac and zinc chloride. The purpose of the zinc chloride is to prevent rotting or drying of the absorbing paste. The muslin bag is covered by pasteboard plates *6* with sawdust *7* sandwiched between them. The negative terminal is attached to the zinc jar and the positive terminal to the carbon rod. The gases that may evolve in operation escape by a glass tube *8*. The top of the cell is closed with a sealing compound (tar) *9* and the cell is placed in a pasteboard container.

Dry Leclanché cells have an emf of 1.4 to 1.6 V and an internal resistance of 0.1 to 0.5 ohm.

The quantity of electricity that a cell is able to give up during discharge is called its *capacity* and is measured in ampere-hours. The capacity of primary cells depends on the following factors:

(1) Discharge rate: the greater the current, the smaller the ampere-hour capacity of the cell.

(2) Operation (continuous or intermittent).

(3) Temperature: the lower the temperature, the smaller the ampere-hour capacity.

(4) Final voltage.

In the Soviet Union dry Leclanché cells are designated as follows (Russian letters throughout): the letter C stands for dry; the letter Л for summer (working temperature from −20 to +60°C); the letter Х for cold-resistant (working temperature from −40 to +40°C); and the letter У for universal (working temperature between −40 and +60°C).

The first number (from 1 to 4) indicates the size of the cell; the letters describe its application; and the last figures indicate its ampere-hour capacity.

For example, the designation "3СЛ-30" reads as follows: "cell size No. 3, dry, summer, 30 Ah capacity".

Soviet manufacturers also make cells in which depolarization is effected by air and manganese dioxide (the Russian type designation is СМВД). The oxygen necessary for depolarization is breathed in from the air through glass tubes arranged at the top of the cell which only has a pasteboard cap. Since some of the electrolyte evaporates through the glass tubes, it has to be replenished by pouring sal ammoniac (about 20 cm^3 per cell). The emf of СМВД cells is twice as great as that of cells using manganese dioxide alone. СМВД cells are available in two sizes: 3СМВД (initial capacity, 45 Ah; rated discharge current 50 mA) and 6СМВД (initial capacity, 150 Ah; rated discharge current, 150 mA).

Dry cells are frequently assembled in series and sealed in cases or blocks of insulating compound. Block assemblies of cells are in common use for radio *A* (heater) supply. Among them are Type БНС-100 dry-cell batteries (100 Ah capacity, 1.5 V initial emf; 150 mA maximum discharge current;

12 dry cells connected in series) and Type БНС-МВД-500 dry-cell batteries (500 Ah capacity, 1.4 V emf; 4 Type 6СМВД cells). Dry-cell batteries such as Type БАС are used for radio *B* (anode) supply.

Dry cells are also used to power signal circuits, telephone lines, flash lights (Type КБС), etc.

The convention for cells and other chemical sources of electricity is given in Fig. 58.

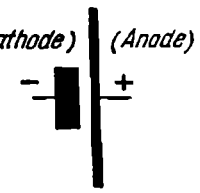


Fig. 58. Conventions for primary cells

40. Storage Cells

The distinction between an electric storage cell (also called secondary cell or accumulator) and a primary cell is that a storage cell after being discharged can be brought back to a full state of charge by passing an electric current through it. Storage cells owe their name "secondary" to the fact that they can give electricity only after they have been charged.

Several storage cells are usually connected in series to make up a storage battery. They may be of the lead-acid or the alkaline type.

Uses for storage cells include the supply of control circuits, relays, signalling, emergency lighting, the operating mechanisms and holding coils of circuit breakers, and electric-station auxiliaries. In all of these applications it is vital that the source of direct current be independent of any power supply from a station or a substation both under normal and emergency conditions.

Storage cells also find use in cars, railway carriages, submarines, shop trucks, radio, laboratory equipment, etc.

In order to charge a storage cell, it must be connected to a charging source, e.g., a dynamo yielding direct current, or rectified alternating current. The positive electrode is connected to the positive lead of the charging source and functions as an anode. The negative electrode is connected to the negative lead and functions as a cathode. The ensuing electrolysis changes the chemical state of the plates so that a potential difference is set up between them.

41. Lead-Acid Storage Cells

A lead storage cell consists of several negative and positive plates immersed in an electrolyte of sulphuric acid and distilled water with a specific gravity of 1.08 to 1.21.

The plates may be either of the Planté (formed) type or of the Faure (pasted) type.

Planté plates are made of pure lead, and a highly developed surface is obtained by cutting, spinning or rolling upstanding fins or leaves on the faces of a pure lead blank.

Faure plates consist each of a lead grid into which the active material is applied (pasted) by hand or machine. The active material may be litharge (PbO) or lead minium. The active material is retained in the pockets of the grid by a sheathing of perforated lead sheets on opposite sides of the plate.

It is customary to make positive plates of the Planté (formed) type and negative plates of the Faure (pasted) type.

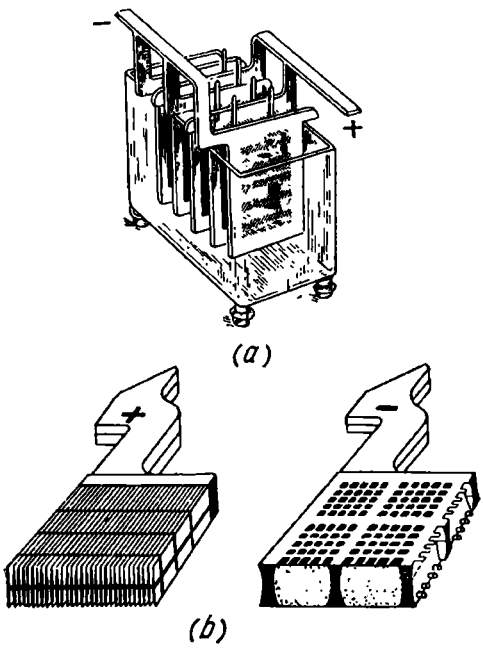


Fig. 59. Lead-acid cell:
a—general view; *b*—cell plates

The positive and negative plates of a battery are connected into respective plate groups insulated from each other. There is one more negative plate than positive, the two outside plates being negative. The reason for this is that all the positive plates can work equally on both sides.

Containers for lead storage batteries are made from glass, lead-lined wood, ceramic materials, hard rubber, or plastics.

During manufacture, storage-cell plates are given a treatment, called "forming". In this process, lead peroxide (PbO_2) is formed on the positive plates and pure sponge lead (Pb) on the negative plates. The polarity of plates can be determined by their colour: positive plates are reddish brown while negative plates are gray.

Fig. 59 shows a general view of a lead storage battery cell and its plates.

The chemical reactions taking place in a lead storage cell can be represented by the following general equation:

$$PbO_2 + 2H_2SO_4 + Pb \underset{\text{charge}}{\overset{\text{discharge}}{\rightleftharpoons}} 2PbSO_4 + 2H_2O$$

The equation should be read from left to right for discharge and from right to left for charge.

On charging (Fig. 60), the charging current should not exceed the maximum value specified by the manufacturer. When the voltage across a cell reaches 2.3 to 2.4 V, appreciable gassing begins at the plates. The charging current should then be brought down by 50 to 60 per cent and the charging continued until the voltage reaches 2.5 to 2.7 V.

Fig. 61 shows a typical charge-discharge curve of a lead-acid storage cell. Referring to the curve, when a charged storage cell has just been discon-

nected from a charging source, its terminal voltage rapidly falls from 2.7 to 2.1 V. On discharge, the voltage of a cell drops off slowly to 1.9 V and then more rapidly to 1.8 V. Lead storage cells should never be discharged below 1.75 to 1.8 V (called the final voltage), otherwise it may be damaged due to the formation of poorly soluble lead sulphate and water:

$$PbO + H_2SO_4 = PbSO_4 + H_2O$$

Lead sulphate reduces the specific gravity of the electrolyte and hampers chemical reactions. The conversion of the active material into lead sulphate is termed *sulphatization.*

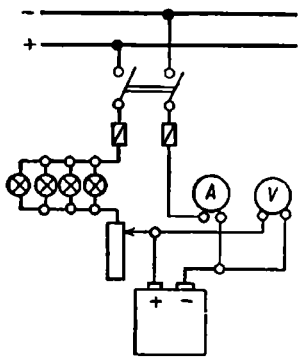


Fig. 60. Circuit for charging a cell

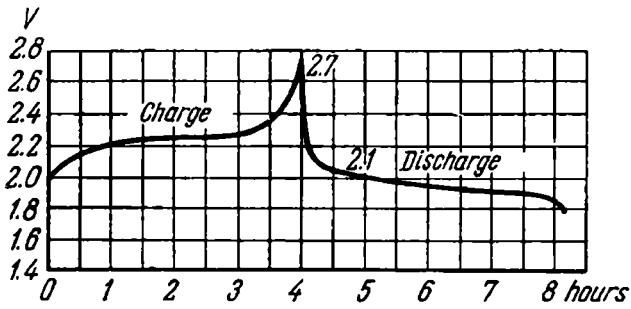


Fig. 61. Charge-discharge curve for a lead-acid cell

In operation, storage cells are subject to *local action* (or *standing loss*) which may be defined as the discharge of the active material due either to some impurity in the electrolyte, which reacts with the active material chemically, or to the deposit of some impurity on a plate. This gives rise to sufficient differences of emf generated at different points of the plate producing local currents between these points through the electrolyte.

The internal resistance of lead storage cells is relatively low. It is about 0.001 ohm for a fully charged state and doubles towards the end of discharge.

The overall efficiency of lead storage cells is anywhere between 75 and 84 per cent.

42. Alkaline Storage Cells

There are two distinct types of alkaline cells: the nickel-iron type and the nickel-cadmium type (the Russian type designations are ЖН and КН, respectively). Both types use the same active material of nickel hydrate on positive plates, while the nickel-iron type has negative plates of pure iron and the nickel-cadmium type negative plates of cadmium mixed with a small proportion of iron.

The construction of both types is basically the same. The active material is enclosed in pockets which are perforated over the entire surface with a

large number of minute holes. A number of such pockets (steel 0.1 mm thick) are assembled in nickel-plated steel retaining frames to form complete plates. In the nickel-iron type there is one more negative plate than positive, and the negative plate group is connected to the container. In the nickel-cadmium type there is one more positive plate than negative, and the positive plate group is connected to the container.

The whole assembly is fitted into a welded sheet steel container, the terminal being brought through the lid in suitably insulated glands. The lid has a filler neck which is also used as a vent.

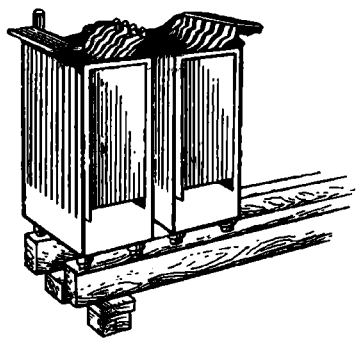


Fig. 62. General view of a storage battery

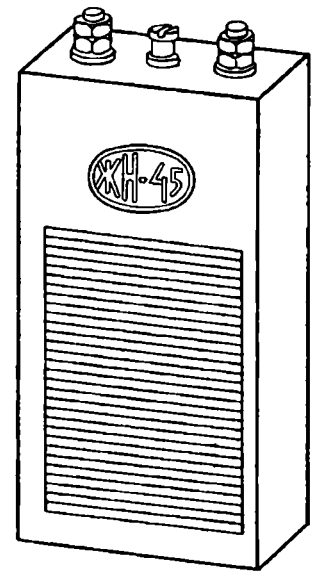


Fig. 63. General view of an alkaline cell

Both types use the same electrolyte of dilute potassium hydroxide of 1.18 to 1.20 specific gravity.

A general view of an alkaline storage cell is shown in Fig. 63.

During discharge in the nickel-iron type the higher oxide on the positive plate changes to a lower oxide, while the chemically pure iron on the negative plate changes to iron hydroxides.

On charging, the chemical reactions are reversed. The combined reactions on charge and discharge may be written as follows:

$$2Ni(OH)_3 + 2KOH + Fe \underset{\text{charge}}{\overset{\text{discharge}}{\rightleftharpoons}} 2Ni(OH)_2 + 2KOH + Fe(OH)_2$$

For the nickel-cadmium type, the charge-discharge reactions can be represented by the following equation:

$$2Ni(OH)_3 + 2KOH + Cd \underset{\text{charge}}{\overset{\text{discharge}}{\rightleftharpoons}} 2Ni(OH)_2 + 2KOH + Cd(OH)_2$$

During charge, the voltage per cell is about 1.55 V and is gradually raised to 1.75-1.8 V at the end of charge. A fully charged alkaline storage cell disconnected from the charging source has a voltage of 1.25 to 1.3 V.

Alkaline storage cells may be discharged to a final voltage of 1.0 to 1.1 V. As the temperature drops off from normal (+25°C) the ampere-hour capacity of alkaline storage cells decreases by 0.5 per cent per degree of temperature fall.

Alkaline storage cells offer a number of advantages over the lead-acid type. These advantages are:

(1) lead, which is expensive, is not used;

(2) they are more durable and mechanically strong (they can be discharged at very high rates, subjected to jarring and shocks, and short-circuited with no harm);

(3) they can stand idle for a long time without injury and incur a very small standing loss; their service life is long;

(4) a smaller amount of fumes is evolved in operation;

(5) they require less skilled attendance;

(6) they weigh less than lead-acid batteries.

On the other hand:

(1) they have a lower emf;

(2) their overall efficiency is also lower (52-55 per cent);

(3) they are more expensive to make.

43. Combinations of Energy Sources

Energy sources (primary or secondary cells) can be combined for operation in series, parallel, or series-parallel. When combined, the constituent cells make up a battery.

1. Cells in Series. Fig. 64*a* shows three secondary cells connected in series: the negative pole of each preceding cell is connected to the positive pole of each following cell.

The convention for cells connected in series is shown in Fig. 64*b*. Since the emfs of the component cells are in one direction, the total emf of the battery will be equal to their sum:

$$E = E_1 + E_2 + E_3.$$

The internal resistance of a battery is equal to the sum of the internal resistances of the individual cells:

$$R_0 = R_{01} + R_{02} + R_{03}.$$

When a battery is connected across an external resistance R, the current flowing around the circuit can be found from the equation

$$I = E/R_0 + R = \frac{E_1 + E_2 + E_3}{R_{01} + R_{02} + R_{03} + R}.$$

Cells are connected in series when the voltage of the load is higher than that of a single cell, while the load current does not exceed the discharge current of the cells.

In practice, batteries are made up of identical cells, i.e., cells with identical emf, internal resistance and ampere-hour capacity.

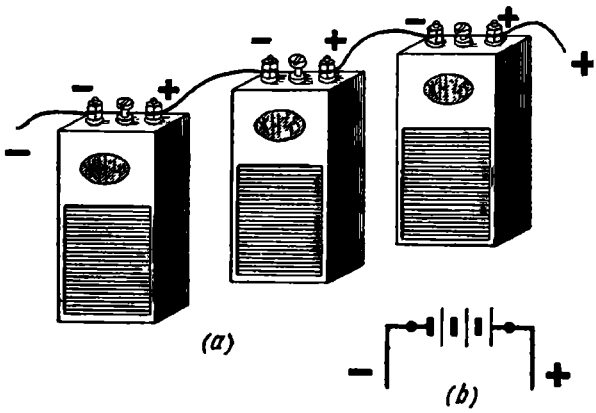


Fig. 64. Storage cells connected in series

Then the emf of a battery consisting of n cells will be

$$E_b = En.$$

The internal resistance will be

$$R_0 = R_{01}n.$$

The current flowing in the circuit will be

$$I = En/R_{01}n+R.$$

Example 3. Find the current delivered by a battery consisting of five storage cells of 1.2 V each and with an internal resistance of 0.2 ohm. The external resistance is 11 ohms.

$$I = En/R_{01}n+R = 1.2\times5+(0.2\times5+11) = 0.5 \text{ A}.$$

2. Cells in Parallel. Cells are said to be connected in parallel when their positive and negative poles are connected to respective common terminals. Fig. 65*a* shows a parallel combination of three storage cells. The conventional diagram for a parallel connection of cells is given in Fig. 65*b*.

Cells connected in parallel must be of the same emf, internal resistance and ampere-hour capacity, or detrimental circulating currents will flow around the circuit.

The emf of parallel-connected cells is equal to that of one cell:

$$E_b = E_1 = E_2 = E_3 = \ldots E_n.$$

With all the cells delivering current in one direction, the total current of a battery will be greater than the current of a single cell.

The internal resistance of a battery consisting of n cells is the internal resistance of a single cell divided by n:

$$R_0 = R_{01} \div n.$$

The current delivered by a battery into the circuit will be

$$I = \frac{E}{R_{01}/n+R}.$$

Cells are connected in parallel where the voltage of a load is equal to the emf of a single cell, while the load current exceeds the discharge current of the cell.

Example 4. Find the current delivered into a circuit by a battery consisting of two parallel-connected storage cells each of 2 V emf and 0.02 ohm internal resistance. The external resistance is 1.99 ohms.

$$I = \frac{E}{R_{01}/n+R} =$$

$$= 2 \div (0.02 \div 2 + 1.99) = 1 \text{ A}.$$

3. Cells in Series-parallel. Cells in series-parallel are shown in Fig. 66*a*; the convention for a series-parallel combination is given in Fig. 66*b*. The emf of a series-parallel combination is equal to the sum of the emfs of the series-connected cells (n) in each parallel branch:

$$E_b = En.$$

The internal resistance of each branch is

$$R_{0br} = R_{01}n.$$

The internal resistance of a combination consisting of m branches will be

$$R_{0b} = R_{0br}/m = R_{01}n/m.$$

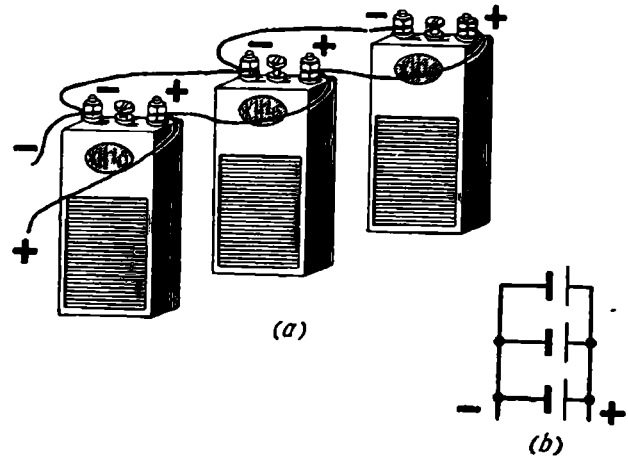


Fig. 65. Storage cells connected in parallel

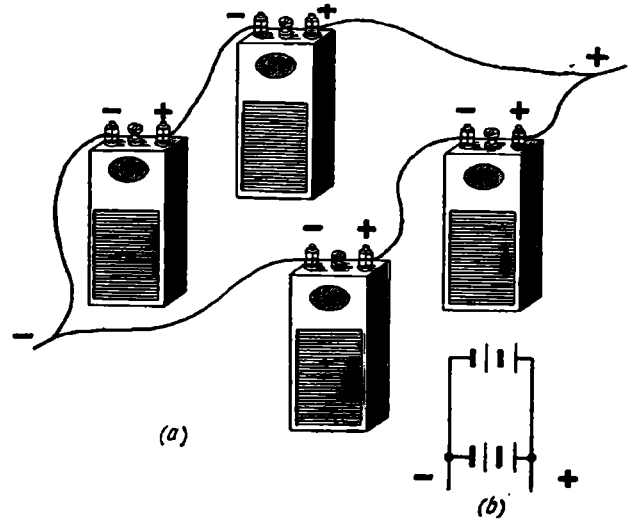


Fig. 66. Storage cells connected in series-parallel

The current delivered by a series-parallel battery into a circuit of R ohms resistance will be

$$I = \frac{E_b}{R_{0b}+R} = En/(R_{01}n/m+R).$$

Cells are connected in series-parallel where the voltage an d current o the load exceed the emf and discharge current of a single cell.

Example 5. What current will be delivered by a battery consisting of two parallel branches with three series-connected cells in each. The external resistance is 1.65 ohms; the emf of a single cell is 1.2 V and the internal resistance is 0.1 ohm.

$$I = En(R_{01}n/m+R) = \frac{1.2 \times 3}{(0.1 \times 3)/2+1.65} = 2 \text{ A}.$$

For the maximum effective power in an external circuit, the internal and external resistances of the circuit must be equal:

$$R_0 = R.$$

Exercises

1. How much zinc (in milligrams) will be deposited from a solution of zinc salt by the passage of 50 coulombs of electricity?
2. How much copper will be liberated by the passage of 5 A during 20 minutes through a solution of copper sulphate?
3. A nickel-plating cell uses a nickel plate 20 grams in weight as the anode. When will the plate be used up if the current through the cell is 10 A?
4. A metal plate 200 cm^2 in area is to be given a zinc coating 0.05 mm thick. Find the time necessary for the operation if the current through the cell is 1 A and the specific gravity of zinc is 7.1?
5. A metal object 10 mm by 40 mm by 60 mm is to be silver-plated. What current has to flow through the cell if a silver coating 0.01 mm thick is to be deposited in 30 minutes? The specific gravity of silver is 10.5.
6. What current will be supplied by a battery consisting of four storage cells of 1.2 V emf and 0.2 ohm internal resistance each, connected (a) in series and (b) in parallel? The external resistance is 4 ohms.
7. What will be the current and voltage of a circuit supplied by four storage cells of 1.2 V emf and 0.3 ohm internal resistance each, connected in series? The external resistance is 8.4 ohms.
8. What will be the current, voltage and power of a battery consisting of three parallel combinations, each combination having five series-connected cells of 2 V emf and 0.003 ohm internal resistance each? The external resistance is 4.995 ohms.

Review Questions

1. What is electrolysis?
2. What factors govern the quantity of substance liberated at the electrodes in electrolysis?
3. What is the electro-chemical equivalent of a substance?
4. What is the essence of Faraday's first and second Laws?
5. What are the industrial applications of electrolysis?
6. What is the construction of a simple primary cell?
7. What is the construction of a storage cell and how does it operate?
8. In what ways are cells connected?

Chapter Four

THERMAL EFFECTS OF ELECTRIC CURRENT

44. Heating of Conductors

All bodies consist of molecules which are in constant random motion. The faster the motion, the higher the temperature of a body. The passage of an electric current through a body (which may be a resistor or a conductor) speeds up the motion of the molecules, and the body is heated.

The quantity of heat is measured in calories (cal). One calorie is the quantity of heat required to raise one gram of water one degree Celsius. 1000 calories = 1 kcal.

Example 1. How much heat is required to raise 250 grams of water from 10° to 100°C?

One calorie is required to raise one gram of water one degree. So 250 calories will be required to raise 250 grams one degree. Thus, to heat 250 grams of water from 10° to 100°C (by 90°) will require $250 \times 90 = 22{,}500$ cal = 22.5 kcal of heat.

45. Joule's Law

The English physicist Joule first established experimentally (in 1841) that the amount of heat produced when an electric current flows through a conductor is dependent on the resistance of the conductor, the magnitude of current and the time of current flow.

This law was also established, independently, by Lenz of Russia in 1844. The quantitive relations between current flow and heat dissipation in a conductor are therefore known as Joule's (or Joule-Lenz's) Law.

As was stated earlier,

$$1 \text{ kW} = 1.36 \text{ hp.}$$

Hence

$$1 \text{ kg-m/sec} = 1/75 \text{ hp} = 9.81 \text{ W} = 9.81 \text{ joules/sec}$$

$$1 \text{ joule/sec} = 0.102 \text{ kg-m.}$$

$$\text{Since } 1 \text{ cal} = 0.427 \text{ kg-m,}$$

$$1 \text{ joule} = 0.24 \text{ cal.}$$

The energy of an electric current is given by

$$W = I^2Rt.$$

Hence, the amount of heat dissipated in a conductor by an electric current is

$$H = 0.24\ I^2Rt \text{ calories.}$$

Thus, Joule's Law states that the amount of heat in calories produced by the passage of a current through a conductor is equal to the square of the current in amperes multiplied by the resistance in ohms and the time in seconds and the proportionality factor 0.24.

Example 2. How much heat will be produced in 3 minutes by the passage of a current of 6 A through a conductor of 2 ohms resistance?

$$H = 0.24\ I^2Rt = 0.24 \times 36 \times 2 \times 180 = 3110.4 \text{ cal.}$$

The formula of Joule's Law can be rewritten as follows:

$$H = 0.24\ IIRt.$$

and since $IR = V$,

$$H = 0.24\ IVt \text{ cal.}$$

Example 3. How much heat will be produced in two hours by a current of 5 A at 120 V flowing through an electric range?

$$H = 0.24\ IVt = 0.24 \times 5 \times 120 \times 7200 = 1{,}036{,}800 \text{ cal} = 1036.8 \text{ kcal.}$$

46. Temperature Rise of Conductors due to Electric Current

Some of the heat produced in a conductor by the passage of an electric current will be given up to the surrounding medium (which may be solid, liquid or gaseous). The temperature of the conductor will rise until the amounts of heat imparted to and given up by the conductor balance.

The temperature rise of a conductor depends on the magnitude of the current flowing through it, the material and cross-sectional area of the conductor, and the cooling conditions. For a given current and material, the temperature rise of a conductor will be independent of its length—the longer a conductor, the greater its cooling area.

For a given material and cooling conditions, the temperature will increase with increasing current density in the conductor.

The largest possible current through a conductor depends on its maximum safe temperature. Above this value, the temperature rise is dangerous. Rubber- or cotton-covered wires, for example, cannot be heated to above 50°C, since this may injure their insulation. To avoid damage, the current density for a given conductor is chosen according to its cross-sectional area. Table 10 gives the maximum current densities for insulated copper wires and cables not buried in ground.

Table 10

Maximum Current Densities for Insulated Copper Conductors

Cross-sectional area, mm²	Current, A	Current density, A/mm²	Cross-sectional area, mm²	Current, A	Current density, A/mm²
0.75	13	17.4	50	192	3.8
1	15	15.0	70	242	3.5
1.5	20	13.3	95	292	3.1
2.5	27	10.8	120	342	2.8
4	36	9.0	150	392	2.6
6	46	7.7	185	450	2.4
10	68	6.8	240	532	2.2
16	92	5.7	300	614	2.0
25	123	4.9	400	737	1.8
35	152	4.3			

Table 11

Current Densities for Bare Conductors

Cross-sectional area, mm²	Indoors		Outdoors	
	current, A	current density, A/mm²	current, A	current density, A/mm²
4	57	14.2	58	14.5
6	73	12.2	76	12.6
10	103	10.3	108	10.8
16	130	8.1	150	9.4
25	165	6.6	205	8.2
35	210	6.0	270	7.7
50	265	5.3	335	6.7
70	340	4.8	425	6.1
95	410	4.3	510	5.4

Referring to the table, the current density decreases with increasing cross-sectional area. This is because conductors of small cross-section give up some of their heat to the surroundings while the internal material of conductors of large cross-section can only give up their heat to the outer material already heated.

Noninsulated (or bare) conductors are better cooled and can carry higher current densities (Table 11).

As can be seen, while an insulated copper wire of 25 mm² cross-section can carry a current of 123 A, an aluminium wire for the same current must have a cross-section 1.5 times as great, otherwise the conductor will grow dangerously hot due to the higher resistivity of aluminium.

The electric energy converted to heat in conductors is wasted. Therefore, every effort is made to keep heat losses in wires to 5-10 per cent of the total energy.

On the other hand, there are applications where the thermal effects of electric current are the primary objective. Some of these applications are outlined below.

47. Incandescent (Filament) Lamps

The incandescent lamp was invented by Lodygin of Russia who demonstrated his invention in 1873, i.e., well before Edison.

The incandescent lamp consists of a filament which is a highly refractory conductor mounted in a glass bulb. The filament is heated by the passage of an electric current through it to such a high temperature that it becomes incandescent (white-hot) and emits light.

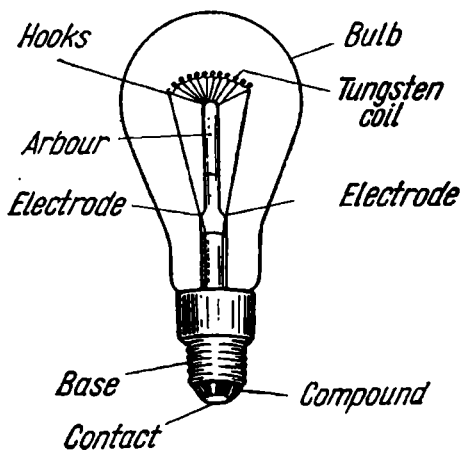


Fig. 67. Incandescent lamp

To prevent rapid burning of the filament, the air is exhausted from the glass bulb, to form a partial vacuum, hence the name vacuum lamp.

Originally, incandescent lamps used a carbon filament made from vegetable fibres by charring. Carbon-filament lamps produced a weak yellowish light and used up much electricity. Heated to 1700°C, the carbon filament gradually burned, and the service life of such lamps was short. Now carbon-filament lamps are no longer in use.

At present, the filament is made of refractory metals such as osmium or tungsten (Fig. 67). Heated to 2200°C in vacuum lamps, the tungsten filament emits a brighter light and consumes less energy than did the carbon filament.

To retard the evaporation of the filament, the bulb of the lamp is filled with an inert gas (nitrogen or argon) which does not affect the filament chemically. This type is known as the gas-filled lamp. In gas-filled lamps, the filament is heated to 2800°C.

In the Soviet Union, general service lamps are made for voltages of 36, 110, 127 and 220 V. Specials may be made for other voltages.

The incandescent lamp has a very low luminous efficiency, only about 4 or 5 per cent of the electric energy consumed being converted to light while the remainder is lost as heat.

Another group of lamps includes gaseous discharge lamps now widely used for decorative purposes, advertising, signs, etc. For its operation, a gaseous discharge lamp depends on the luminescence of a rarefied gas or vapours during the passage of an electric current through it. This principle is employed in mercury (high-pressure) vapour, sodium vapour and neon lamps. The term "neon tubes" is loosely applied to several different types. In point of fact, only one type of tube can be strictly called neon, because the others employ other gases or vapours. Neon produces a red-orange colour, argon blue-violet, and helium ivory white or yellow. Commercial "neon" lamps are made to operate on high-voltage alternating current supplied by suitable transformers.

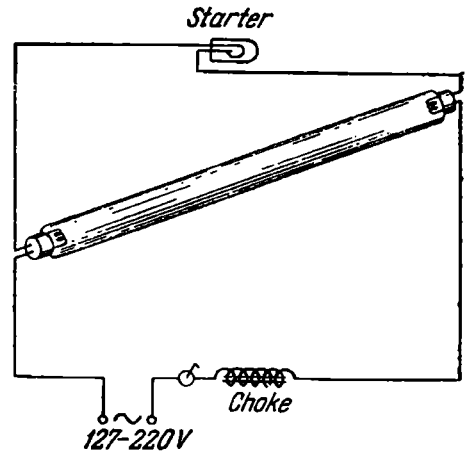


Fig. 68. Connection of a fluorescent lamp in a circuit

Still another class of lamps embraces fluorescent lamps. A fluorescent lamp is a quartz tube filled with low-pressure mercury vapours. The passage of an electric current through it causes the mercury vapours to emit strong invisible ultraviolet radiation. The inside of the tube is coated with a fluorescent powder, or phosphor, which converts the ultraviolet radiation of the mercury discharge into visible light.

The three commercial types of fluorescent lamps available in the Soviet Union are: ДС (daylight) lamps for use where good colour rendition is essential (print shops, textile mills, etc.); БС (white) lamps for industrial, office and residential interiors; and ТБ (warm white) lamps for museums, theatres and art galleries. The luminous efficiency of fluorescent lamps is four times as great as that of ordinary incandescent lamps.

Fig. 68 shows a circuit for a fluorescent lamp.

48. The Electric Arc

An electric arc is an electric discharge in gases accompanied by high heat and a bright glow.

An electric arc is formed when two conductors (electrodes) are brought together to make an electric contact and then separated. This introduces a considerable resistance to the flow of current, and the tips of the electrodes and the air gap are raised to a high temperature (Fig. 69).

Fig. 69. An electric arc

In Russia, the electric arc was obtained for the first time by Petrov in 1802.

Arc electrodes are usually made of carbon, because they do not melt at the high temperature of an arc discharge. Incidentally, the temperature of an arc may be as high as 3500°C and will melt most high-refractory metals.

In operation on direct current the positive electrode will wear out quicker than will the negative. To make up for this difference, positive electrodes are made twice as thick as negative electrodes. In operation on alternating current electrodes of equal size are used. Adding some metal salts to the electrodes alters the colour of the arc.

As the tips of the electrodes burn off, the distance between them increases. To keep the arc alive, the elctrodes must be continuously fed towards each other. This may be done either manually (as in projection lanterns) or automatically (as in floodlights and searchlights).

In Russia, the first automatic arc, voltage and current regulators were developed by Chikolev (1845-1898).

For a steady and quiet arc, the voltage should be 40 to 60 V at 10 to 12 A or higher. Where the energy source has a higher voltage or current, a series resistor is necessary to limit them.

Example 4. Determine the series resistor that should be used for the arc lamp of a motion-picture projector which operates on 50 V at 12 A? The generator voltage is 110 V (Fig. 70).

The voltage drop across the series resistor will be

$$110-50 = 60 \text{ V}.$$

With a current of 12 A, the desired voltage drop will be produced across a resistor with the following resistance:

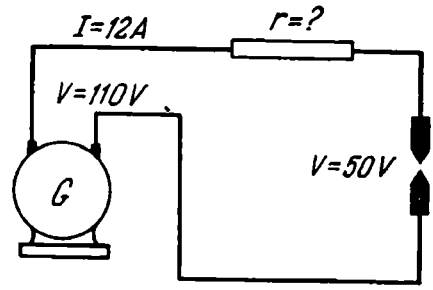


Fig. 70. See Ex. 4

$$R = V/I = 60 \div 12 = 5 \text{ ohms}.$$

The electric arc was first used for illumination by Yablochkov of Russia (1847-1894), who developed what was called an "electric candle".

Yablochkov's candle consisted of two carbon electrodes set up side by side and separated by a piece of burned kaolin. As the electrodes burned down, the partition melted, leaving the separation between the electrodes constant.

The strong brightness and great luminous flux of the electric arc have made it an efficient light source of searchlights and floodlights capable of illuminating objects at distances up to several kilometres.

The heat generated by the electric arc is utilized in arc furnaces for melting highly refractory materials.

49. Electric Welding

All electric welding processes may be classed into two broad systems: arc welding and resistance welding.

Arc welding was invented by Bernardos of Russia in 1882. In the Bernardos process, an electric arc is drawn between the metal to be welded (which is on one side of the welding circuit) and a carbon electrode (which is on

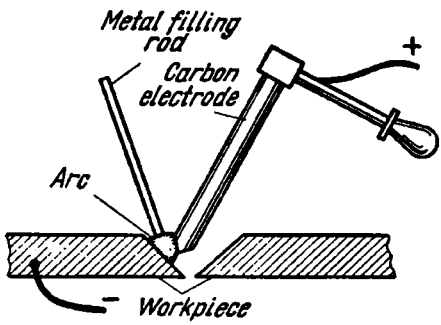


Fig. 71. The Bernardos (carbon-electrode) welding process

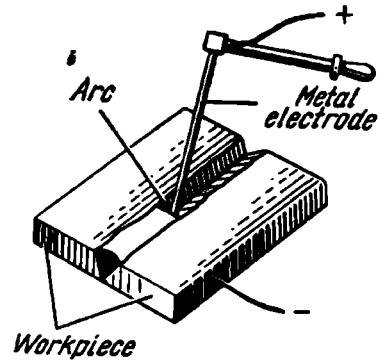


Fig. 72. The Slavianoff (metal-electrode) welding process

the other side of the circuit) (Fig. 71). The metal is fused by the high temperature of the arc, and filling material is provided by a metal filling rod, the end of which is held and melted in the arc. On solidifying, the filler metal builds up a weld.

In 1891, another Russian engineer, Slavianoff patented a process of arc welding which is a modification of the Bernardos process. Instead of using a carbon electrode, the electrode is of the same material as the metal to be welded (Fig. 72). The electrode melts, thereby supplying the filling metal to build up the weld. When an electrode is consumed it is replaced by a new one.

Welding electrodes are often made with a coating of suitable mixtures. These coatings or coverings serve to facilitate the establishment and maintenance of the arc and to obtain a stronger weld. In welding, the coating

material melts, producing a shield of gas or a blanket of slag on the molten metal which protects it from oxidation and other undesirable effects.

Resistance welding utilizes the heat generated by the resistance to the flow of an electric current passed directly through the workpiece. The heat generated at the point of contact between the parts to be welded reduces the metal to a plastic state; the pieces are then pressed together to complete the weld.

At the present time, both arc and resistance welding processes are widely used for production and repair purposes. They are applied to steel, cast-iron and nonferrous sheet, plate and shapes, to make pipes, towers, trusses, boilers, ships, etc.

In recent years, many new processes have been added to the art of welding. Among them are the underwater welding process developed by Khrenov and first used during the construction of the Saratov-Moscow gas pipeline; automatic submerged arc welding developed by E. Paton for high-speed mass-scale production; high-frequency, high-voltage welding developed by Nikitin for light and heavy sections.

Spark machining is another process related in a way to the electric arc. Spark machining is based on erosion by arcing. Outside of the production field, erosion by arcing often affects contacts. In the case of d-c circuits, erosion results in loss of material from one contact and the deposit of some of this material on the other. This action tends to eat away (hence the name "erosion" which comes from the Greek "erogo" for "gnaw away") one contact and build up the other. In the case of a-c circuits, both contacts are gradually eroded. As a result, contacts may "freeze" to each other, thereby upsetting the operation of the entire installation. The effect of erosion can be overcome by including a capacitor in the contact circuit.

In the Soviet Union, a spark-machining equipment has been developed by B. R. and I. N. Lazarenko. It works as follows. One side of a source is connected to a metal rod and the other side to the workpiece immersed in an oil bath. The metal rod is caused to vibrate. The arcing that sets up between the rod and the work eats away the metal, leaving a hole having the contour of the rod (which may be hexagonal, square, triangular, etc.).

50. Electric Heating Appliances

Electric heating appliances are widely used both in the home (electric heaters, ranges, kettles, boiling pans, electric irons, water heaters, etc.) and in industry (crucible and muffle furnaces, drying cabinets, etc.).

Every heating appliance is built around a heating element (or elements). The heating element (Fig. 73) consists essentially of a spiral of high-resistance alloy which generates heat when an electric current is passed through it. The spiral is mounted on a refractory support made of ceramics, asbestos or mica. Heating elements exposed to the atmosphere in operation are usually made of nichrome. Water-immersed heating elements are wound

with rheotan or nickeline wire. If turned on in the air, such elements will burn since the air will not be able to abstract heat as fast as water.

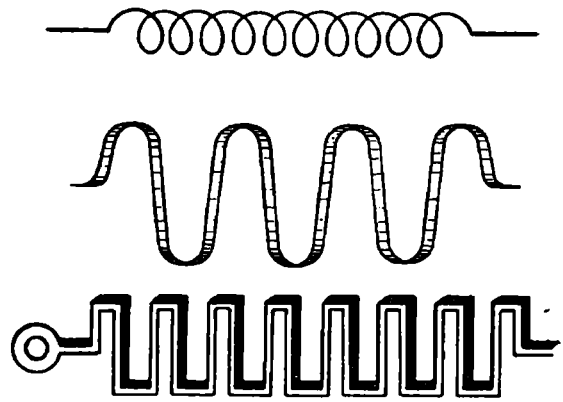


Fig. 73. Heating elements

51. Thermal Relays

The thermal type of relay usually employs a bimetal strip as the operating mechanism. A bimetal is produced by firmly bonding together two metals or alloys, the composite product being rolled into strip.

When two similar metals are bonded together, they will expand by the same amount on heating (Fig. 74*a*). If, on the other hand, a strip of two

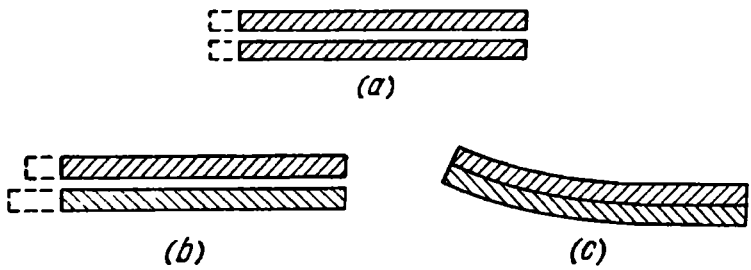


Fig. 74. Effect of heating on metal strips

dissimilar metals (Fig. 74*b*) is heated, the metals will expand differently. Bimetal strips are usually made of invar (an alloy of iron and nickel) and brass. On heating, a bimetal strip will bend on the side of the metal having the lower coefficient of expansion (Fig. 74*c*).

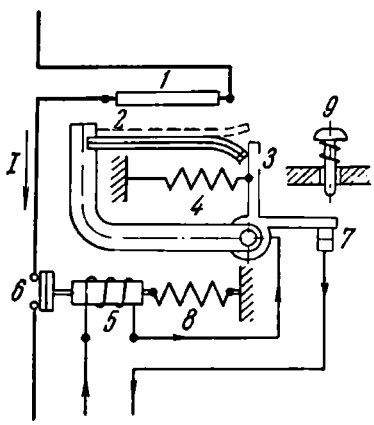


Fig. 75. Diagram of a thermal relay

Fig. 75 is a diagrammatical representation of a thermal relay using a bimetal strip.

Pressure on the START button energizes a solenoid *5* which closes the main contacts *6* of the motor, and the latter starts running. The heater coil *1* carrying the motor current is placed near a bimetal strip *2* so as to heat the latter indirectly. The bimetal strip and heater are so chosen that the amount of heat dissipated under normal conditions will not cause the bimetal strip to flex.

When an overload causes sufficient heating of the coil *1*, the bimetal strip flexes upwards at its outer end to release the catch *3*, which it holds locked under normal conditions. Pulled by the spring *4*, the catch will turn to break the trip contacts *7*, thereby de-energizing the solenoid and causing the main contacts to break too. The motor will stop. The relay is hand-reset after tripping, by rotating the RESET knob *9* which will rotate the catch and close the trip contacts. This may be done a half minute to three minutes after the tripping, when the bimetal strip has cooled off and has returned to lock the catch. It is only then that pressure on the start button will start the motor again. The motor can be stopped at will by pressing the STOP button.

52. Fuses

A fuse is a length of low-melting wire or of strip (copper, lead or silver) placed in a circuit to melt when the current in it has reached a predetermined value.

There are generally three types of fuses: (1) plug (Fig. 76*a*); (3) cartridge (76*c*); and (2) open-link (76*b*).

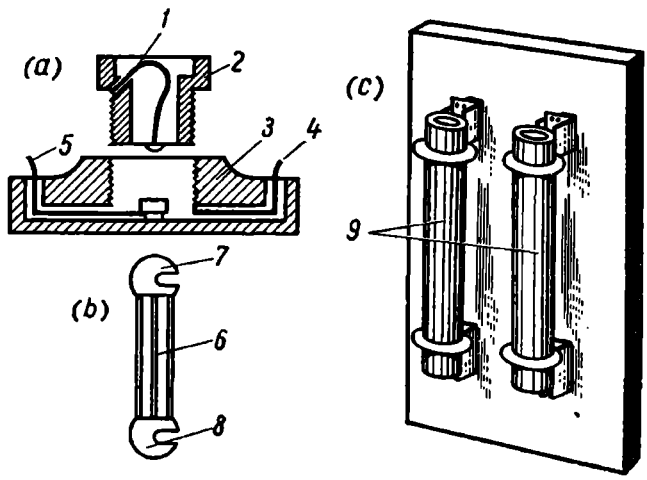


Fig. 76. Fuses

In a plug fuse, the fuse link *1* is placed inside a porcelain plug *2* which is screwed or inserted into a receptacle *3*, to which the leads *4* and *5* of the circuit to be protected are connected.

In an open-link fuse, the ends of the fuse link *6* are fastened to terminals for convenience in connection to a base.

In a cartridge fuse, the fuse link is placed inside an easily removable porcelain container *9* so that the link can be renewed when blown.

Fuses are chosen so that any increase in the circuit current above the safe limit will blow the link, thereby opening the circuit, before the wires become dangeronsly hot.

Current ratings of fuses for rubber-insulated wires laid in a common conduit, and also for two- and three-core rubber-covered cables and tubular wires will be found in Table 12.

Table 12

Current Ratings of Fuses

Cross-sectional area of wire, mm²	Current rating of fuses, A	Cross-sectional area of wire, mm²	Current rating of fuses, A
1	10	25	100
1.5	10	35	125
2.5	20	50	160
4	25	70	200
6	35	95	225
10	60	120	260
16	80	150	300

53. Jointing of Conductors

Poor joints offer an increased resistance to the flow of current. The resulting heat dissipation may be great enough to burn or melt the defective joint. This is why utmost care must be taken in jointing conductors.

The ends of wires to be jointed should be thoroughly cleaned, tinned, twisted, and the joint soldered. In the case of copper, aluminium or iron busbars, their ends are cleaned of rust and oxides with a file and emery paper. The ends are overlapped and bolted reliably together. The joint is then given a coating of petroleum jelly to protect it from oxidation.

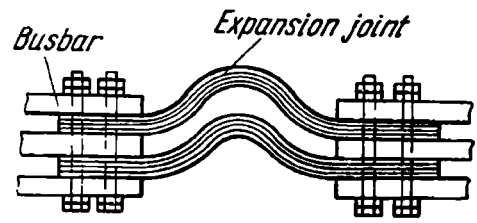


Fig. 77. An expansion joint

On long straight busbars it is customary to employ expansion joints spaced at 15 m in the case of aluminium and at 25 m in the case of copper. An expansion joint is made of the same material as the busbars but in separate strips 0.4 to 1 mm thick (Fig. 77). Expansion joints take up the stresses set up in busbars by heating, thereby preventing the busbars from distortion or bending.

54. Thermo-EMF Thermocouples

An effect known as the thermoelectric effect is that by which an emf is developed due to a difference of temperature between two junctions of dissimilar conductors in the same circuit.

To prove it, solder two dissimilar conductors at one end, connect their opposite ends to a galvanometer, and heat the junction. The galvanometer will deflect, showing the presence of a current. The two different conductors form what is known as a *thermocouple*. The current flowing in the circuit is called *thermoelectricity*, and the emf which causes it to flow is referred to as *thermo-emf*.

The value of thermo-emf is small and approximately proportional to the difference in temperature between the junction and the surroundings. Table 13 gives the emfs of common thermocouples for a temperature difference of 100°C between the hot and cold junctions.

Table 13

EMF of Thermocouples

Junction	Emf, V
Antimony/bismuth	−0.011
Copper/iron	+0.001
Copper/constantan	−0.0047
Silver/constantan	−0.0041
Silver/nickel	−0.0024
Platinum/90% Pt-10% Rh	−0.001

The "+" sign indicates that the current in the hot junction flows from the first to the second metal.

The most important application of thermocouples is the measurement of temperature in inaccessible places. A thermocouple-based thermometer consists of two wires (say, platinum and Pt-10% Rh) placed inside a porcelain sheath. The "hot" junction is placed where the temperature is to be taken, and the "cold" junction is connected to the terminals of a galvanometer calibrated to read degrees of temperature. Thermocouples can accurately take both elevated (2000° and higher) and low temperatures. Their accuracy is greater than that of any other type of thermometer.

Exercises

1. How much heat will be liberated in one hour by a current of 5 A flowing through an electric iron with a 24-ohm spiral?
2. How much heat will be dissipated in a conductor of 3.5 ohms resistance in half an hour if the conductor is connected across a storage cell of 2 V emf and 0.5 ohm internal resistance?
3. How much heat will be liberated by a 250-ohm lamp and a 100-ohm lamp during 3 minutes if they operate at 100 V and are connected (a) in series and (b) in parallel?
4. How much heat will be generated in 10 minutes by the spiral of an electric plate wound with nichrome wire 10 m long and 0.5 mm^2 in cross-section and operating on 110 V?
5. How much heat per second will be generated in each of three resistors of 2, 3 and 6 ohms, respectively, connected in parallel if the total current is 18 A?
6. How much heat will be generated by a heater wound with nickeline wire 10 metres long and 0.2 mm^2 in cross-section, connected across an energy source of 20 V emf and 0.2 ohm internal resistance?
7. How much nickeline wire 0.5 mm^2 in cross-section must be taken to wind an electric plate capable of raising 2.5 litres of water from 10°C to boiling in 40 minutes if the heat losses by the plate, kettle and air are 40 per cent? The plate will operate on 120 V.

Review Questions

1. State Joule's Law and give the formula.
2. What is the number 0.24 in the formula of Joule's Law?
3. What are the practical applications of the thermal effects of electric current?
4. What are fuses used for and how are they constructed?
5. What is the construction of thermal relays and where are they employed?
6. What is electric welding?
7. What is current density and why should it be reduced for conductors of large cross-section?
8. Why is a current-carrying conductor heated?
9. What is thermoelectricity and where is it utilized?

Chapter Five

ELECTROMAGNETISM

5. The Magnetic Field of a Straight Isolated Wire Carrying a Current

As was stated in Chapter I, the electromagnetic field is the result of the interplay of an electric and a magnetic field. Now we shall take up the magnetic field in more detail.

A compass needle brought close to a long straight isolated wire carrying an electric current will tend to align itself at right angles to the plane passing through the axis of the wire and the pivot of the needle. This is an indication that some force is acting upon the needle. This force is termed the magnetic field. The magnetic field also exerts force on electric charges in motion and current-carrying conductors placed in the field. The magnetic field induces an emf in current-carrying conductors cutting across the field or in stationary conductors when they are placed in a varying field.

Summing up, we may define *the magnetic field as a component of the electromagnetic field. It is set up by electric charges in motion or by a variable electric current and is described by the force it exerts on charges in motion and, consequently, on electric currents.*

Thus, the magnetic field is a consequence of electric current and cannot be obtained independently of the latter.

The magnetic field can be mapped by applying iron filings to a piece of pasteboard through which a heavy current-carrying conductor is passed (Fig. 78). The filings will form circles representing lines of magnetic induction. Moving the pasteboard up or down the conductor will not change the arrangement of the lines of magnetic induction. This proves that the magnetic field is set up along the whole length of the conductor.

The direction of lines of magnetic induction will, however, change if the direction of the current through the conductor is changed (Fig. 79). This can be demonstrated by arranging small compass needles around the

conductor and alternately reversing the current through the conductor — the needles will rotate differently.

The properties of the lines of magnetic induction around a current-carrying wire may be summed up as follows: (1) lines of magnetic induction are circles symmetrical about, and concentric with, the axis of the wire; (2) the spacing between the lines of induction decreases as we move closer to the conductor;

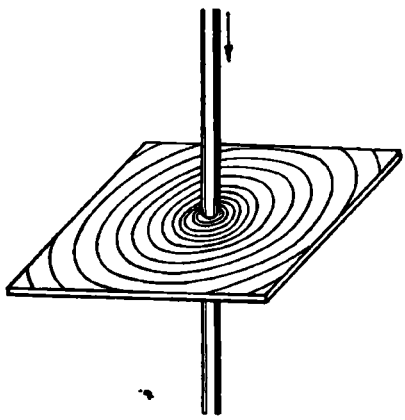


Fig. 78. Magnetic field around a current-carrying conductor

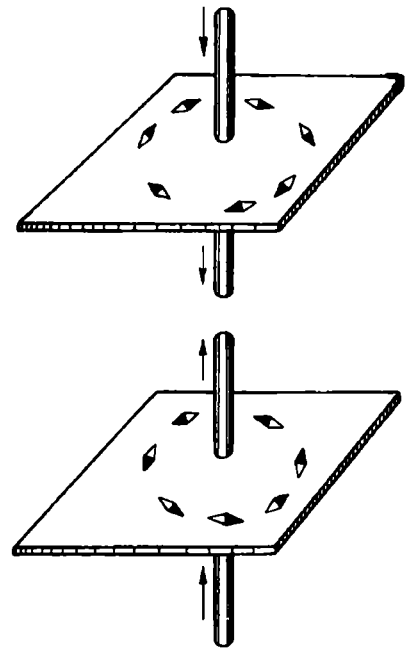


Fig. 79. Direction of lines of magnetic induction

(3) magnetic induction (or flux density) depends on the magnitude of the current in the wire; (4) the direction of the lines of magnetic induction depends on that of the current in the wire.

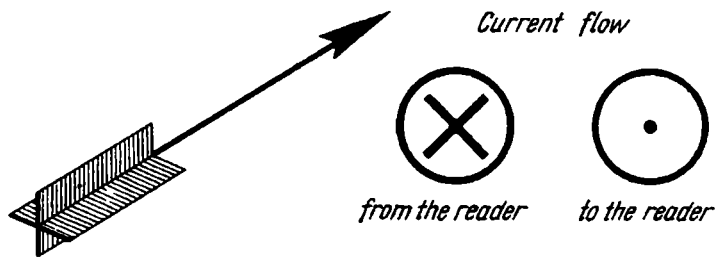


Fig. 80. Symbols for the direction of current i n a conductor

Graphically, the direction of the current in a conductor is shown by a cross if the current flows away from the reader and by a dot if the current flows towards the reader (Fig. 80), the conductor being shown in cross-section as a circle.

The direction of the lines of magnetic induction around a conductor can be determined by what is known as the right-hand screw rule. If a right-hand screw is moved progressively in the direction of the current, the rotation of its handle will give the direction of the lines of magnetic induction around the conductor (Fig. 81). The same rule applies to a compass needle placed in the field of a current-carrying conductor (Fig. 82).

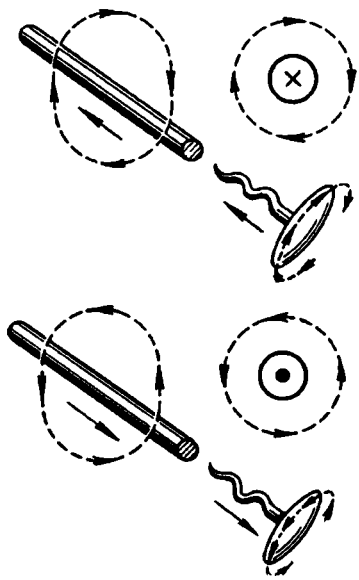


Fig. 81. Application of the right-hand screw rule to a current-carrying conductor

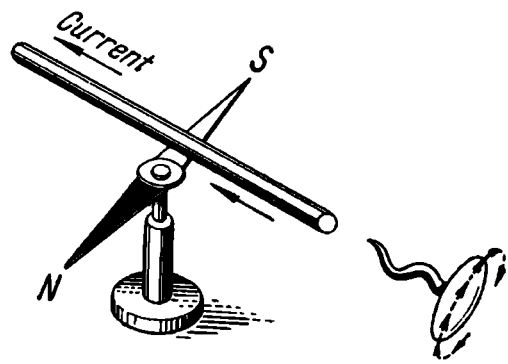


Fig. 82. Application of the right-hand screw rule to a compass needle

The magnetic field is described by a vector of magnetic induction and therefore has both direction and magnitude.

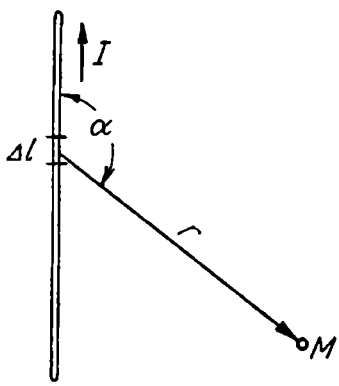


Fig. 83. The Biot-Savart Law

The quantitative expression for magnetic induction (or flux density) was derived experimentally by Biot and Savart (Fig. 83). Taking the deflection of a compass needle as a measure of the intensity of magnetic fields of a current, varying in magnitude and shape, the two scientists concluded that any current element projects into space a magnetic field, *the magnetic induction (flux density) of which, dB, is directly proportional to the element of length, dl, the current I, the sine of the angle α between the direction of the current and the vector, R, connecting a given point of the field and the current element, and is inversely proportional to the square of the length of the vector*, or

$$dB = K\frac{I\,dl\sin\alpha}{R^2},$$

where K is the proportionality factor dependent on the magnetic properties of the medium and the system of units used.

In the rationalized m.k.s.a. system

$$K = \mu_r \mu_0 / 4\pi,$$

where μ_0 is the permeability of free space (vacuum) in the m.k.s.a. system, or

$$\mu_0 = 4\pi 10^{-7} \text{ henries/metre},$$

where μ_r is *relative permeability*, a dimensionless quantity giving the ratio of the permeability of a given material to that of free space.

The dimensions of magnetic induction can be found from the equation

$$[dB] = \left[\frac{\mu_r \mu_0 Idl \sin \alpha}{4\pi R^2}\right] = \frac{\text{ohm} \times \text{sec}}{\text{m}} \times \frac{\text{Am}}{\text{m}^2} = \frac{\text{V sec}}{\text{m}^2}.$$

One volt-sec is equal to one weber. Therefore

$$B = \text{Wb/m}^2.$$

In practice, use is made of the gauss—a smaller unit of magnetic induction:

$$1 \text{ Wb/m}^2 = 10^4 \text{ gauss}.$$

The Biot-Savart Law gives the magnetic induction of an indefinitely long straight wire carrying a steady current:

$$B = \mu_r \mu_0 \frac{I}{2\pi a},$$

where a is the distance from the wire to the point where the magnetic induction is measured.

The ratio of magnetic induction to the product of permeabilities, $\mu_r \mu_0$, is termed *magnetic field strength* (or magnetizing force, or magnetic field intensity) and is designated by the letter H:

$$H = \frac{B}{\mu_r \mu_0}$$

or

$$B = H \mu_r \mu_0.$$

The dimensions of H are

$$[H] = \left[\frac{B}{\mu_r \mu_0}\right] = \frac{\text{V} \times \text{sec} \times \text{m}}{\text{m}^2 \times \text{ohm} \times \text{sec}} = \text{A/m}.$$

Sometimes use is made of another unit of magnetic field strength called the oersted:

$$1 \text{ oersted} = 79.6 \text{ A/m} = \text{approx. } 80 \text{ A/m} = 0.8 \text{ A/cm}.$$

Like magnetic induction B, magnetic field strength H is a vector quantity.

Now we can define a line of magnetic induction as follows: *this is a line drawn in a magnetic field such that its direction at every point is the direction of the magnetic flux at that point.*

The product of magnetic induction and the area normal to the direction of the field (or the vector of magnetic induction) is known as the *magnetic flux* and is designated by the letter Φ:

$$\Phi = BA.$$

The dimensions of magnetic flux are

$$[\Phi] = [BA] = \frac{\text{V} \times \text{sec}}{\text{m}^2}\,\text{m}^2 = \text{V-sec, or the weber.}$$

A smaller unit of magnetic flux is the maxwell:

$$1 \text{ weber} = 10^8 \text{ maxwells}$$

$$1 \text{ maxwell} = 1 \text{ g(f)} \times 1 \text{ cm}^2.$$

56. The Magnetic Field in Air due to a Closed Circular Conductor

If a conductor carrying a current is bent in the form of a ring (Fig. 84), the lines of magnetic induction around it will again be concentric circles, leaving the plane of the ring, or loop on one side and entering it on the other.

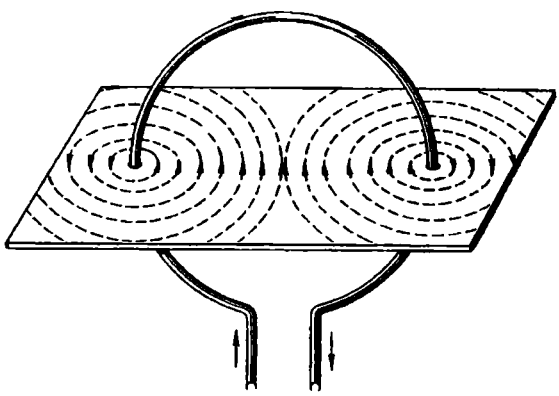


Fig. 84. Circular conductor carrying current

The magnetic field due to a closed circular conductor resembles that of a very short bar magnet whose axis coincides with a perpendicular to the middle of the plane of the loop. Its direction can be determined by the right-hand screw rule. The screw should be placed along the axis of the loop and at right angles to its plane. If the screw is rotated in the direction of the current in the loop, the direction of its progressive motion will give the direction of the magnetic field. The strength of the magnetic field in the centre of a loop is given by

$$H = I/2R;$$

its magnetic induction is

$$B = \mu_r \mu_0 I/2R = \mu_r \mu_0 I/D,$$

where R is the radius of the loop and D is its diameter.

57. Solenoids and Electromagnets

A *solenoid* is a coiled wire carrying an electric current (Fig. 85*a*).

Let us imagine a solenoid cut into two halves lengthwise and the direction of the current through the solenoid turns designated as given earlier. Then the magnetic field of the solenoid, as determined by the right-hand screw rule, will have the shape shown in Fig. 85*b*.

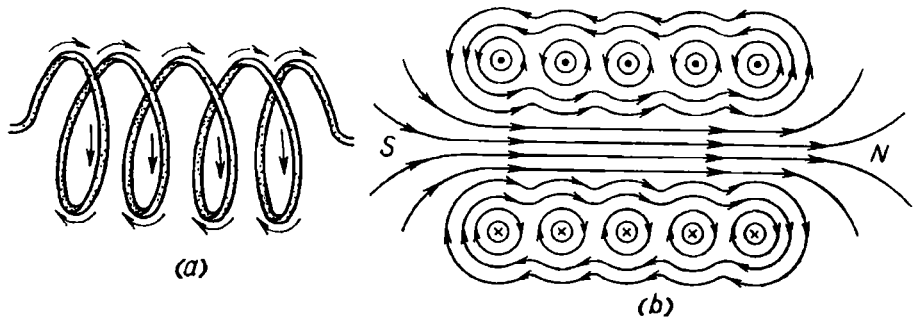


Fig. 85. A solenoid and the magnetic field around it

The strength of the magnetic field on the axis of a very long solenoid with n_0 turns per unit length will be

$$H = In_0.$$

The lines of magnetic induction enter a solenoid at the south pole and leave it at the north pole.

Fig. 86. Determining the poles of a solenoid

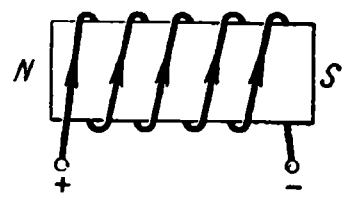


Fig. 87. An electromagnet

The poles of a solenoid can be identified by the right-hand screw rule as follows. The screw is placed along the axis of the solenoid and rotated in the direction of the current in the solenoid turns: the progressive motion of the screw will indicate the direction of the magnetic field (Fig. 86).

A solenoid having a steel (or iron) core inside it is called an *electromagnet* (Fig. 87). An electromagnet develops a stronger magnetic field than does a

solenoid, because the core is magnetized. The poles of an electromagnet can be determined in exactly the same way as those of a solenoid.

Electromagnets are widely employed to produce magnetic fields in electric generators, motors, instruments, etc.

Instead of fuses, protection of big electric installations is provided by automatic oil or air circuit-breakers. They are tripped by a variety of relays. A *relay* is a device operating in response to variations in current, voltage, power, frequency or some other electric variable.

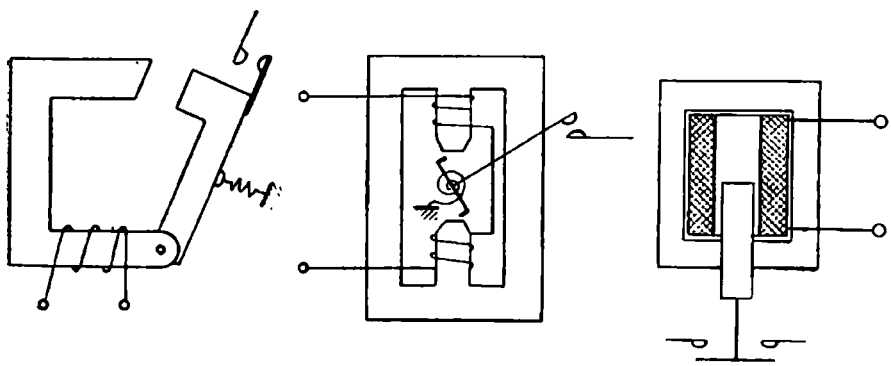


Fig. 88. An electromagnetic relay

Here is a brief outline of an electromagnetic relay (Fig. 88). For their operation, electromagnetic relays depend on the interaction of the magnetic field set up by a fixed coil carrying an electric current and a movable steel armature. If there is any change in the main current circuit, the relay coil is energized and pulls the armature. The armature rotates or is pulled in to close the contacts in the tripping coil of the oil or air circuit-breaker or pilot relay.

Relays are also widely used in automatic and remote control.

The magnetic flux of a solenoid (or electromagnet) increases with increasing turns and current. The product of current and turns (or ampere-turns) of a solenoid is known as the magnetomotive force (mmf) and is designated by the letter *F*.

A solenoid carrying a current of 5 A and having 150 turns will have an mmf of $5 \times 150 = 750$ ampere-turns. The same mmf will be produced by a solenoid of 1500 turns carrying a current of 0.5 A: $0.5 \times 1500 = 750$ ampere-turns.

The mmf of a solenoid can be increased in several ways: (1) by inserting an iron core, thereby turning it into an electromagnet; (2) by increasing the cross-sectional area of the core in the case of an electromagnet; (3) by decreasing the air gap of the electromagnet (this will reduce the reluctance, or magnetic resistance, of the magnet).

58. Ampere's Law. Magnetic Ohm's Law

Let a closed path l link a current loop (Fig. 89) like two links of a chain. The current flowing around the loop establishes a magnetic field around it. The field strength H at the point A on the path l will be given by its vector. If the path links several current loops, as many vectors will have to be

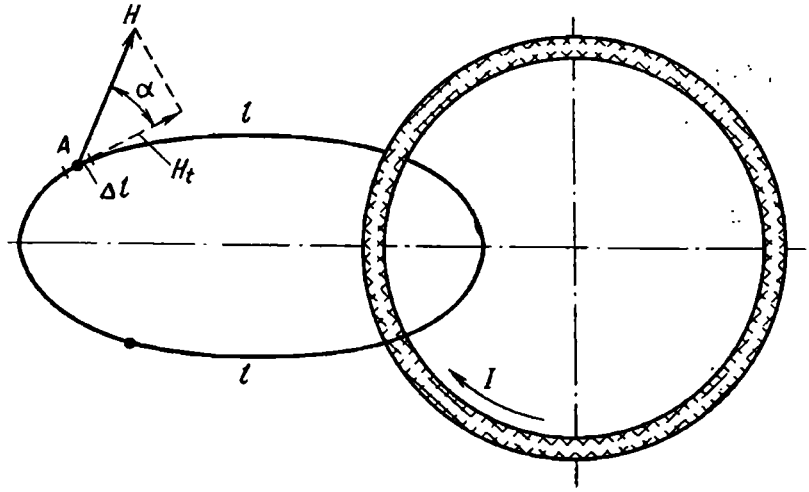


Fig. 89. Ampere's Law

drawn at a given point on the path. Adding the individual vectors geometrically will give the vector of the resultant field strength.

In a general case, the vector of the resultant field strength H will make an angle α with an element of length dl. Therefore, the longitudinal, or tangential, component of the resultant field strength H will be

$$H_t = H \cos \alpha.$$

In the case of a nonuniform magnetic field, the path can be divided arbitrarily into n elements of length dl, along which the field strength is deemed constant. Then the sum of the products of all the elemental paths and the tangential components will be

$$H_1 dl_1 \cos \alpha_1 + H_2 dl_2 \cos \alpha_2 + \ldots + H_n dl_n \cos \alpha_n,$$

or

$$\sum_{k=1}^{k=n} H_k dl_k \cos \alpha_k,$$

where $\sum_{k=1}^{k=n}$ indicates that the summation is over values from $k=1$ to $k=n$.

Electromagnetic theory proves that the summation of the field strength H round a closed path is equal to the total current flowing across the surface bounded by the path, or

$$\sum H_k dl_k \cos \alpha_k = \Sigma I.$$

This expression is one of the forms given to *Ampere's Law* (which is also known in other forms).

Where the same current flows through the surface bounded by the path repeatedly as in the case of a coil having T turns, the total current will be

$$\Sigma I = TI.$$

If the closed path of the summation coincides with a line of magnetic induction, the field vector at any point in the path will be directed tangentially towards the element of length dl.

In such a case,

$$H_{tk} = H_k; \quad \alpha_k = 0; \quad \cos \alpha_k = 1,$$

and Ampere's Law may be rewritten as follows:

$$\sum_{k=1}^{k=n} H_k dl_k = \Sigma I.$$

If the value of H is the same at any point of the path and the sum of dl's around the path is l, Ampere's Law may be expressed thus:

$$Hl = \Sigma I.$$

Ampere's Law provides a basis for the calculation of magnetic circuits and makes it possible in some cases to determine readily the strength of a magnetic field.

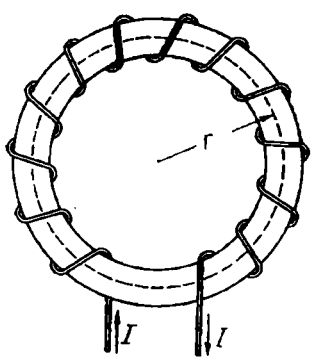


Fig. 90. Determinating the field strength of a toroid

In the case of a straight wire carrying an electric current the field strength, or magnetizing force at a distance a can be found from Ampere's Law as follows:

$$\Sigma I = I,$$

$$l = 2\pi a.$$

Therefore, $H2\pi a = I$,

whence $H = I/2\pi a$ (A/m).

The very same expression is deducible from the Biot-Savart Law.

In the case of a toroid (ring solenoid) (Fig. 90), the field strength can again be found from Ampere's Law. The path in this case is a circle with a radius r. The path links T turns carrying currents of the same direction:

$$H2\pi r = IT.$$

Taking the length of the centre line of the toroid equal to $l=2\pi r$ gives

$$Hl = IT,$$

whence

$$H = \frac{IT}{l} \text{ A/m.}$$

Thus, the strength of the magnetic field due to a toroid is directly proportional to the ampere-turns, or magnetomotive force F, and inversely proportional to the length of the path of the magnetic flux.

The magnetic induction (or magnetic flux density) inside a toroid will be

$$B = \mu_r \mu_0 H = \mu_r \mu_0 \frac{IT}{l} \text{ Wb/m}^2.$$

If the cross-sectional area of a toroid is the same the whole length of it and is equal to A, and the magnetic induction B is known, the magnetic flux due to the toroid will be

$$\Phi = BA = \mu_r \mu_0 HA = \mu_r \mu_0 \frac{ITA}{l} \text{ Wb.}$$

The expression may be rewritten as follows:

$$\Phi = \frac{TI}{l/\mu_r \mu_0 A}.$$

The product IT is known as the mmf, the expression $l/\mu_r \mu_0 A$ in the denominator gives the *reluctance* (designated R_m in the Soviet Union). Therefore, we may write

$$\Phi = F/R_m.$$

This expression resembles the familiar $I=E/R$ and is known as the magnetic Ohm's Law, or Bosanquet's Law.

59. Ferromagnetic, Paramagnetic and Diamagnetic Materials

On the basis of magnetic properties, all materials may be divided into three groups: ferromagnetic, paramagnetic and diamagnetic.

1. Ferromagnetic materials are those which are strongly attracted by a magnet. They include iron, steel, nickel, cobalt, the rare-earth element gadolinium and some of their alloys. Their relative permeability ranges from several hundred to several thousand (cobalt, 150; nickel, 300; iron, up to 5000; permalloy, up to 100,000).

2. Paramagnetic materials include those materials which are not strongly attracted by a magnet. These are aluminium, magnesium, tin, platinum, manganese, oxygen, etc. Their relative permeability is slightly higher than unity. Air, for example, has a permeability of 1.0000031.

3. Diamagnetic materials are slightly repelled by a magnet. These are zinc, mercury, lead, sulphur, copper, chlorine, silver, water, etc. Their relative permeability is less than unity. Copper, for example, has a relative permeability of 0.999995.

60. Ferromagnetic Materials in a Magnetic Field

The relationship between B and H for ferromagnetic materials is usually plotted graphically in the form of magnetization curves. The values of H in A/m, A/cm or oersteds are laid off on the axis of abscissas (the horizontal axis), and the values of B in Wb/m^2, Wb/cm^2 or gauss on the axis of

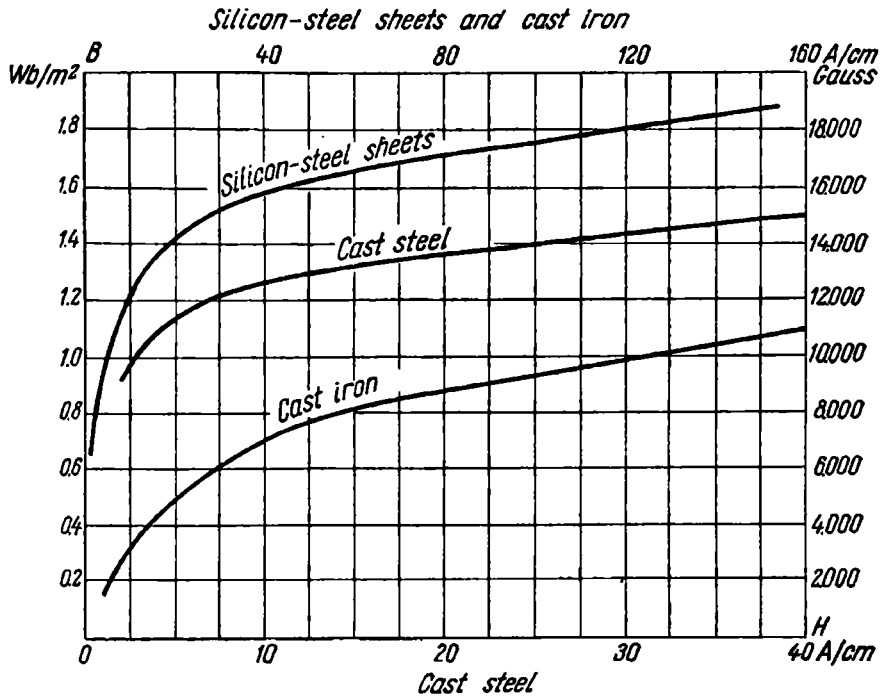


Fig. 91. Magnetization curves for electrical sheets, and cast steel and iron

ordinates (the vertical axis). Fig. 91 presents magnetization curves for silicon-alloy (electrical-sheet) steel, cast steel and cast iron. Referring to Fig. 91, magnetic induction B first increases rapidly with increasing field strength H as far as the knee on the curve, and then levels off. The flat part of the curve corresponds to magnetic saturation of the material. It will be noted that the relationship $B/\mu_0 H = \mu$ is not constant; it continuously varies with changes in B and H.

Among the factors that affect the permeability of a material are magnetic induction, chemical composition, past history (heat treatment and machining), and temperature. Permeability also depends on the shape and size of a given part.

Magnetization curves are obtained experimentally for every material and every grade of a material.

Some materials remain magnetized when the magnetizing force has been removed. This magnetization is called residual, and the flux density associ-

ated with it is called remanent flux density, or residual induction or residual magnetism. Low residual induction is displayed by pure iron, soft steel, silicon-alloy (electrical-sheet) steel, and permalloy (the name given to a group of iron-nickel alloys). These metals and alloys are readily magnetized and demagnetized. For this reason they go to make magnet cores, transformer circuits, pole pieces, generator armatures, etc.

The highest residual magnetization is displayed by steels alloyed with tungsten, chromium, cobalt, nickel and aluminium. They go to make permanent magnets.

61. Hysteresis

When a ferromagnetic material such as an iron core is placed in a coil carrying an electric current, it becomes magnetized. As the current, I, in the coil is increased, the magnetizing force H and the magnetic induction or flux density B also increase. This relation between H and B is shown by the so-called virgin or neutral curve Oa in Fig. 92. If, after it has reached its saturation value at a the magnetizing force H is reduced, the flux density B will also drop off. In doing so, however, it will lag behind H, so that at $H=0$ there will be some magnetism left in the core. This residual magnetism (or residual flux density, or remanence as it is variously called) is represented by Ob on the Y-axis and is due to a phenomenon called hysteresis which is caused by some irreversible processes.

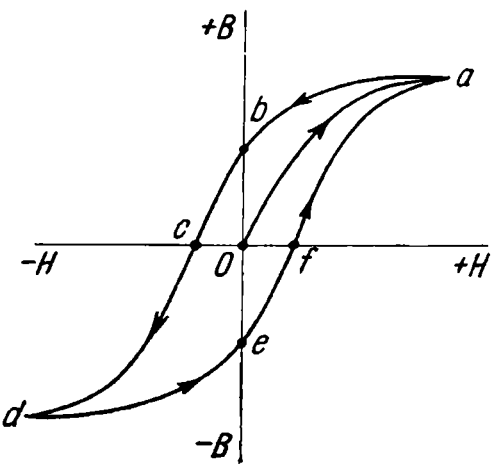


Fig. 92. Hysteresis loop

In order to reduce the residual flux density to zero, a negative magnetizing force, called the coercive force, H_c, must be applied. By the time $B=0$, the coercive force will have become $H_c=Oc$.

Referring to Fig. 92, it can be seen that the virgin curve is not retraced either in demagnetization or in repeated magnetization. Instead, a closed loop, *abcdefa*, called the hysteresis loop, is traced out.

Hysteresis is especially pronounced in materials of high residual magnetism, such as hard steel. In most cases, hysteresis is a liability. It results in dissipation of heat, waste of energy, and humming due to change of polarity and rotation of elementary magnets in the material.

62. Calculation of Magnetic Circuits

A magnetic circuit is a closed path of magnetic flux.

Fig. 93*a* shows a solenoid. In this case the magnetic circuit is through air. The reluctance of air is very high, and so the magnetic flux remains negligible even if the mmf is great.

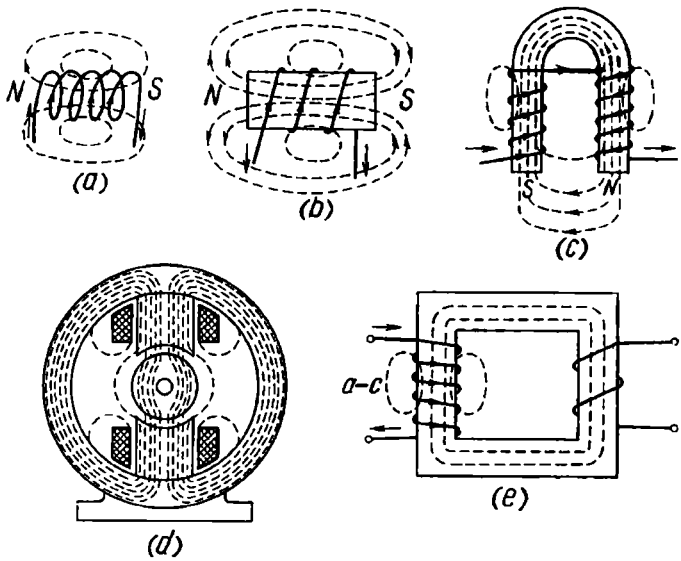


Fig. 93. Types of magnetic fields

To increase the magnetic flux, magnetic circuits include some ferromagnetic material (usually, cast steel or silicon-alloy steel) which have a lower reluctance.

Fig. 93*b* shows a straight electromagnet with an open core. Only a portion of the path is through the core, the remainder being through air.

A horseshoe-shaped electromagnet, such as shown at *c* in Fig. 93, provides a better path for magnetic flux. In this case, most of the magnetic flux is confined in the metal, the remainder being in air between the poles N and S.

At *d* in Fig. 93 is the magnetic circuit used in electric machines and instruments. A steel armature is placed between the poles of an electromagnet. The greater part of the path is through steel and only a small part is through air (2 or 3 mm).

Transformers use closed magnetic circuits of steel (at *e* in Fig. 93). Although magnetic circuits for transformers are built up of separate parts, care is taken during assembly to reduce the air gaps between the component parts as far as practicable.

Some of the magnetic flux produced by mmf fails to pass through the magnetic circuit. This part, known as the leakage flux, is shown at *b*, *c*, *d* and *e* in Fig. 93. Therefore, the total magnetic flux produced by the field

coil of an electromagnet is equal to the sum of the useful flux and the leakage flux.

It would seem that a magnetic circuit could then be calculated from the equation

$$\Phi = \frac{IT}{l/\mu_r\mu_0 A}.$$

The relative permeability, μ_r of ferromagnetic materials is, however, a variable dependent on many factors. Therefore, the above equation holds only when the magnetic circuit contains nonmagnetic materials (including air) with a predetermined value of relative permeability.

In practice, magnetic circuits are preferably calculated graphically.

The procedure is this. First, the desired magnetic flux is fixed. Then the magnetic circuit is divided into parts of constant cross-section and uniform material; for each part of the path the flux density is

$$B = \Phi/A.$$

Next, the value of H is found from magnetization curves for each value of B thus obtained. If there are air gaps in the magnetic circuit, the relation between B_0 and H_0 is determined from the formula

$$H_0 = B_0/\mu_0 = B_0 10^7/4\pi = 80 \times 10^4 B_0 \text{ (A/m)},$$

where B_0 is in Wb/m^2, μ_0 is in henries/m, and H_0 in A/cm.

If the flux density is in gauss and the magnetizing force in A/cm, the relation between B_0 and H_0 will be

$$H_0 = 0.8B_0.$$

With H for each part of the path found, the mmf can be determined by Ampere's Law:

$$IT = H_0 l_0 + H_1 l_1 \times H_2 l_2 + \ldots + H_n l_n.$$

Example. Determine the mmf of the electromagnet coil shown in Fig. 94 (all dimensions are in millimetres). The core is made of silicon-alloy steel. The required flux in the core is 60,000 maxwells. The leakage flux may be neglected.

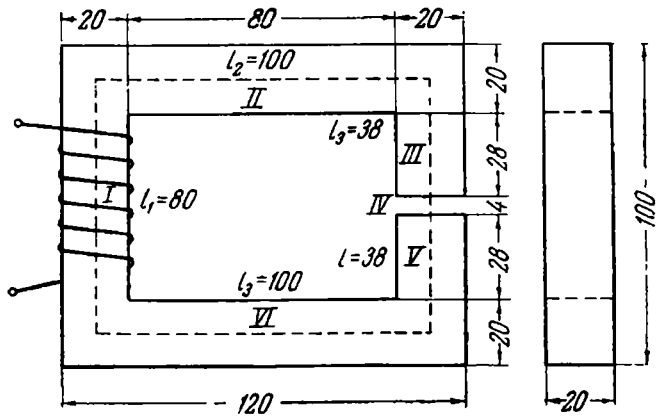


Fig. 94. Calculation of a magnetic circuit

Draw a centre line the whole length of the magnetic circuit. Divide the circuit into five parts and measure the length of each part.

Since the magnetic flux is deemed the same in each of the parts and the cross-section is also the same (2 cm square), the magnetic flux density will be constant throughout the circuit.

$$B = \Phi/A = 60{,}000 \text{ maxwells} \div 4 \text{ cm}^2 = 15{,}000 \text{ gauss.}$$

From the magnetization curve in Fig. 91 for silicon-alloy steel we find that a magnetic flux density of 15,000 gauss corresponds to a magnetizing force $H = 30$ A/cm. For the air gap we have

$$H_0 = 0.8 \times 15{,}000 = 12{,}000 \text{ A/cm.}$$

Multiplying the magnetizing forces by the lengths of the respective parts of the circuit will give the product Hl for these parts.

Now tabulate the results (Table 14):

Table 14

Part of circuit	Material	B, gauss	l, cm	H, A/cm	Hl, A
I	Silicon-alloy steel	15,000	8	30	240
II and VI	Ditto	15,000	10×2	30	600
III and V	Ditto	15,000	3.8×2	30	228
IV	Air	15,000	0.4	12,000	4800

$IT = \Sigma(Hl) = 5868$ ampere-turns.

It should be noted that while for the silicon-alloy-steel parts (I, II, III, V and VI) with a total length of 35.6 cm (8+20+7.6 cm) the desired magnetic flux will be produced by an mmf of 1068 ampere-turns (240+600+228 A), the air gap as little as 4 cm across (or 1/89 the steel circuit) will require an mmf of 4800 ampere-turns. This explains why it is essential to keep air gaps in magnetic circuits to a minimum.

63. Permanent Magnets

The discovery of the magnetic field around a wire carrying an electric current led to the observation that magnetic fields and electric currents produce similar effects. Later, it was established that every magnetic field was traceable to an electric current and not to concentrations of positive and negative magnetic masses at the north and south poles of a magnet, as had been thought before.

According to present-day concepts, the magnetic field of a magnetized ferromagnetic material (i.e., of a permanent magnet) is mainly due to the spin of the electrons about their own axes, which gives then the effect of a minute current-turn. In a demagnetized material the directions of the elementary current-turns and their magnetic fields are random and cancel one another so that no external field can be detected. The application of

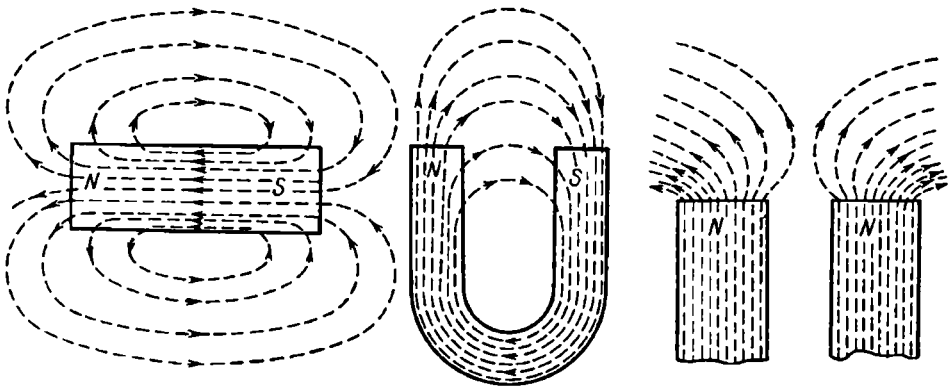


Fig. 95. The magnetic field of permanent magnets

an external field causes the parallel alignment of these current-turns and of their magnetic fields, thereby producing a resultant magnetic field.

Ferromagnetic materials possessing high residual magnetization are called *permanent magnets.*

If a permanent magnet is covered with a piece of pasteboard or glass sprinkled with iron filings, the filings will arrange themselves in curves extending from pole to pole (Fig. 95). These curves (lines of magnetic induction) are considered to emerge from the north pole and to enter the south pole of a magnet.

Lines of magnetic induction exhibit the follow ng properties:

(1) they form closed paths;

(2) inside a magnet they are directed from the south to the north pole;

(3) they tend to shrink, i.e., display longitudinal tension;

(4) they tend to act upon one another at right angles to their direction, i.e., display lateral expansion;

(5) they never intersect.

64. A Current-carrying Wire in a Magnetic Field

If a current-carrying wire is placed in a magnetic field (Fig. 96*a*), the addition of the magnetic fields of the magnet and wire will build up the resultant magnetic field on one side and reduce it on the other side (top and bottom of drawing). The two magnetic fields will bend the lines of induction, which will tend to shrink, thereby expelling the wire (Fig. 96*b*).

The direction of the force acting upon a current-carrying wire in a magnetic

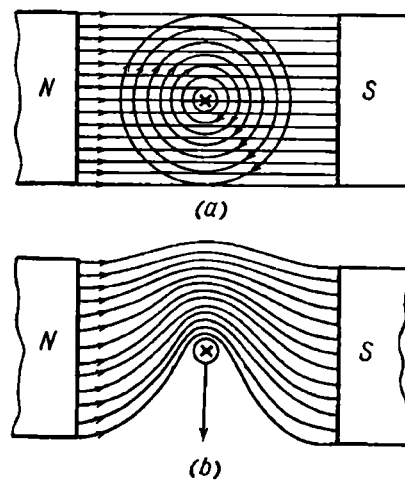


Fig. 96. Interaction of a current-carrying conductor and a magnetic field

field may be determined by the left-hand rule *by placing the palm so that the lines of induction leaving the north pole enter the palm, and the four fingers are in the direction of the current in the wire; then the thumb will give the direction of the force* (Fig. 97).

Referring to Fig. 98, the direction of the force acting upon a current-carrying wire can be changed by either reversing the poles, thereby reversing the magnetic field (at *a*, *b*, *c* and *d* in Fig. 98) or reversing the current in the wire (at *a* and *c* and also at *b* and *d*).

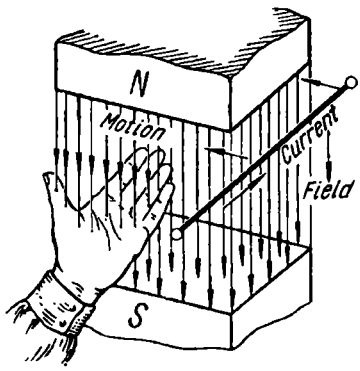


Fig. 97. Application of the left-hand rule

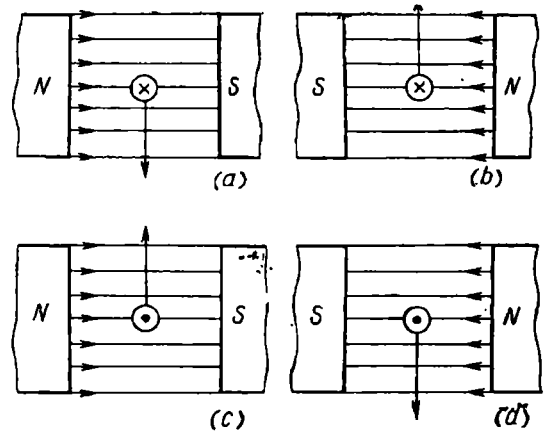


Fig. 98. Relationship between the force acting upon a current-carrying conductor placed in a magnetic field, the direction of the field, and the direction of the current in the conductor

If the poles and the current are both reversed, the direction of the force acting upon the wire will remain unchanged (at *a* and *d* and also at *b* and *c* in Fig. 98).

The force dF acting upon an element of length dl of the wire is proportional to the magnetic induction B, the current I, the element of length dl and the sine of the angle α between the direction of the element of length dl and the direction of the magnetic field. In symbols

$$dF = BIdl \sin \alpha.$$

For a finite straight wire placed at right angles to the direction of a uniform magnetic field, the force acting upon the wire will be

$$F = BIl.$$

This expression may be used in order to determine the dimensions of magnetic induction.

The dimensions of force are

$$[F] = \text{joule/m} = \text{watt-sec/m} = \text{VA-sec/m}.$$

Hence

$$[B] = \frac{\text{VA-sec/m}}{\text{A-m}} = \text{V-sec/m}^2,$$

or the same as deduced from the Biot-Savart Law.

65. Operating Principles of the Direct-current Motor

An electric motor is a machine which converts electrical energy into mechanical energy. In Russia, the first electric motor was constructed by Yakobi in 1838.

Any electric motor depends for its operation on the interaction of a current-carrying wire and a magnetic field. Fig. 99 gives a diagrammatic representation of a d-c motor.

The magnetic field is set up by poles *1* and *2* of an electromagnet. The wires that carry an electric current are placed in the slots of a steel barrel-shaped armature *3*.

When an electric current is allowed to flow through the wires in the upper half of the armature in an outward direction and in the lower half in an inward direction, the magnetic field will tend (by the left-hand rule) to push out the top wires to the left and the bottom wires to the right. Since the wires are firmly embedded in the armature slots, the force acting on them will rotate the armature.

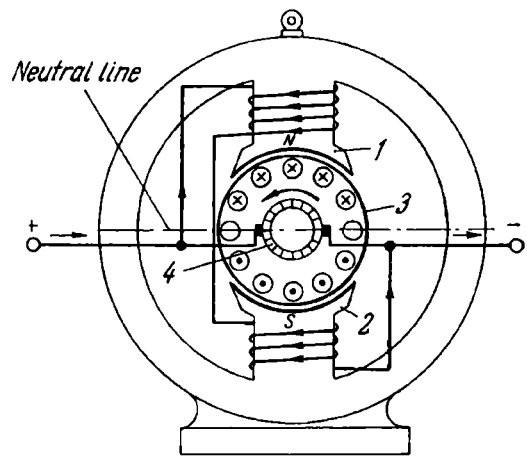


Fig. 99. Diagram of an electric motor

Referring to the figure, when the wires in which the current flows from the reader move down into a position opposite the south pole, they will be pushed to the left, and further rotation of the armature will be impeded. The same will happen when the wires in which the current flows towards the reader move up into a position opposite the north pole. To avoid this, the current should be reversed as soon as the respective wires have crossed the neutral line. In d-c motors this purpose is served by a commutator *4*, to which the leads from the armature winding are taken.

66. Magnetic Field due to Two Parallel Conductors

If two wires carrying electric currents in the same direction are placed close to each other, the lines of magnetic force enveloping the two wires will pull them together (Fig. 100*a*).

If the currents in two wires placed nearby flow in opposite directions, the magnetic lines between them will push the wires apart (Fig. 100*b*).

Let two parallel wires of length l be spaced a distance a apart. The magnetic induction due to the current I_1 on the second wire will be

$$B_1 = \mu_r\mu_0(I_1/2\pi a),$$

and the force acting on the second wire

$$F_2 = lI_2B_1 = (\mu_r\mu_0I_1I_2l)/2\pi a.$$

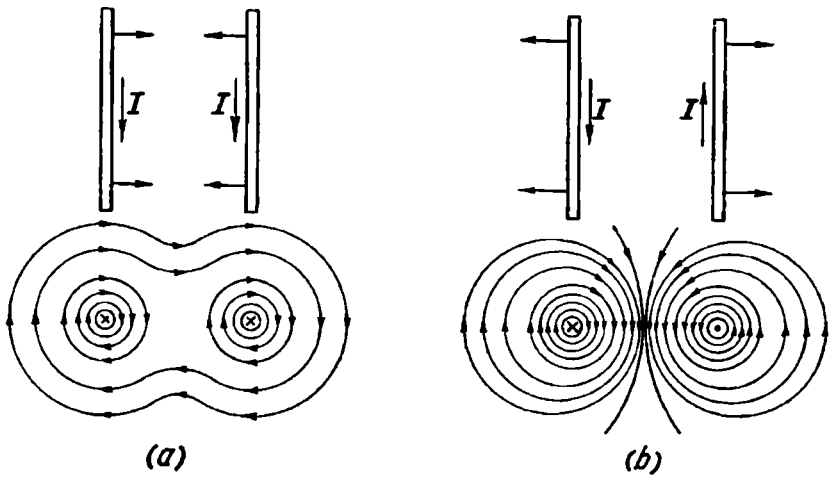


Fig. 100. Interaction of two wires carrying current:
a—in same direction; *b*—in opposite directions

The magnetic induction on the first wire due to the current I_2 will be

$$B_2 = \mu_r\mu_0(I_2/2\pi a),$$

and the force acting on the first conductor

$$F_1 = B_2lI_1 = \mu_r\mu_0I_1I_2l/2\pi a,$$

or equal to F_2.

67. Magnetic and Nonmagnetic Materials

Both magnetic and nonmagnetic materials are used in electrical equipment.

The chief factors, apart from cost, affecting the choice of magnetic materials are magnetic field strength, magnetic flux, magnetic induction (flux density), permeability, and electrical losses.

The relation between magnetic induction and magnetic field strength can be expressed graphically as a *hysteresis loop*. The hysteresis loop serves

as a means for comparing the hysteresis losses of different materials and is also useful in determining the residual induction and the coercive force of materials.

Apart from the hysteresis loss (P_h), there is an eddy-current loss ($P_{e\text{-}c}$) which is due to currents circulating in magnetic materials as the result of emfs induced in them by an alternating magnetic field (or varying induction).

The two loss components (hysteresis loss and eddy-current loss) make up what is known as the core (or iron) loss, P. It is the power expended in a magnetic material subjected to an alternating magnetic field.

In magnetic-circuit calculations, account is taken of the so-called *standard core loss* ($P/B/f$). This is the total power in watts consumed in each kilogram of material at a specified temperature when the material is subjected to an alternating magnetizing force, so as to produce an induction varying harmonically between specified limits of $+B$ and $-B$ and of a frequency of 50 c/s. Hence the symbol: $P/B/f$.

The magnetic induction of a material should not exceed a maximum depending on the nature and quality of the material. An increase over and above this maximum will only result in greater power losses and excessive heating.

For practical purposes all magnetic materials may be classed into soft (or temporary) and hard (permanent).

Soft magnetic materials are those which do not retain their magnetism for any appreciable time after the magnetizing force has been removed. For this reason they have:

(1) high relative permeability, μ_r, which permits obtaining high magnetic induction with the fewest possible ampere-turns;

(2) very low hysteresis and eddy-current losses;

(3) high magnetic stability.

They are made into magnetic circuits for electric machines, transformers, chokes, relays, instruments and the like. Here is an outline of some of these materials.

Electrolytic iron is obtained by the electrolysis of iron sulphate or chloride and remelting the products in a vacuum. Finely divided electrolytic iron is fabricated into parts by methods of powder metallurgy.

Carbonyl iron is prepared as a powder by the thermal decomposition of $Fe(CO)_5$. The powder is sintered at temperatures up to 1200°C into magnetic cores for telephone, radio, television and similar equipment.

Both electrolytic and carbonyl irons contain not over 0.05 per cent of total impurities.

Silicon alloy is chiefly used for high-grade electrical sheets and strips from 0.3 to 0.5 mm thick, made into punchings (or stampings) for electric machines and transformers. The silicon content improves the magnetic properties, decreases hysteresis, increases the electrical resistivity and, consequently, decreases the eddy-current loss. A recent development in the

production of high-quality materials for these purposes is cold-reduced oriented silicon iron (designated ХВП in the Soviet Union). A very pure silicon iron is reduced from the ingot into sheet by a sequence of cold reductions and annealed in a hydrogen atmosphere. In this condition the crystallites of the material are preferentially oriented in the direction of rolling, which fact improves their magnetization. The material as made in the Soviet Union is available in sheets measuring from 1000 mm by 700 mm to 2000 mm by 1000 mm.

Permalloy is a term applied to a number of nickel-iron alloys with the following composition: nickel, 30 to 80 per cent; iron, 10 to 18 per cent; the remainder being copper, molybdenum, manganese, and chromium. Permalloy is easy to machine and is available in sheet form. It has a very high permeability in weak magnetic fields (up to 200,000 henries/cm). Permalloy is manufactured into parts for telephone and radio equipment, cores for transformers and inductors, and parts for electrical instruments.

Alsifer is an alloy of aluminium, silicon and iron of the composition: silicon, 9.5 per cent; aluminium, 5.6 per cent; iron in the balance. Alsifer is a hard and brittle material which is difficult to work. Its advantages are high permeability in weak magnetic fields (up to 110,000 henries /cm), high electrical resistivity (0.81 ohm-mm^2/m), and absence of expensive component materials. It is made into cores for high-frequency service.

V-Permendur is an alloy of 50 per cent cobalt, 1.8 per cent vanadium, the remainder being iron. It is available in sheet, strip and tape form. It goes to make cores for electromagnets, dynamic loudspeakers, telephones, oscilloscopes, etc.

All of the materials listed above and also magnetite (the mineral of the formula $FeO \cdot Fe_2O_3$) can be prepared as powders in which the particles are insulated by shellac, phenol-formaldehyde resins, polystyrene, water-glass, etc., and mixed with a suitable binder. The mix is then pressed in moulds to obtain cores for transformers, chokes, and radio equipment. Because of their grainy structure, such parts develop low eddy-current losses when used in magnetic fields produced by high-frequency currents.

Hard magnetic materials are those which retain their magnetism for a long time after the magnetizing force has been removed. For this reason they are made into permanent magnets, and hence another name for them is permanent-magnet materials. These materials have (1) high residual induction; (2) very high maximum magnetic energy; and (3) high magnetic stability.

The cheapest of them is carbon steel (carbon, 0.4 to 1.7 per cent; the remainder being iron). Carbon-steel magnets, however, have poor magnetic properties and easily lose them when heated, jarred, or struck.

Alloy steels show better magnetic properties and are, therefore, used for permanent magnets more often than is carbon steel. This group includes chromium, tungsten, cobalt and cobalt-molybdenum steels.

Still better magnetic properties are displayed by alloys obtained from the nickel-aluminium-iron-cobalt series. They are characterized by high available energy, high coercive force and high magnetic stability against the effects of heat, vibration and demagnetizing fields, but mechanically they are very hard, impossible to forge, and difficult to machine, except by grinding.

The characteristics of a number of the more important permanent magnet materials are summarized in Table 15.

Table 15

Characteristics of Hard Magnetic Materials

Material	Analysis, wt per cent	Weight per unit available energy
Carbon steel	0.45 C; remainder Fe	26.7
Chromium steel	2-3 Cr; 1 C	17.2
Tungsten steel	5 W; 1 C	15.8
Cobalt steel	5-30 Co; 5-8 Cr; 1.5-5 W	5.1-12.6
Cobalt-molybdenum steel	13-17 Mo; 10-12 Co	3.8
Alni	12.5 Al; 25 Ni; 5 Cu	3.6
Alnisi	14 Al; 34 Ni; 1 Si	3.4
Alnico	10 Al; 17 Ni; 12 Co; 6 Cu	3.1
Magnico	24 Co; 13 Si; 8 Al; 3 Cu	1

Nonmagnetic materials used in electrical equipment may be plastics and nonferrous metals, such as aluminium, brass, bronze, etc. In some applications, where mechanical strength or low cost is essential, they are usually replaced by nonmagnetic steel and nonmagnetic cast iron. The approximate composition of nonmagnetic steel is: carbon, 0.25-0.35 per cent; nickel, 22-25 per cent; chromium, 2-3 per cent; the remainder iron. Nonmagnetic steel is used in the mechanical parts of transformers, chokes, inductors, etc.

The approximate composition of nonmagnetic cast iron is: carbon, 2.6-3 per cent; silicon, 2.5 per cent; manganese, 5.6 per cent; nickel, 9-12 per cent; the remainder iron. It is made into covers, casings, bushings of oil circuit-breakers, cable boxes and welding transformers.

Exercises

1. Determine the magnetic field strength due to a current of 100 A flowing in a long straight wire at a distance of 10 cm from the wire?
2. Determine the strength of a magnetic field, which is set up by a current of 20 A flowing around a closed 5-cm-radius circular conductor, at a point in the centre of the loop.

3. Determine the magnetic flux in a piece of nickel placed in a uniform magnetic field of 500 A/m. The cross-sectional area of the nickel piece is 25 cm^2. The relative permeability of nickel is 300.

4. A straight wire 40 cm long is placed in a uniform magnetic field at an angle of 30° to the direction of the field. The current flowing in the wire is 50 A. The flux density of the field is 5000 gauss. Determine the force pushing the wire out of the magnetic field.

5. Determine the force with which two straight parallel wires placed in air repel each other. The wires are each 2 metres long, spaced 20 cm apart and carry 10-A currents.

Review Questions

1. What experiment demonstrates that a magnetic field is established around a current-carrying conductor?
2. What are the properties of lines of magnetic induction?
3. How can the direction of lines of magnetic induction be determined?
4. What is a solenoid and what is its magnetic field?
5. How can the poles of a solenoid be determined?
6. What is an electromagnet and how can its poles be determined?
7. What is hysteresis?
8. What shapes do electromagnets have?
9. How do current-carrying conductors act upon each other?
10. What acts upon a current-carrying conductor placed in a magnetic field?
11. How do we determine the direction of the force acting upon a current-carrying conductor in a magnetic field?
12. What is the operating principle of electric motors?
13. What materials are called ferromagnetic, paramagnetic and diamagnetic?

Chapter Six

ELECTROMAGNETIC INDUCTION

68. Induced Electromotive Force

When a permanent magnet (*1* in Fig. 101*a*) is inserted into a solenoid *2*, the pointer of a galvanometer *3* connected as shown in the figure will deflect. This shows that there is an emf and a current in the solenoid. When the magnet is held stationary, the galvanometer reads zero (Fig. 101*b*). This experiment shows that for an emf to be developed, or *induced*, there must be a magnetic field and a conductor and they must be in motion with respect to each other.

When the magnet is removed from the solenoid (Fig. 101*c*) the galvanometer pointer deflects in the opposite direction, indicating that the direction of induced emf depends on the direction in which the magnetic field moves when cutting across the conductor or (which is the same) on the direction in which the conductor moves in cutting across the magnetic field.

In our experiment the pointer deflects to the right when the magnet is with its north pole down (Fig. 101*a*) and to the left when it is with its north pole up (Fig. 101*d*). This is proof that the direction of the induced emf also depends on the direction of the magnetic field.

The production of an emf in a circuit by a change in the magnetic flux linking the circuit is called *electromagnetic induction*. It was discovered by Faraday in 1831.

When a current-carrying conductor (at *2* in Fig. 102) is brought close to another conductor carrying no current (*1* in Fig. 92), an emf is induced in the latter. Similarly, when conductor *2* remains stationary but the current in it is varied or interrupted repeatedly, an emf is induced in conductor *1*. The generation of an emf in a conductor due to variations in the current flowing in another conductor is called *mutual induction*. It is the basis of transformers, inductors, etc.

There is one more method for inducing emf. As will be recalled, a current-carrying conductor is always surrounded by a magnetic field. Any change in the magnitude or direction of the current or repeated interruptions of the circuit supplying the current to the conductor causes the magnetic field to vary. In varying, the magnetic field of the conductor will cut across the latter, inducing an emf in it. This phenomenon is known as *self-induction* and the emf due to it is called the *emf of self-induction.*

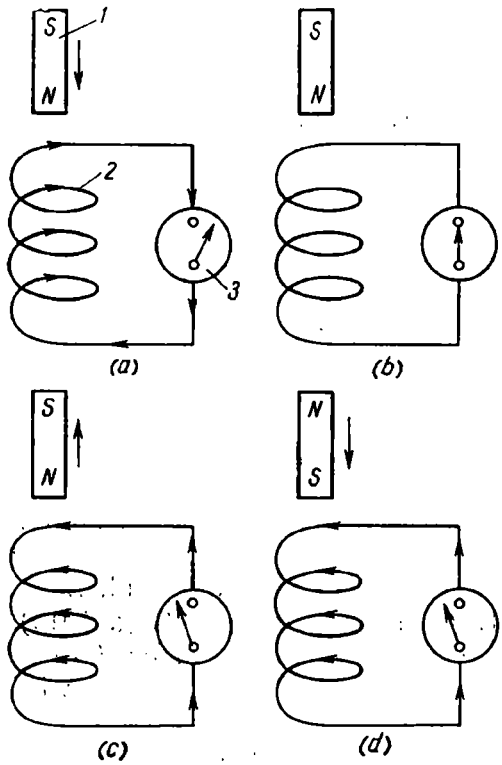


Fig. 101. Relationship between induced emf, the direction of the magnetic field, and the movement of the field relative to the conductor

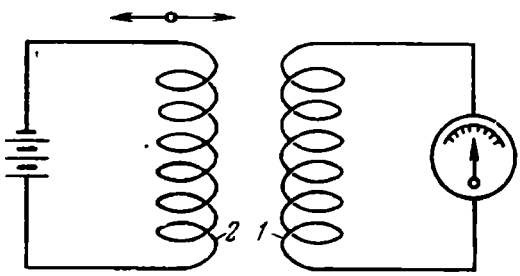


Fig. 102. Mutual induction

69. The Direction and Magnitude of Induced Electromotive Force

Any change in a magnetic field linking a closed electric circuit (a coil or a loop) is accompanied by the induction of an emf in the latter, and the direction of the induced emf depends on both the direction in which the circuit moves and the direction of the magnetic field.

The direction of an induced emf may be determined by the right-hand rule: *When the right hand is so placed in a magnetic field along the conductor that the lines of magnetic induction emerging from the north pole enter the palm and the thumb points in the direction in which the conductor moves, the other four fingers will give the direction of the induced emf* (Fig. 103).

If the conductor remains stationary and the magnetic field moves, the direction of the induced emf may likewise be determined by the right-hand rule; we assume that the field is stationary and the conductor moves in a direction opposite to the motion of the field.

Induced emf can be explained in terms of the free electron theory. Ordinarily, the free electrons in a conductor move at random due to thermal agitation, and the positive and negative charges distributed uniformly

throughout the conductor cancel one another. When the conductor is moved in a uniform magnetic field with a certain speed in the direction *n* (Fig. 104) at right angles to the vector of magnetic induction, the electrons will also move in the same direction. The lines of magnetic induction are shown directed towards the reader.

In such a case, a force will act upon the electric charges to speed them up along the conductor. The positive charges associated with the crystal lat-

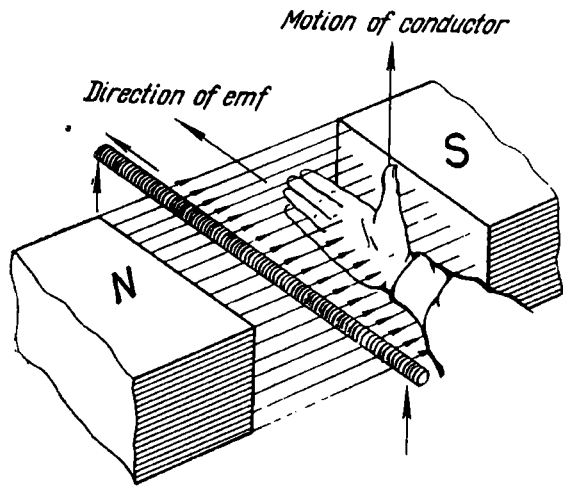


Fig. 103. Determining the direction of induced emf by the right-hand rule

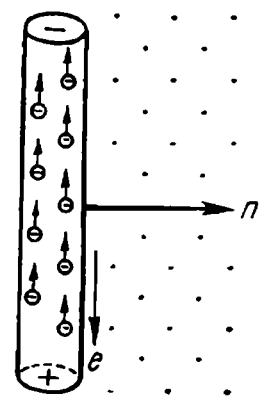


Fig. 104. Electromagnetic induction in a conductor

tice of the material do not move relative to the conductor, but the free electrons are able to move with respect to it.

In our example, the electrons move upwards, producing a current flowing downwards. As can be easily proved, the direction of the induced emf and current in the conductor agrees with the right-hand rule.

The magnitude of an induced emf depends on:

(1) the magnetic flux density B: in the case of a denser magnetic flux the conductor will cut across a larger number of lines per unit time (say, in one second);

(2) the speed, u, with which the conductor moves across the magnetic field; the greater the speed, the more lines the conductor will cut across in unit time;

(3) the effective length of the conductor (that part which is in the magnetic field), l, since a longer conductor can cut across a larger number of lines per unit time;

(4) the sine of the angle α between the direction of the conductor and that of the magnetic field (Fig. 105).

The velocity vector of a conductor in a magnetic field can be resolved into two components: u_n, or the normal component ($u_n = u \sin \alpha$) and

u_t ($u_t = u \cos \alpha$), or the tangential component. The tangential component does not contribute to the induced emf, since with u_t alone the conductor would move parallel to the vector B and would not cut across the lines of magnetic induction.

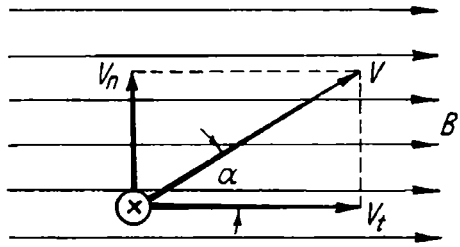


Fig. 105. Resolution of the velocity of a conductor moving in a magnetic field

The magnitude of induced emf can be obtained from the formula

$$e = Blu \sin \alpha \text{ (volts)}.$$

Now we shall apply our knowledge of induced emf to the conversion of electrical energy into mechanical.

Let a straight current-carrying conductor AB (Fig. 106) be placed in an external magnetic field. When the conductor is stationary, the energy supplied by an external source goes solely to heat the conductor:

$$W = VIt = I^2Rt \text{ (joules)}.$$

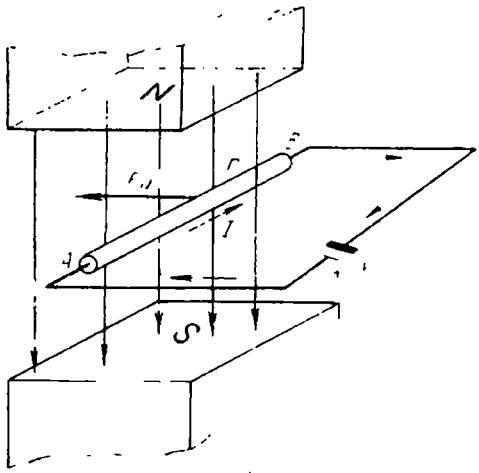


Fig. 106. Conversion of electrical into mechanical energy

The consumed power is

$$P_{el} = VI = I^2R \text{ (watts)},$$

whence the current around the circuit

$$I = \frac{V}{R} \text{ (amperes)}. \qquad \text{(a)}$$

On the other hand, we know that a current-carrying conductor placed in a magnetic field will be acted upon by the field with a force tending to move the conductor in a direction which can be determined by the left-hand rule. The conductor will cut across the lines of magnetic induction and an emf will be induced in it. As given by the right-hand rule, the direction of this induced emf will be opposite to that of the current. Designating it as the counter emf E_b, we find that

$$E_b = Blu \text{ (volts)}.$$

By Kirchhoff's second Law for a mesh we have

$$V-E_b = IR$$

or

$$V = E_b+IR, \tag{b}$$

whence the current in the mesh is

$$I = \frac{V-E_b}{R}. \tag{c}$$

A comparison of Eq. (a) and Eq. (b) will show that, given the same V and R, the current will be greater in the conductor moving in a magnetic field than in a stationary conductor.

Multiplying each term of Eq. (b) by I gives

$$VI = E_bI+I^2R.$$

Since $E_b = Blu$, $VI = BluI+I^2R$.

Noting that $BlI = F$ and $Fu = P_{mech}$, we obtain

$$VI = Fu+I^2R$$

or

$$P = P_{mech}+P_{el}.$$

The latter expression shows that when a current-carrying conductor moves in a magnetic field, the input power is converted into thermal and mechanical power.

70. Operating Principle of the Direct-current Generator

An electric generator is a machine that converts mechanical energy into electrical energy. For its operation, an electric generator depends upon electromagnetic induction. In a generator, the armature *1* (Fig. 107), which carries a winding, is driven by a prime mover in the magnetic field of the north and south poles of electromagnets *2*. The emf induced in the armature winding is collected by means of a commutator *3* and brushes and delivered into an external circuit. The current delivered into the external circuit is direct current due to the commutator.

The prime mover may be a steam engine, a steam turbine, a water wheel, an internal combustion engine, a wind mill, or an electric motor.

71. Lenz's Law

In 1834 Lenz formulated a universal rule for determining the direction of induced emf in a conductor. This rule, known as Lenz's Law, can be stated as follows: *The electromotive force induced by the variation of magnet-*

ic flux is always in such direction that the magnetic effect of the current it produces opposes the flux variation.

The validity of Lenz's Law may be proved as follows:

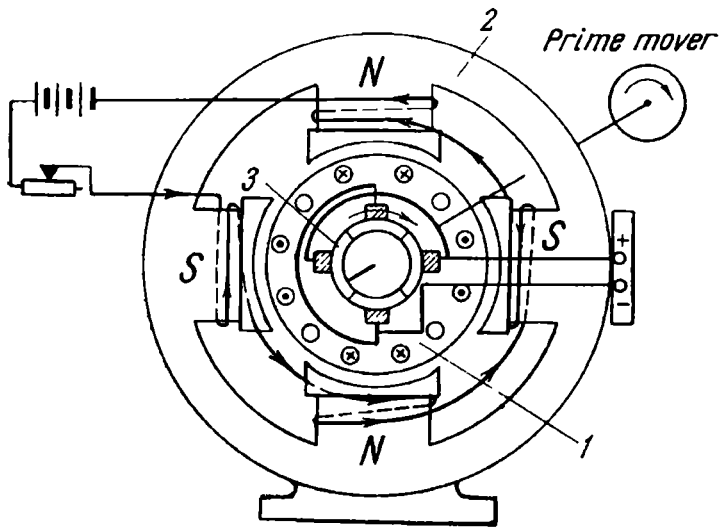


Fig. 107. Diagram of a d-c generator

1. Let a conductor be placed in a magnetic field as shown in Fig. 108. When moving downwards the conductor will cut across the magnetic field and an emf will be induced in it. Its direction can be determined by the right-hand rule. In our case, both the induced emf and the current will be directed towards the reader. From the foregoing it follows that the conductor will be pushed out of the magnetic field and the direction of this action will be determined by the left-hand rule. In our case, the conductor will be pushed upwards. To put it another way, the induced current interacts with the magnetic field so as to oppose the motion of the conductor, i.e., the cause of the current.

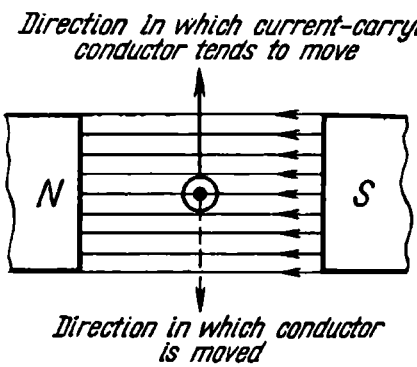


Fig. 108. Illustration of Lenz's Law

2. Let a permanent magnet be lowered into a coil with its north pole down (Fig. 109). The direction of the current in the coil will be as shown by arrows in Fig. 109*a* and we may assume that the pointer of the galvanometer connected in the circuit deflects to the left. With this current flowing in it, the coil becomes a solenoid, with its north pole at the top and its south pole at the bottom. Since the like poles of the magnet and solenoid repel each other, the current induced in the coil will oppose the motion of the

permanent magnet, i.e., it will be opposed to the force that produced it.

When the permanent magnet is withdrawn from the coil, the galvanometer deflects to the right, and the direction of the induced current will be as shown in Fig. 109*b*, or opposite to that in Fig. 109*a*.

By the right-hand screw rule, the south pole is now at the top and the north pole at the bottom. The unlike poles of the magnet and solenoid

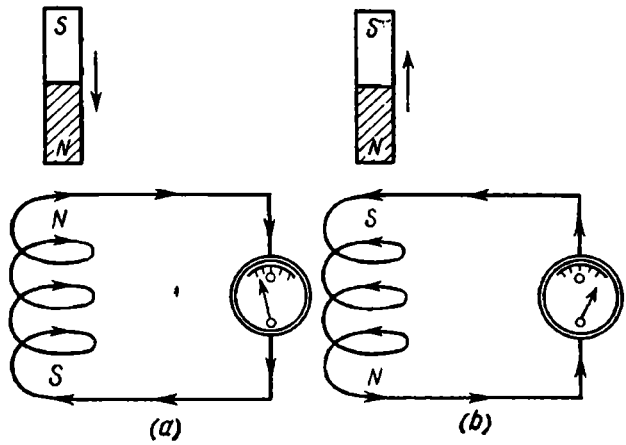


Fig. 109. Solenoid opposing the movement of a magnet;
***a*—downward; *b*—upward**

will attract each other and tend to slow down the withdrawal of the magnet. Again, the induced current opposes the effect that produced it.

3. Let a current flow around circuit *I* (Fig. 110*a*) from *A* to *B*. The magnetic field produced by the current around the conductor *AB* will cut across the conductor *CD*, and an emf will be induced in circuit *II*. As circuit *II* is closed via a galvanometer, a current will begin to flow around it. The galvanometer is connected as in the previous experiment.

Deflecting to the left, the galvanometer indicates that the current flows through the instrument from top to bottom. A comparison of the currents in the conductors *AB* and *CD* will show that they flow in opposite directions.

As will be recalled, two parallel conductors carrying currents in opposite directions repel each other. Therefore, the conductors *CD* and *AB* will tend to repel each other so as to eliminate the effect of the magnetic field around the conductor *AB*, which is the cause that gave rise to the induced current.

The induced current will flow around circuit *II* for only a short time. As soon as the magnetic field around the conductor *AB* has reached a steady state, it will no longer cut across the conductor *CD* and there will be no current induced in circuit *II*.

When circuit *I* is opened, the current will begin to decrease. The decreasing current will cause the magnetic field to collapse, and it will cut across the conductor *CD* inducing in it a current in the same direction as the one in the conductor *AB* (Fig. 110*b*). Since two parallel conductors carrying currents in the same direction attract each other, the conductors *AB* and *CD* will tend to maintain the collapsing magnetic field.

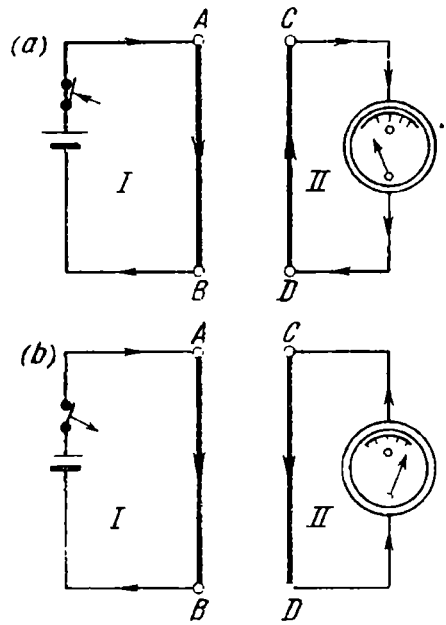


Fig. 110. Current induced in circuit *II*:
a—when circuit *I* is being completed;
b—when circuit *I* is being broken

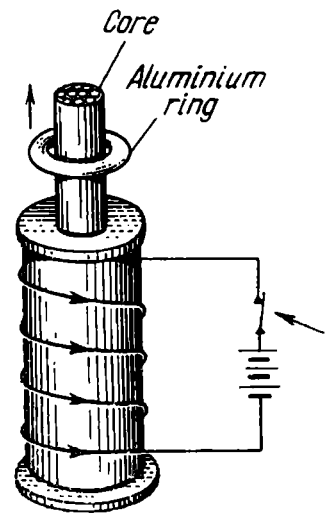


Fig. 111. An aluminium ring loosely put on the steel core of a coil jumps up when current is applied

4. Let a coil have a core built up of lengths of steel wire with a light aluminium ring put around the core (Fig. 111). Just as the circuit is closed, an electric current will traverse the coil setting up a magnetic field around it. The lines of magnetic induction will cut across the aluminium ring to induce in it a current opposite in direction to the one in the coil. Since conductors carrying currents in opposite directions repel, the aluminium ring will jump up.

The relation between a magnetic flux varying with time and the resultant emf may be expressed thus:

$$e = -\frac{d\Phi}{dt}.$$

The ratio $d\Phi/dt$ gives the average rate of change of the magnetic flux with time. The smaller the time interval dt, the smaller the difference be-

tween this emf and its instantaneous value. The negative sign is indicative of the direction of the induced emf as specified in Lenz's Law.

With the magnetic flux varying at a uniform rate with time, the ratio $d\Phi/dt$ will be constant. Then the absolute value of the emf in the conductor will be

$$e = \Phi/t.$$

For more than one turn in a coil, the induced emf will be

$$e = -T(d\Phi/dt) \text{ volts.}$$

The product of the coil turns and the linked magnetic flux is known as flux-linkage and is designated in the U.S.S.R. by the Greek letter ψ.

Then the above expression known as the change-of-linkage law can be presented thus:

$$e = -d\psi/dt.$$

This is Faraday's Law of electromagnetic induction.

72. Eddy Currents

In addition to the emf induced in the armature winding, so-called eddy, or Foucault, currents are induced in the steel armature itself as it cuts across the magnetic field.

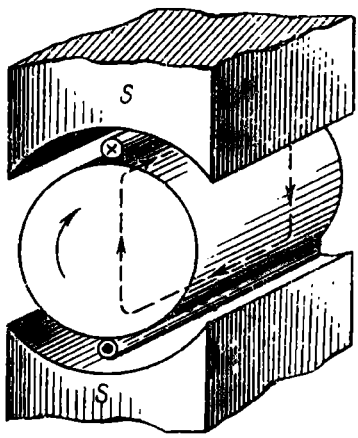


Fig. 112. Eddy currents induced in the solid armature of an electric machine

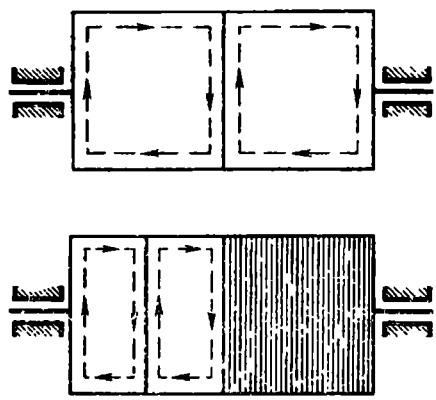


Fig. 113. Effect of a split magnetic circuit on eddy currents. Direction of eddy currents is shown by arrows

Fig. 112 shows diagrammatically an armature rotating in a magnetic field. The eddy currents induced in the armature flow as indicated by the dotted arrows. In passing through the armature, they heat it, thereby dissipating some energy. If measures are not taken to minimize eddy currents they may dangerously heat the armature and damage the winding

insulation. Though eddy currents and the associated loss of energy cannot be eliminated completely, they can and must be reduced to a minimum.

This can be done by building up the armatures of generators and motors and the cores of transformers with separate stampings 0.35 to 0.5 mm thick. They are assembled in the direction of the lines of magnetic induction and are insulated from one another either by varnish or tissue paper. As a result, the magnetic flux in each stamping and, consequently, the emf and current in them are reduced. Fig. 113 shows the paths of eddy currents through a laminated armature.

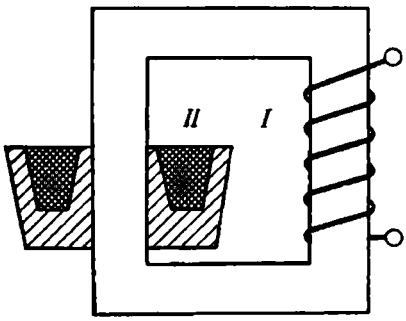


Fig. 114. An induction furnace (schematic).

Eddy-current loss may be further reduced by adding about 4 per cent silicon to the armature or core steel.

In some cases, eddy currents can be put to useful work. Their uses include the hardening of steel by high-frequency currents, operation of induction-type instruments and meters, a-c relays (described elsewhere in this book), etc.

In induction furnaces (Fig. 114) the eddy currents induced by primary *I* in secondary *II*, which is the charge of the furnace, are large enough to generate heat sufficient to melt metal.

In measuring instruments, eddy currents serve as an electromagnetic brake or damper (Fig. 115). When a pointer (Fig. 115*a*) oscillates or a disc

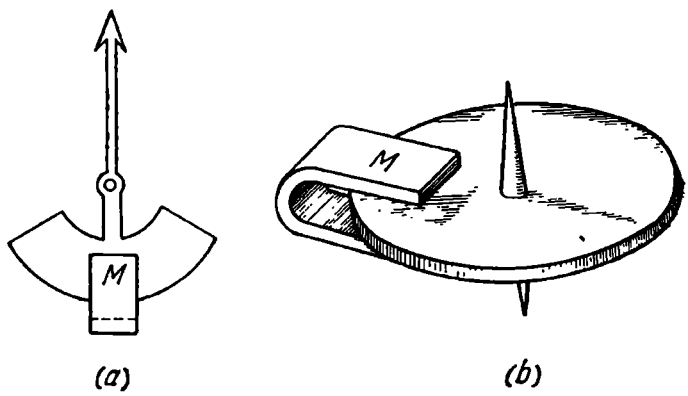


Fig. 115. Electromagnetic brakes (dampers)

(Fig. 115*b*) rotates, they cut across the magnetic field of a magnet *M*, and eddy currents are induced in them. According to Lenz's Law, these currents are so directed as to oppose the cause producing them. As a result, the disc will be slowed down and the pointer damped.

73. The Electromotive Force of Self-induction and the Inductance of a Circuit

When the switch in the circuit shown in Fig. 116 is closed, an electric current begins to flow as shown by the single-headed arrows. This current gives rise to a magnetic field whose lines of induction will cut across the

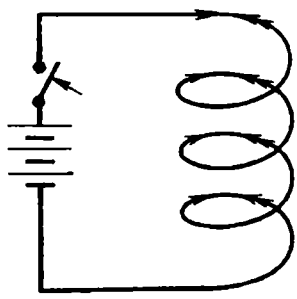


Fig. 116. Emf of self-induction acts against emf of supply when the supply circuit is closed

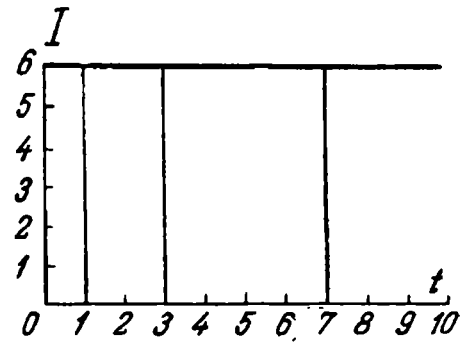


Fig. 117. Graph of direct current

conductor to induce in it an emf. This is known as the emf of self-induction. According to Lenz's Law, this emf will oppose the emf of the cell. The direction of the emf of self-induction is shown by the double-headed arrows (Fig. 116).

It is not difficult to realize that a steady current begins to flow around a circuit only after a steady-state magnetic flux has been set up. Then the lines of magnetic induction will no longer cut across the conductor, and there will be no emf of self-induction.

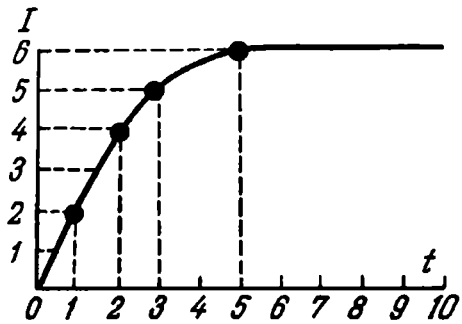


Fig. 118. Graph showing the rise of direct current in a circuit due to emf of self-induction

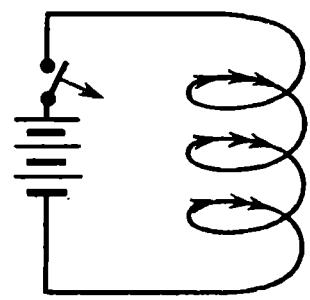


Fig. 119. Emf of self-induction aids emf of supply when the circuit is closed

A steady, or unvarying, current is graphically shown in Fig. 117. Time is plotted along the X-axis and the current, along the Y-axis. Referring to the figure, the current is 6 A at time 1 and remains at 6 A at times 3, 7, etc.

Fig. 118 shows how the current flowing around a circuit attains a steady-state value after the switch in the circuit has been closed. When the switch is closed, the emf of self-induction opposes the emf of the cell and the current in the circuit is zero. At time 1 it has risen to 2 A, then to 4 A, 5 A and finally to 6 A.

When the circuit is interrupted (Fig. 119), the decreasing current, whose direction is shown by the single-headed arrow, will act to reduce its mag-

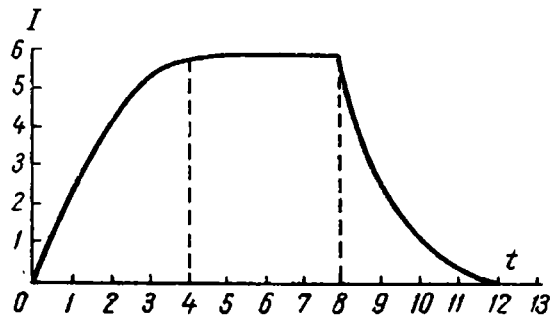


Fig. 120. Graph showing the rise and collapse of current around a circuit due to emf of self-induction

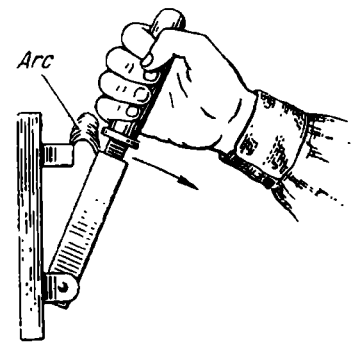


Fig. 121. Current induced when the circuit is opened

netic field. In collapsing from a certain value to zero, this magnetic field will again cut across the conductor to induce an emf of self-induction. This emf will be in the same direction as the emf of the source (shown by the double-headed arrow in Fig. 119), and will tend to maintain the current in the circuit.

In view of the fact that the emf of self-induction always acts against the cause which gives rise to it, it is said to be reactive.

Graphically, variations in the current flowing around a circuit due to the emf of self-induction induced when the circuit is closed and opened are shown in Fig. 120.

In circuits containing many turns and substantial steel cores (i.e., having a large inductance), the opening of the circuit may produce an emf of self-induction which is many times the emf of the source. If not guarded against, the resulting current can break down the air gap between the jaw and blade contacts of a knife switch, producing an electric arc which is capable of melting the copper parts of the switch and burning the hand of the operator if the switch has no casing around it (Fig. 121).

In the circuit itself, the emf of self-induction can rupture the insulation of coils, electromagnets, etc. This can be prevented by providing a special arc-breaking contact in circuit-breakers or switches. This contact shorts out the winding on opening.

It should be remembered that an emf of self-induction manifests itself not only when a circuit is opened or closed, but also during any current changes.

The emf of self-induction depends on the rate of change of the current flowing around the circuit. If, in one and the same circuit, the current drops off from 50 A to 40 A during, say, 1 second at one moment and from 50 A to 20 A during 1 second at another moment, the emf of self-induction will be three times greater in the latter case.

The emf of self-induction also depends on the inductance of the circuit. Circuits of high inductance are the windings of generators, motors, transformers and inductors which have steel cores. Straight conductors have lower inductance. Short straight conductors, incandescent lamps and heating appliances have hardly any inductance, and so almost no emf of self-induction is induced in them.

The magnetic flux linking a loop and inducing an emf of self-induction in it is directly proportional to the current flowing in the loop:

$$\Phi = LI,$$

where L is a proportionality factor called the coefficient of self-induction, or, more usually, the inductance. Its dimensions are:

$$[L] = \left[\frac{\Phi}{I}\right] = \frac{V \times \text{sec}}{\text{A}} = \text{ohm-sec}.$$

The unit of inductance is known as the henry.

1 henry $= 10^3$ millihenries $= 10^6$ microhenries,

1 henry $= 10^9$ cm (in the c.g.s. *emu* system).

The inductance of one kilometre of telegraph wire is 0.002 henry. The inductance of large electromagnets is as much as several hundred henries.

With the current in a circuit changing by di, the magnetic flux will change by $d\Phi$:

$$d\Phi = Ldi.$$

The emf of self-induction in the circuit will be

$$e_L = -(d\Phi/dt) = -L(di/dt).$$

If the current changes with time uniformly, the expression di/dt will be constant, and it may be replaced by the ratio I/t. Then the absolute value of the emf of self-induction induced in the circuit can be expressed thus:

$$E_L = L(I/t).$$

From the latter expression

$$1 \text{ V} = 1 \text{ henry} \times (1 \text{ A}/1 \text{ sec}),$$

or a circuit has unit inductance, one henry, if the emf induced is 1 volt when the current is changed at the rate of one ampere per second.

As has been shown, an emf of self-induction is induced whenever a circuit is closed or opened or the circuit current is changed. Conversely, if the current in a circuit remains constant, the magnetic flux of the conductor is also constant, and no emf of self-induction can be induced (since $di/dt = 0$).

In practice, it is often desirable to have coils without inductance (these may be multiplier resistors for voltmeters, resistors of plug-type rheostats,

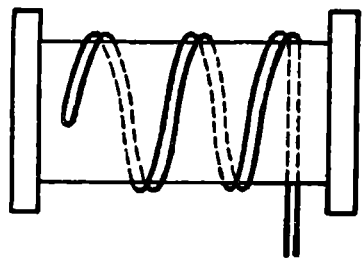


Fig. 122. Bifilar winding of a coil

etc.). This requirement is satisfied by a bifilar winding (Fig. 122). Here, the currents in adjacent conductors flow in opposite directions, and their magnetic fields cancel one another. The total magnetic flux and inductance of such a coil will be zero.

74. Mutual Induction

As has been stated, when two independent circuits or coils are in proximity to each other, a change in current in one is accompanied by a change in its magnetic field, which induces an emf in the other, a property called mutual induction. Let there be two conductors *I* and *II* (Fig. 123), two coils, or two circuits. The current, i, in one is supplied by a source not shown in the figure. The current produces a magnetic flux Φ, a portion of which links the other conductor, the rest following a closed path outside it. Then

$$\Phi_1 = \Phi_{12} + \Phi_{11}.$$

If instead of straight conductors we have two coils with turns T_1 and T_2, respectively, the flux-linkage in the second circuit will be

$$\psi_{12} = T_2\Phi_{12}.$$

Since the flux Φ_{12} is proportional to the current i_1, the flux-linkage, ψ_{12}, and the current, i_1, will be related thus:

$$\psi_{12} = M_{12}i_1,$$

whence

$$M_{12} = \psi_{12}/i_1 = T_2\Phi_{12}/i_1,$$

where M_{12} is the proportionality factor, called the *mutual inductance.*

The dimensions of mutual inductance are

$$[M] = [T\Phi/i] = \text{V} \times \text{sec/A} = \text{ohm-sec or henries.}$$

Thus, the unit of mutual inductance M is the same as that of inductance L.

Mutual inductance depends on the number of turns, size, and relative position of the coils, and the magnetic permeability of the medium in which they are placed.

If the current in the second conductor is i_2, we may write by analogy

$$\psi_{21} = T_1\Phi_{21}$$

and

$$\psi_{21} = M_{21}i_2,$$

whence

$$M_{21} = \psi_{21}/i_2 = T_1\Phi_{21}/i_2.$$

From Ohm's Law for a magnetic circuit, we can prove that

$$\Phi_{12} = i_1T_1/R_m$$

and $$\Phi_{21} = i_2T_2/R_m,$$

where R_m is the reluctance of the closed circuit linked by magnetic fluxes Φ_{12} and Φ_{21}.

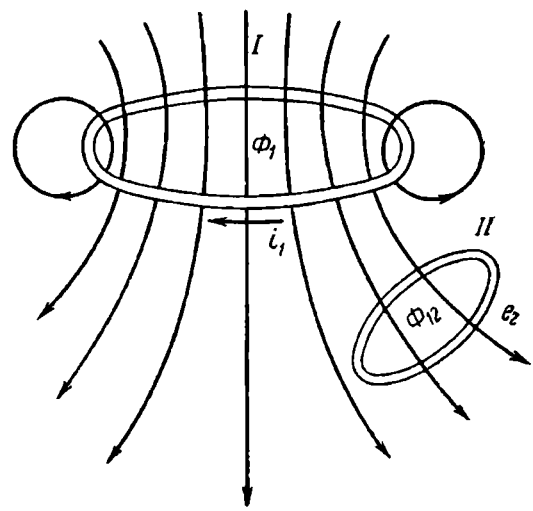


Fig. 123. Mutual induction

The expressions

$$M_{12} = \psi_{12}/i_1 \text{ and } M_{21} = \psi_{21}/i_2$$

may be rewritten as follows:

$$M_{12} = T_2\Phi_{12}/i_1 = T_2i_1T_1/i_1R_m = T_1T_2/R_m;$$
$$M_{21} = T_1\Phi_{21}/i_2 = T_1i_2T_2/i_2R_m = T_1T_2/R_m.$$

Thus, $M_{12} = M_{21} = M$.

In other words, the mutual inductance of two circuits coupled inductively or magnetically is the same whether current flows in the first circuit and voltage is observed in the second, or current flows in the second and voltage is observed in the first.

A change in the current i_1 will produce a change in the magnetic fluxes Φ_{11} and Φ_{12} and an emf will be induced in the second circuit:

$$e_{M_2} = -d\Phi_{12}/dt = -T_2(d\Phi_{12}/dt) = -M(di_1/dt).$$

Similarly,

$$e_{M_1} = -d\Phi_{21}/dt = -T_1(d\Phi_{21}/dt) = -M(di_2/dt).$$

These emfs are emfs of self-induction. If one of the two circuits has a resistance R_1 and an inductance L_1, the voltage V_1 applied to this circuit must be high enough to balance the emfs of self- and mutual induction and also the voltage drop across the resistance R_1 of the circuit:

$$V_1 = L_1(di_1/dt)+M(di_2/dt)+i_1R_1.$$

For the second circuit

$$V_2 = L_2(di_2/dt)+M(di_1/dt)+i_2R_2.$$

The inductances L_1 and L_2 of the two circuits and their mutual inductance M are related thus:

$$M = \sqrt{L_1L_2}.$$

This formula, however, holds only when the entire flux produced by the first circuit interlinks the second circuit. In practice, the linkage is less complete, and $M<\sqrt{L_1L_2}$. The ratio $M/\sqrt{L_1L_2} = k$ is the *coefficient of coupling*, giving the proportion of the flux in one circuit which links the other. It is less than unity. It would be equal to unity if $\Phi_{12} = \Phi_1$ and $\Phi_{21} = \Phi_2$.

The coupling between two circuits can be varied by moving them towards, or away from, each other, or by varying their relative position.

Let two coils be so positioned that their axes are parallel and their magnetic fields are in one and the same sense (the coils are said to be connected in series aiding). Then

$$V = i(R_1+R_2)+L_1(di/dt)+L_2(di/dt)+2M(di/dt) =$$
$$= i(R_1+R_2)+di/dt(L_1+L_2+2M) = iR+L'(di/dt),$$

and the inductance of the system

$$L' = L_1+L_2+2M.$$

If the connection is in series opposing (one of the coils is turned through 180° with respect to the other), the ampere-turns will be in opposition. Then

$$V = i(R_1+R_2)+L_1(di/dt)+L_2(di/dt)-2M(di/dt) = iR+L''(di/dt),$$

where,

$$L'' = L_1+L_2-2M.$$

So, relative rotation of the two coils can vary the inductance of the system from L' to L''. This principle is utilized in *inductometers (variometers)*, or variable inductance standards, used in inductance measurements. The Ayrton-Perry inductometer, among the most commonly used instruments, employs two pairs of coaxial coils, the outer pair being stationary while the other pair rotates within it.

75. Operating Principle of the Transformer

Two coils having mutual inductance make up a *transformer*, which is a device for transferring energy from one circuit to another without direct connection and usually, but not always, with a change in voltage or current, although without change of frequency (Fig. 124).

A transformer consists essentially of two independent electric circuits (or windings) linked with a common magnetic circuit (or the core). The

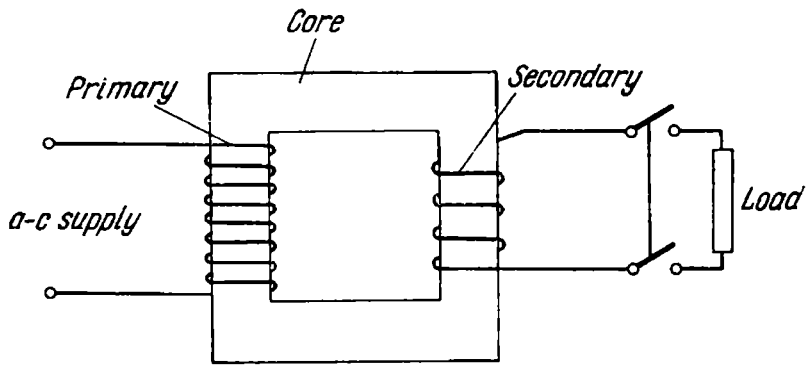


Fig. 124. Diagram of a transformer

winding connected to the supply circuit is referred to as the primary winding (or simply the primary), while the winding connected to the receiver circuit is termed the secondary winding (or the secondary).

A transformer can be used only with alternating current, since no voltage will be induced in the secondary if the magnetic field does not change. If direct current is applied to the primary, a voltage will be induced in the secondary only at the instant of closing or opening the primary circuit, since it is only at these times that the field changes.

76. Induction Coils

Induction coils may be divided into two groups: those with a primary winding only; and those with both primary and secondary windings (Fig. 125). The secondary-type coil is a special type of transformer. It consists of a core *1* (built up of lengths of steel wire), a primary wound with several turns of heavy wire *2*, and a secondary *3* having a large number of turns (from 16,000 to 1,000,000 or more) wound with fine wire. The primary is connected via a mechanical interrupter *4* to a d-c source (a battery of primary or secondary cells) *5*.

When the circuit is closed by a switch, *6*, current is established in the primary. This current produces magnetic flux in the core. When the flux reaches a certain value, the pull on the interrupter contact arm is sufficient to overcome the pull of the spring and thus opens the circuit at *4*. When the circuit is open, the flux quickly decreases to zero and thus induces a con-

siderable emf in the secondary. The spring then returns the interrupter contact arm to its original position, the primary circuit is closed, and the sequence is repeated.

The large number of turns in the secondary generates a voltage of several thousand or even hundred thousand volts. The air gap between the terminals of the secondary is broken down, and an arc strikes which may be up to 1 metre long in large induction coils.

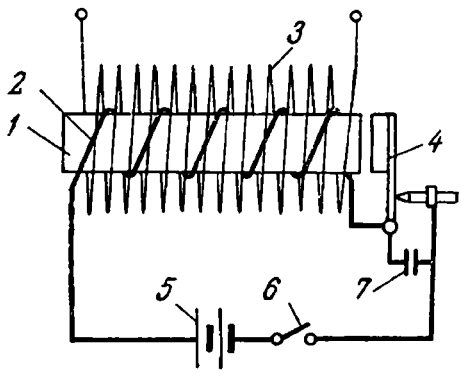


Fig. 125. Diagram of an induction coil

If a high value of emf is to be obtained in the secondary, it is essential that the primary circuit be interrupted as quickly as possible. The spark which appears between the contacts of the mechanical interrupter hampers quick interruption of the circuit. To prevent sparking, a capacitor, 7, is connected in parallel with the contacts of the interrupter.

If the primary of an induction coil is connected to an a-c source, the interrupter may be dispensed with.

Physics owes many of its discoveries to the induction coil. Induction coils are widely employed in telephony, in internal-combustion engines for ignition, etc.

Exercises

1. A conductor cuts across a uniform magnetic field of 6000 gauss at an angle of 30° to the direction of the field. The length of the conductor is 50 cm. It is moving at the rate of 2.5 m/sec. Find the induced emf.
2. A magnetic flux of 0.6×10^6 maxwells threads a coil having 200 turns. The flux collapses to zero in 0.05 sec. Determine the emf induced in the coil.
3. The current traversing a coil decreases at a steady rate of 200 A per 0.01 sec. Determine the emf of self-induction, if the inductance of the coil is 0.05 henry.
4. Determine the inductance of a coil having 100 turns if the current changes at a steady rate by 20 A and the magnetic flux by 0.6 weber during the same period of time.

Review Questions

1. What is electromagnetic induction?
2. When is an induced emf produced?
3. Name the factors governing the polarity of the induced emf.
4. How is the direction of the emf induced in a conductor determined?
5. Name the factors governing the magnitude of the emf induced in a conductor.
6. What is the operating principle of a d-c generator?
7. State Lenz's Law and describe experiments that prove it.
8. What is mutual induction?
9. What is the operating principle of the transformer?
10. What is an induction coil?
11. What are eddy currents and how can they be reduced?
12. What is self-induction?
13. How does self-induction manifest itself in d-c circuits?

Chapter Seven

SINGLE-PHASE ALTERNATING CURRENT

77. Generation of Alternating Current

Let there be a uniform magnetic field produced between the north and south poles of an electromagnet (Fig. 126*a*). A straight conductor is made to trace out a circular path in the clockwise direction within the field. As

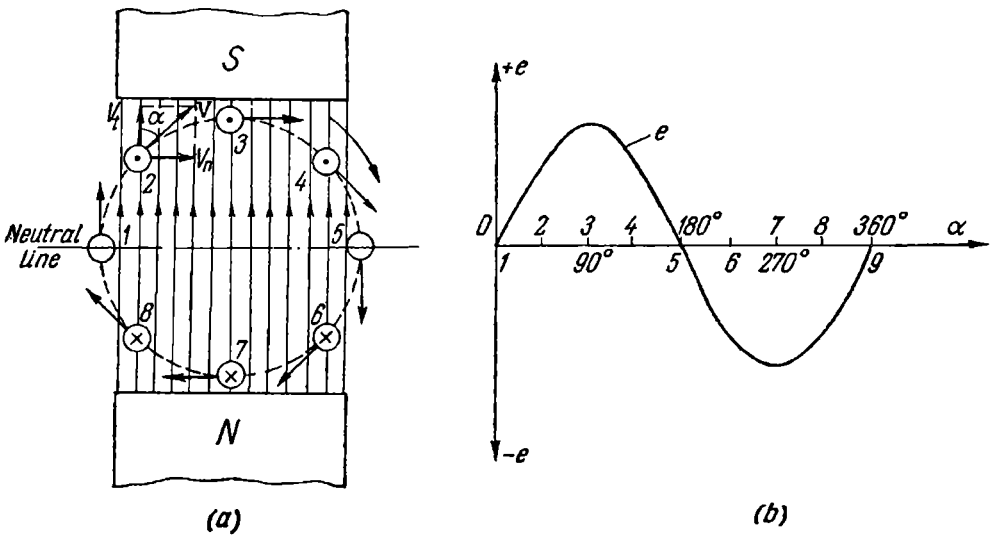


Fig. 126. Generation of alternating current:
a—rotation of a conductor in a uniform magnetic field; *b*—graph of an alternating current

the conductor cuts across the lines of magnetic induction, an emf is induced in it. As has been stated earlier, the value of this emf depends on the magnetic induction (or flux density) B, the effective length of the conductor l,

the speed u with which the conductor cuts across the lines of magnetic induction, and the sine of the angle α between the direction of the conductor and the direction of the magnetic field:

$$e = Blu \sin \alpha.$$

Let us resolve the circular speed u into two components – the normal speed u_n and the tangential speed u_t, as was explained in Sec. 64. The normal speed u_n is responsible for the induced emf and is equal to

$$u_n = u \sin \alpha.$$

The tangential speed u_t does not contribute to the induced emf and is

$$u_t = u \cos \alpha.$$

At $\alpha = 90°$, the normal speed will be

$$u_n = u \sin \alpha = u \sin 90° = u.$$

In other words, the normal speed is at its highest, and so is the emf induced in the conductor:

$$e = Blu = E_m.$$

Hence, a general expression for the emf induced in a conductor moving in a magnetic field will be

$$e = E_m \sin \alpha \quad \text{or} \quad e = \sin \alpha.$$

Fig. 120 shows the position, every 45°, of a conductor moving in a magnetic field. As can be seen from the figure, the angle α varies. Also, the direction of the induced emf is reversed when the conductor moves past the neutral line (according to the right-hand rule). Table 10 relates the position of the conductor to the angle α and the direction of the induced emf and also its value (which is proportional to $\sin \alpha$).

Referring to the table, the emf induced in the conductor rises from zero to a maximum, falls to zero, to reverse its direction, rises to a maximum in the opposite direction and again drops to zero, repeating this cycle of changes every complete revolution of the conductor continuously. Such an emf is called *alternating*.

The cycle of changes in the induced emf can be represented graphically (Fig. 120*b*). The angles α are plotted on the X-axis and the values of the induced emf on the Y-axis. The emf induced when the conductor is moving past the south pole can be taken as positive and plotted vertically upward of the X-axis. The emf induced when the conductor is moving past the north pole may be taken as negative and plotted vertically downwards from the X-axis. The continuous curve connecting the values of the emf at any given angle α is called a *sinusoidal* curve or wave. Using this curve, it is possible to find the instantaneous value of emf at any given moment of time. To do so, the desired angle α is laid off from the origin on the X-axis

Table 16

Relationship Between the Induced Electromotive Force and the Position of a Conductor in a Magnetic Field

Position of conductor	Angle α	Sin α	Direction of emf in conductor
1	0	0	—
2	45	0.707	towards the reader
3	90	1	ditto
4	135	0.707	ditto
5	180	0	—
6	225	−0.707	away from the reader
7	270	−1	ditto
8	315	−0.707	ditto
9 or 1	360	0	—

and a perpendicular is erected from the point thus obtained. The length of the perpendicular between the X-axis and the curve will give the value of the emf induced in the conductor at this time.

In our example, the conductor rotates in a uniform magnetic field, and the emf induced in the conductor obeys the sine law. Hence, it is called a sinusoidal emf, and the graph is termed a sine waveform. It will be shown later on that electrical engineering gives preference to sine-wave currents and voltages in many applications.

Fig. 127 shows a loop or a single-turn coil rotating in a magnetic field under the action of a mechanical force. The ends of the coil are connected to two copper rings *3* and *4*, upon which two carbon brushes *5* and *6* bear. This arrangement, known as a slip-ring collector, collects the current induced in the coil and delivers it to an external circuit. The current varies both in direction and magnitude regularly with time and is called an *alternating current.*

Of the four sides of the coil, only those numbered *1* and *2* in Fig. 127 contribute to the generation of induced emf. For this reason they are often called *effective* conductors of a coil or winding.

The arrangement shown in Fig. 127 is difficult to realize practically because it is impossible to produce and maintain a uniform magnetic field in the presence of a high reluctance to the magnetic flux, which has to pass through air for the most part of its path.

Real electric machines have a steel drum placed between the poles of an electromagnet. The slots in the drum receive the conductors of the winding.

This type of construction is shown in Fig. 128. The magnetic flux now has to cross only a short air gap between the pole pieces and the drum. It can be proved that the lines of magnetic induction enter and leave the

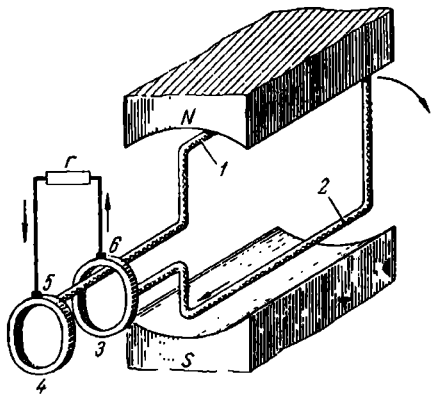


Fig. 127. Slip-ring collector of an a-c generator

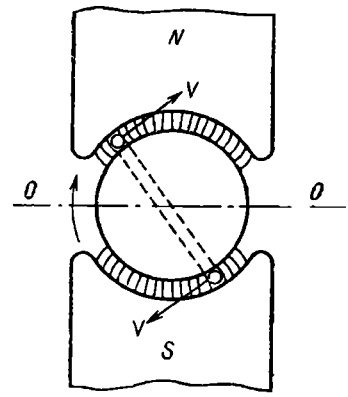


Fig. 128. Magnetic flux due to a steel core

drum radially. Then the direction of the angular speed will be normal to the direction of the magnetic flux at any instant of time, and the speed will always be normal ($u=u_n$).

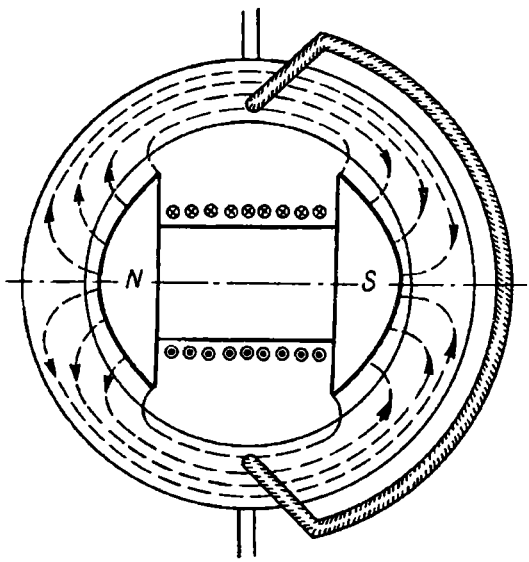


Fig. 129. A two-pole a-c generator

The desire to obtain a sine emf prompts the designer to shape the pole pieces so that the magnetic induction B in the air gap obeys the sine law:

$$B = B_m \sin \alpha,$$

where B_m is the maximum magnetic induction in the air gap at $\alpha=90°$, or

$$B = B_m \sin \alpha = B_m \sin 90° = B_m.$$

As has been shown already, the induced emf will then also be at its highest:

$$e = B_m l u = E_m$$

and $$e = E_m \sin \alpha$$

or $$e = \sin \alpha.$$

For an emf to be induced in a generator, it is immaterial whether a moving conductor cuts across a stationary magnetic field, or a moving field cuts across a stationary conductor. In the foregoing examples, the winding where the emf is induced is arranged on the moving part of the

machine, called the *rotor*, and the pole pieces are on the stationary part, termed the *stator*. Better working conditions for an a-c winding are obtained, however, when it is arranged on the stator and the winding that produces a magnetic field (and therefore called the field winding) and its poles are placed on the rotor (Fig. 129).

The direct current necessary to energize the field winding can be supplied by an outside exciter (a d-c generator) mounted on a common shaft with the a-c generator, or by a rectifier.

78. Basic Concepts and Definitions Pertaining to Alternating Current

As was shown in the preceding section, an alternating emf and an alternating current reverse their direction and sign at regular intervals and vary in magnitude continuously. The value of an alternating current, voltage, etc., at any given moment is known as the *instantaneous value* and is designated by a small letter (i for current, v for voltage, and e for emf).

The highest of the instantaneous values are referred to as *peak values*. They are designated by capital letters and a subscript m (I_m for current, E_m for emf, and V_m for voltage).

The time occupied by one complete cycle of change in emf, current or voltage is called the *period*. It is designated by the letter T and is measured in seconds. The number of cycles of complete change repeated within one second is the *frequency*. It is measured in cycles per second and it is designated by f.

A broad spectrum of frequencies is used in electrical and radio engineering. Power or commercial frequency in the Soviet Union is 50 c/s. Induction furnaces operate on frequencies from 1000 to 8000 c/s when powered by motor-generator sets and 150 to 250 kilocycles per second when powered by valve oscillators. The frequencies in use in dielectric heating (which is applicable to drying wood, plastics, foodstuffs and other dielectric materials) range from 2 to 40 megacycles per second, the most common frequencies being 20 to 25 megacycles per second. Telephony uses frequencies from several hundred to a few thousand cycles while radio engineering employs frequencies running into many millions or even thousands of millions of cycles per second.

The frequency of an alternating current is measured with a frequency meter.

The period and the frequency are related as follows:

$$T = \frac{1}{f}; \quad f = \frac{1}{T}.$$

Example 1. Determine the period of a current with a frequency of 50 c/s.

$$T = 1/f = 1 \div 50 = 0.02 \text{ sec.}$$

Example 2. Find the frequency of a current with a period of 5×10^{-8} sec.

$f = 1/T = 1 \div 5 \times 10^{-8} = 20 \times 10^6$ c/s $= 20 \times 10^3$ kilocycles per sec $= 20$ megacycles per second.

79. Sinusoidal Quantities

Practical electrical engineering has given preference to electrical quantities that obey the sine law. When speaking of current, emf, voltage or magnetic flux, we shall take them as obeying the sine law.

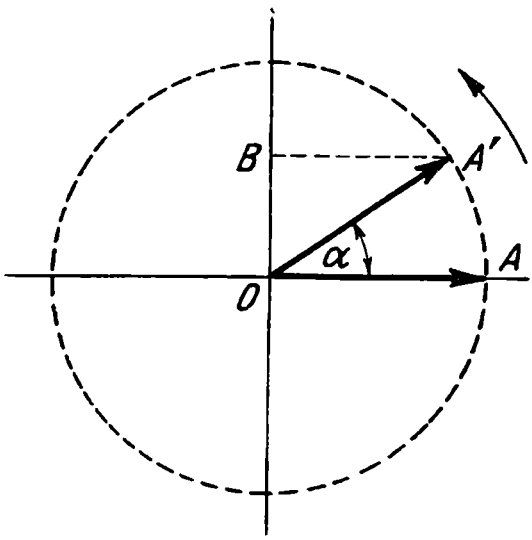


Fig. 130. Rotation of a vector around an axis

Let there be a radius vector OA (Fig. 130) representing some variable (say, a current) to a definite scale. The radius vector is rotated uniformly in a counterclockwise direction around a point O. The terminus of the radius vector describes a circumference, and the angle through which the radius vector is deflected varies with time. The angle per unit time represents *angular velocity* or *angular frequency*, ω (the Greek letter "omega"):

$$\omega = \alpha/t,$$

whence

$$\alpha = \omega t.$$

The angle α is often expressed in radians instead of degrees of arc (a *radian* is that angle whose intercepted arc in a circle equals the radius of the circle. The length of a circle is $C = 2\pi R$ and it contains $2\pi R/R = 2\pi$ radians.)

The radius vector OA completes one revolution, or one period, in T seconds, giving angular velocity or angular frequency:

$$\omega = \alpha/t = 2\pi/T \text{ radians/sec.}$$

Since $1/T = f$, we have

$$\omega = 2\pi f \text{ radians /sec.}$$

The angle of rotation of the radius vector from the origin is

$$\alpha = \omega t = 2\pi f t.$$

The angle α is called the *phase angle.*

A uniformly rotating vector OA has its length projected onto a vertical diameter so that the length $OB = OA \sin \alpha$, i.e., it follows the sine law. If the length of the vector is A_m, the instantaneous values of the projected length a will be:

$$a = A_m \sin \alpha = A_m \sin \omega t.$$

At $\alpha = 0°$, $a = A_m \sin 0° = 0$.

At $\alpha = 90°$, $a = A_m \sin 90° = A_m$.

In the latter case, the instantaneous value of the projected length is equal to its peak value.

Varying the phase angle, and projecting the vector A_m onto a vertical diameter will show a succession of instantaneous values of the sinusoidal quantity (Fig. 131). The portion of the diagram on the left is a vector diagram (also termed a phasor diagram), and that on the right is a sine wave.

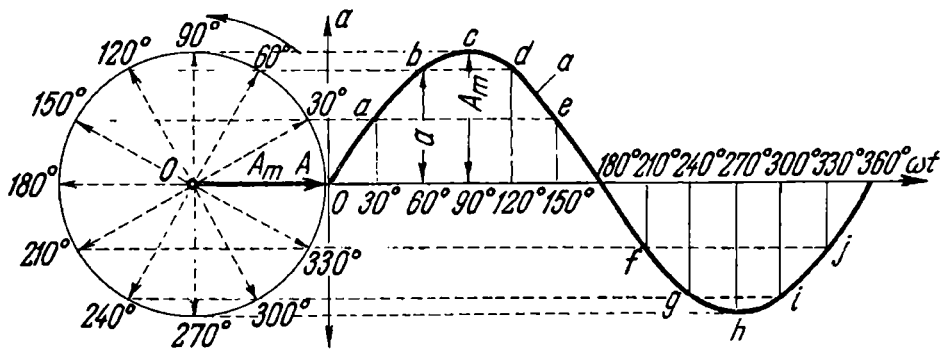


Fig. 131. Sine wave and its development by a rotating vector

The sine wave shown in Fig. 131 is, thus, drawn out by the uniform counterclockwise rotation of a radius vector (or phasor) of length equal to the peak value of the quantity.

To sum up, a quantity obeying the sine law can be represented in equation form, by a vector diagram (phasor diagram), or by a sine-wave form.

If the radius vector at $t = 0$ makes an angle ψ with the X-axis, the instantaneous value of the quantity will then be

$$a = A_m \sin(\omega t + \psi).$$

The angle ψ (the Greek letter "psi") is referred to as the initial *phase angle.*

A vector diagram and its development into a waveform by a rotating vector are shown in Fig. 132 for this case.

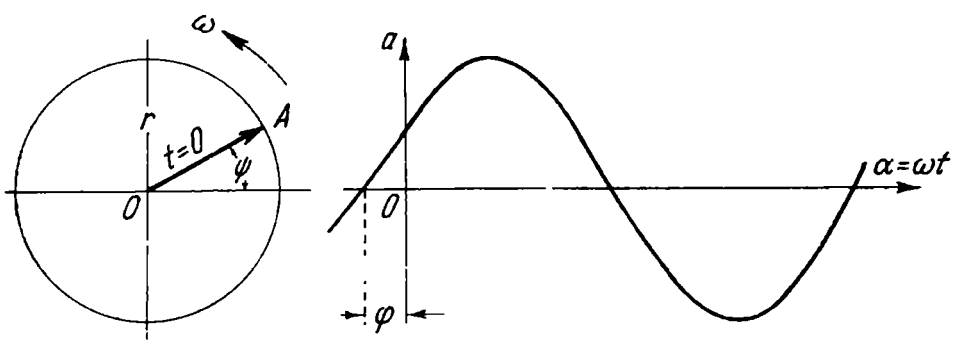


Fig. 132. A sine-wave form with an initial phase displacement

Suppose that two vectors which point in one and the same direction are rotated in synchronism. At any moment of time the two vectors will be turned through the same angle. The vectors and the quantities they represent are then said to be in phase. The vector diagram and the waveforms of two quantities in phase are shown in Fig. 133.

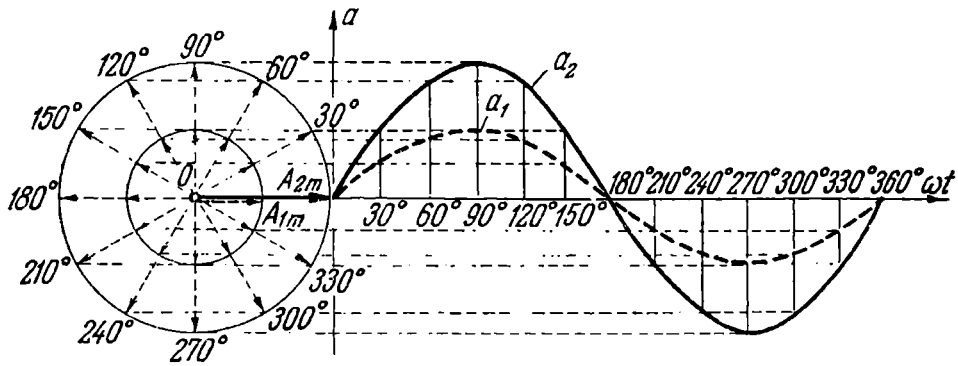


Fig. 133. Development of a sine wave by two coincident rotating vectors

In equation form, the two quantities will be written as follows:

$$a_1 = A_{1m} \sin \omega t;$$

$$a_2 = A_{2m} \sin \omega t.$$

If two rotating vectors are displaced through an angle φ with respect to each other, their waveforms will also be displaced through the same angle φ. Fig. 134 shows two waveforms displaced in phase through 90°. In such a case, the waveform a_1 is said to be in quadrature leading, and the waveform a_2 in quadrature lagging, with respect to the other waveform.

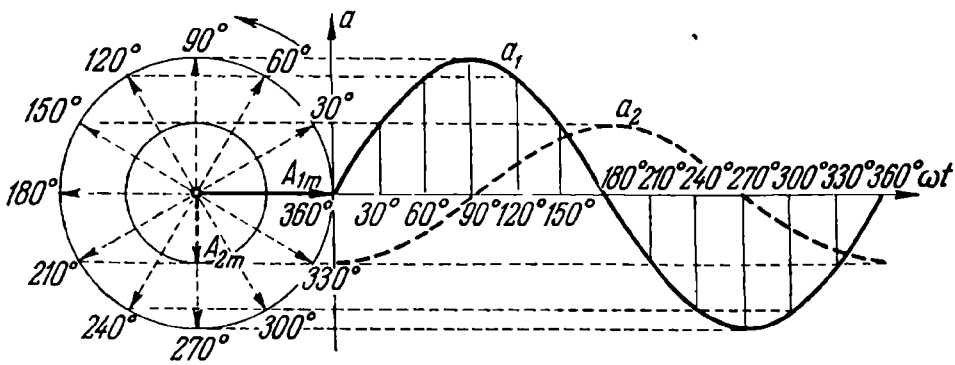


Fig. 134. Development of two sine waves displaced through 90° by two rotating vectors in quadrature

Calculations of a-c circuits involve addition and subtraction of sinusoidal quantities (currents, voltages, etc.).

Let us examine addition of two sinusoidal quantities described by the equations

$$a_1 = A_{1m} \sin (\omega t + \psi_1);$$
$$a_2 = A_{2m} \sin (\omega t + \psi_2).$$

Their sum will be

$$a = a_1 + a_2 = A_{1m} \sin (\omega t + \psi_1) + A_{2m} \sin (\omega t + \psi_2).$$

And we finally get

$$a = A_m \sin (\omega t + \psi).$$

Thus, the sum of two sine waves of the same period is likewise a sine wave of peak value A_m and with an initial phase angle ψ.

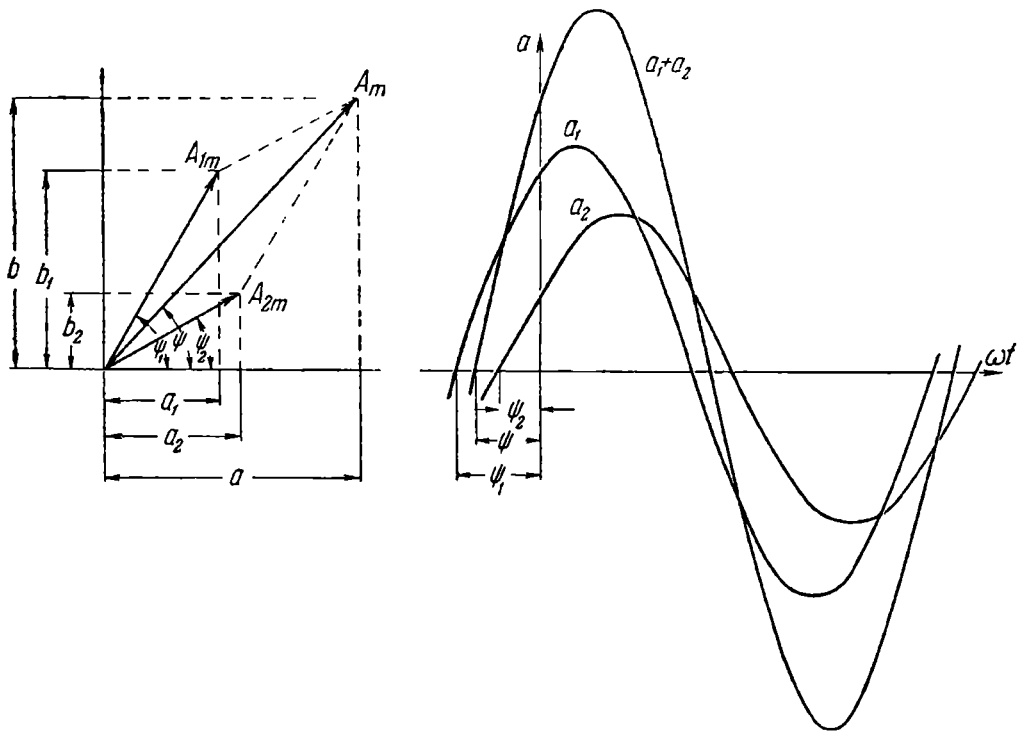


Fig. 135. Addition of two sine waves

Addition of sinusoidal quantities can be done by vector addition (on the left of Fig. 135). As follows from the graph

$$a_1 = A_{1m} \cos \psi_1; \qquad b_1 = A_{1m} \sin \psi_1;$$
$$a_2 = A_{2m} \cos \psi_2; \qquad b_2 = A_{2m} \sin \psi_2;$$
$$a = a_1 + a_2; \qquad b = b_1 + b_2.$$

The resultant vector A_m, equal to the geometrical sum of the vectors A_{1m} and A_{2m} is

$$A = \sqrt{a^2+b^2};$$

$$\tan \psi = b/a.$$

Vectorial addition of two sine waves is shown on the right of Fig. 135. Any instantaneous value of the resultant sine wave is equal to the sum of the instantaneous values of the component sine waves at any given instant of time.

The same technique is applicable to the addition of three or more sine waves.

To distinguish a vector from a scalar, vectors are written with a bar thus:

$$\bar{A} = \bar{A}_1+\bar{A}_2.$$

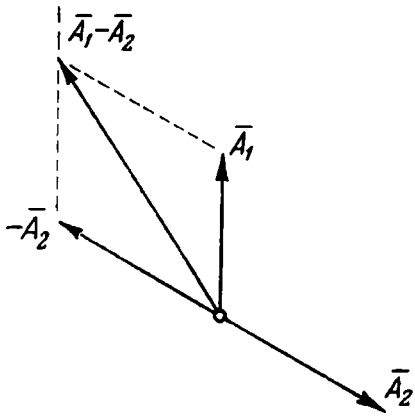


Fig. 136. Vector subtraction

Let us examine the subtraction of two vectors. Let vectors A_1 and A_2 represent some sinusoidal quantities and A_2 is to be subtracted from A_1 (Fig. 136). Subtraction of vectors can always be replaced by addition of the minuend and a subtrahend of equal magnitude and opposite sign:

$$\bar{A}_1-\bar{A}_2 = \bar{A}_1+(-\bar{A}_2).$$

80. Alternating-current Frequency Related to the Number of Poles and Rotor Speed

In a two-pole generator, the cycle of changes in the induced emf is completed during one revolution of the rotor, or within one period. If the rotor completes five revolutions per second, the emf will go through five cycles or changes per second, and the frequency of the current will be 5 cycles per second. Thus, the frequency of the current in a two-pole generator is numerically equal to the revolutions per second of its rotor. The current frequency, f, will be

$$f = n/60,$$

where n is the revolutions per minute of the rotor.

To obtain a current of commercial frequency (50 c/s), the rotor of a two-pole machine will have to make 3000 rpm:

$$f = n/60 = 3000 \div 60 = 50 \text{ c/s}.$$

In a four-pole machine, i.e., a machine having two pair of poles ($p = 2$) (Fig. 137), the cycle of changes in current will be completed during a half-revolution of the rotor (positions *1* through *5* in the diagram). By the time the rotor completes one revolution, the current will have gone through two cycles. In a six-pole machine ($p = 3$), three cycles are completed during one revolution of the rotor.

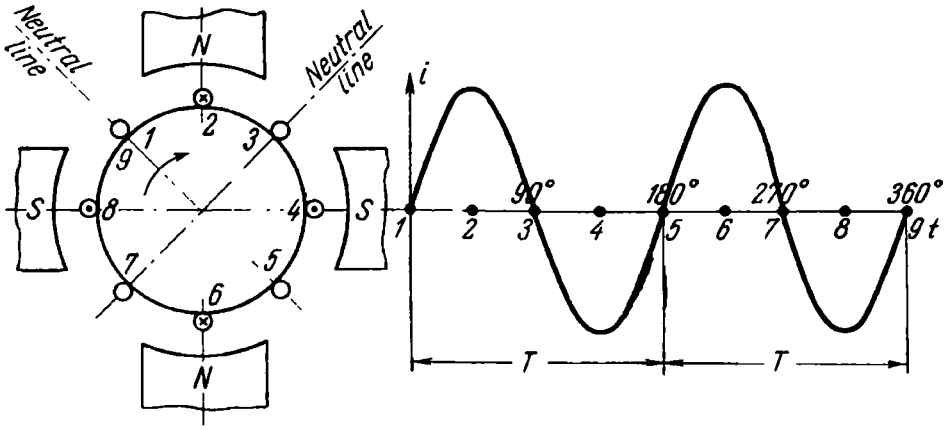


Fig. 137. Alternations of an alternating current in the rotor winding of a four-pole generator

Thus, for a machine having p pairs of poles the frequency of the current at $n/60$ revolutions per second will be p times that of a two-pole machine, or

$$f = pn/60.$$

Then the formula for the speed of an electrical machine will be

$$n = 60f/p.$$

Example 3. Determine the frequency of the alternating current generated by an eight-pole machine ($p = 4$) running at a speed of 750 rpm.

$$f = pn/60 = 4 \times 750 \div 60 = 50 \text{ c/s}.$$

Example 4. Determine the speed of a 20-pole generator ($p = 10$), if a frequency meter reads 25 c/s.

$$n = f60/p = 25 \times 60 \div 10 = 150 \text{ rpm}.$$

Example 5. A generator driven by a water wheel runs at 75 rpm. Determine the number of poles in the generator if the frequency of the current is 50 c/s.

$$p = f60/n = 50 \times 60 \div 75 = 40 \text{ pairs, or } 80 \text{ poles}.$$

81. Active, Virtual or RMS Value of Alternating Current

A "d-c ampere" is a measure of a steady current. On the other hand the "a-c ampere" must measure a current that is continually varying in amplitude and periodically reversing direction. To put the two on the same basis,

an a-c ampere is defined as an alternating current of 1 A which, flowing in a resistance, produces heat energy at an average rate equal to that produced by 1 A direct current in the same resistance.

Since the rate of heat production is

$$P = I^2R \text{ watts},$$

the equivalent alternating current is such that the square root of the average i^2 gives the value required. It is therefore also called the *root-mean-square*, or *rms*, *value*.

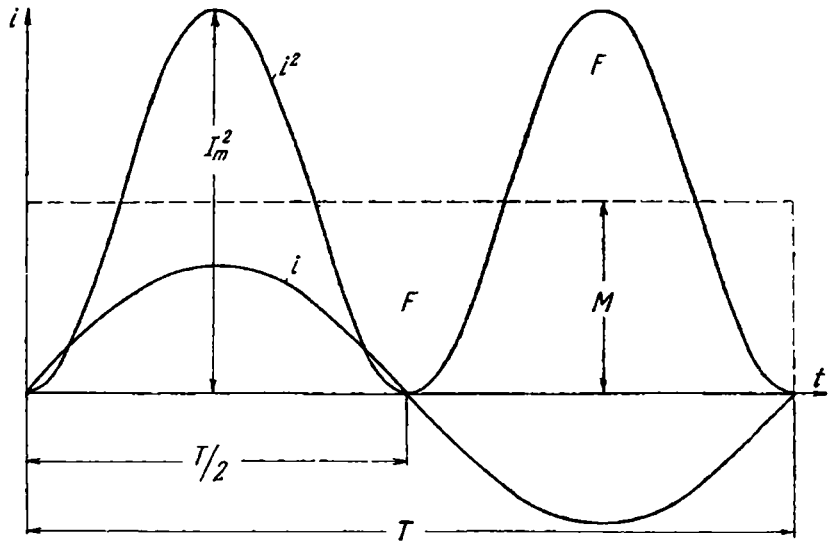


Fig. 138. The effective value of alternating current

Let the average i^2 over a complete period be M. Then for equivalent direct and alternating currents we have

$$I^2R = MR,$$

whence $I = \sqrt{M}$, and I is the rms value.

Higher mathematics gives $M = 1/2\, I_m^2$.

Finally we get

$$I = \sqrt{M} = I_m/\sqrt{2}.$$

The rms value of voltage or emf is found by the same technique:

$$V = V_m/\sqrt{2}; \qquad E = E_m/\sqrt{2}.$$

Rms values, also known as active, effective or virtual, are designated by capital letters (I, V, E) without subscripts.

Ammeters and voltmeters connected in a-c circuits read rms values.

In vector diagrams, it is more convenient to use rms values and not peak values. To this end, the lengths of the vectors are reduced by a factor of $\sqrt{2}$. The positions of the vectors in the diagram remain the same.

82. The Mean Value of a Current

The mean value (or average value) of a current may sometimes be required. The mean value of a sine wave over a complete cycle is zero. Therefore, the half-cycle mean value is intended (Fig. 139).

The instantaneous values over a half-cycle are averaged as follows. A rectangle is constructed, having a base $T/2$ and an area equal to that between the sine wave and the X-axis. The height of the rectangle will give the mean value of the current over a half-cycle.

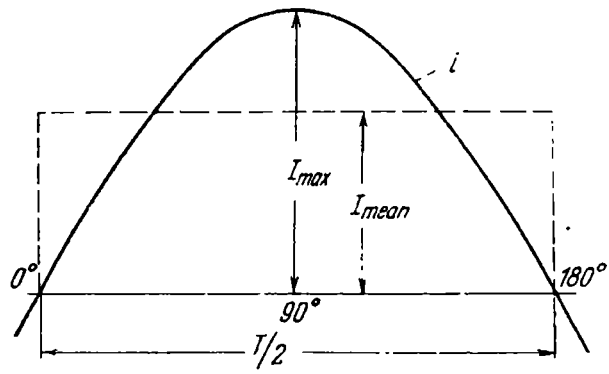


Fig. 139. The mean value of an alternating current

Higher mathematics gives the following relationship between the mean and peak values of a sine wave

$$I_{mean} = (2/\pi)I_{max} = 0.637\, I_{max}.$$

Similarly, for voltage and emf:

$$V_{mean} = 0.637\, V_{max};\ E_{mean} = 0.637\, E_{max}.$$

The ratio of the effective (rms) value to the mean value is termed the *form factor* and is designated by the letter K_f. For sine waveforms:

$$K_f = I/I_{mean} = I_{max}\pi/\sqrt{2} \times 2\, I_{max} = \pi/2\sqrt{2} = 1.11.$$

The ratio of the peak to the rms value is known as the *peak factor* and is designated by the letter K_p.

For sine waveforms

$$K_p = I_{max}/I = \sqrt{2} = 1.41.$$

83. Alternating-current Circuits with Resistance Only

Fig. 140 shows an electric circuit containing a resistance R. For simplicity, the effect of inductance and capacitance may be neglected. A sine-wave a-c voltage is applied to the terminals of the circuit:

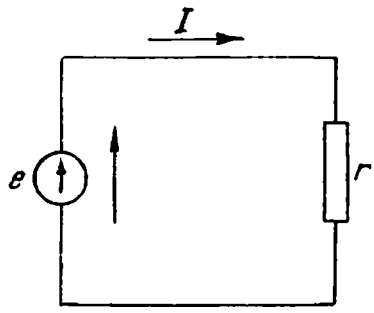


Fig. 140. A circuit with resistance only

$$v = V_{max} \sin \omega t.$$

By Ohm's Law, the instantaneous value of current will be

$$i = v/R = (V_{max}/R) \sin \omega t = I_{max} \sin \omega t,$$

where $I_{max} = V_{max}/R$.

Or, in terms of rms values

$$I_{max}/\sqrt{2} = V_{max}/\sqrt{2}R$$

and

$$I = V/R.$$

As follows from the last expression, in a purely resistive circuit Ohm's Law is just as valid for alternating current as it is for direct current. Also, Ohm's Law shows that the instantaneous value of voltage is proportional to the instantaneous value of current. Therefore, the voltage and the current of an a-c circuit with resistance only are in phase with each other. Fig.

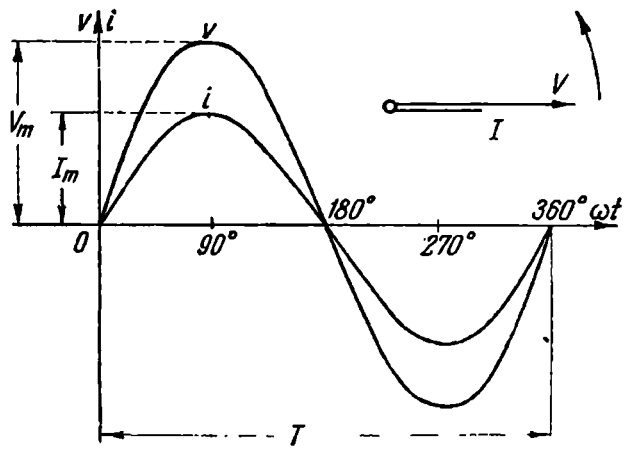


Fig. 141. Graph and vector diagram of an a-c circuit containing resistance only

141 shows the vector diagram and waveforms of the voltage and current in this circuit. The lengths of the vectors represent the rms values of both voltage and current. The resistance of conductors to alternating current is somewhat higher than to direct current due to the skin effect (explained in Sec. 85). To distinguish it from d-c (or ohmic) resistance, it is termed the *effective* resistance. The formula for it is $R = P/I^2$.

In the circuit shown in Fig. 140 the applied voltage is counterbalanced by the voltage drop across the resistance R. It is termed the *resistive voltage drop*, and is designated V_R:

$$V_R = IR.$$

The instantaneous power delivered to the circuit in question is the product of the instantaneous values of voltage and current:

$$p = vi.$$

Fig. 142 shows the instantaneous power averaged over a complete cycle. As can be seen, the power is a pulsating quantity with twice the current frequency.

The power averaged over a complete cycle, or simply the average power is designated by the letter P and can be derived from the formula (the proof of which is omitted for simplicity)

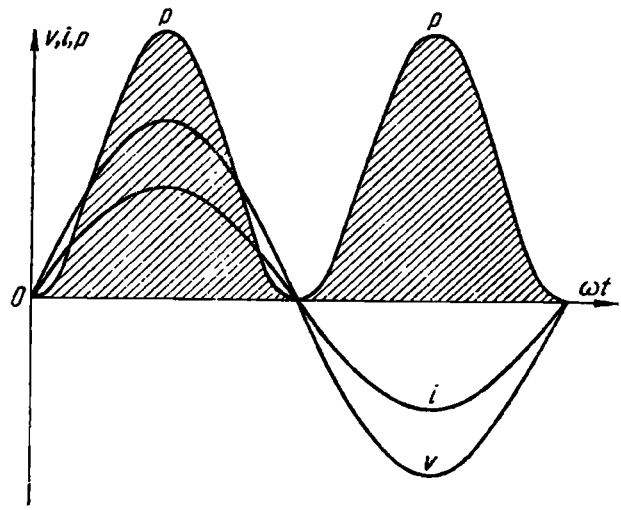


Fig. 142. Graph of instantaneous power in a resistive circuit

$$P = VI \cos \varphi,$$

where φ is the phase angle or phase difference between the voltage and the current.

The average power is also termed the true power. The formula of true power given above is valid for all a-c circuits. In purely resistive circuits, the voltage and current vary in step with each other (i.e., they are in phase). Therefore, the angle φ is zero and $\cos \varphi = 1$. Then, in a purely resistive circuit $P = VI$, or $P = I^2R$. Thus, for a-c circuits containing resistance only, the formula for power is the same as for d-c circuits.

84. Alternating-current Circuits with Inductance Only

As has been stated earlier, whenever an electric circuit is being closed or opened or its current is being varied, an emf of self-induction is induced in it due to the magnetic field set up around the conductor. This emf is reactive. When the current in the circuit increases, it opposes the applied emf, for which reason it takes some time for the current in the circuit to attain a steady-state value. Conversely, when the current in the circuit is decreased, the emf of self-induction tends to prevent the current from decreasing.

It has also been shown that the emf of self-induction *depends on the rate of change* in the current and the inductance (number of turns, presence, or otherwise, of a steel core) of the circuit:

$$e_L = -L(di/dt).$$

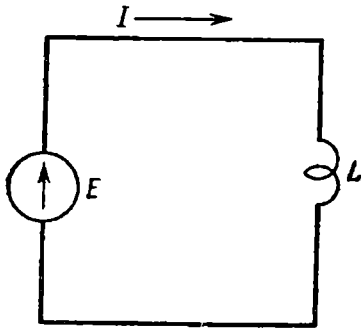


Fig. 143. An a-c circuit with inductance only

In a-c circuits, the emf of self-induction is induced continually since the current is varying all the time.

Fig. 143 shows an a-c circuit containing an inductor (a coil) of inductance L, without an iron core. For simplicity, it may be assumed that the resistance of the inductor is negligibly small, or that the coil is a pure inductance.

Fig. 144 shows changes in the current flowing through a pure inductance during one cycle. The first half-cycle is divided into equal time intervals.

During the first interval, *0-1*, the current rises from zero to *1-1'* by an increment *a*. During the second interval, the instantaneous value of current rises to *2-2'* by an increment *b*. During the third interval, the current rises to *3-3'* by an increment *c*, etc.

Finally, the current rises to its maximum or peak value (at 90°). However, the increments become progressively smaller until, at the peak value of current, the increment is zero.

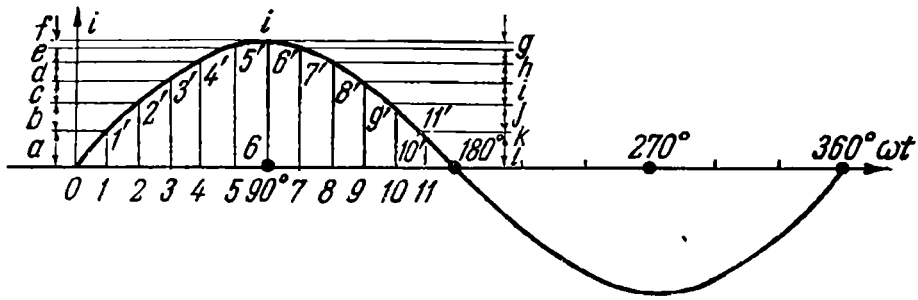


Fig. 144. Rate of change of an alternating current

As the current drops from the peak value to zero, the decrements grow progressively larger, until at zero current the decrement is the largest. The current falls rapidly to zero, then reverses its direction and rises towards the maximum again.

Thus, the rate of change is highest near the zero value of current and lowest at the peak values. In other words, not only does the current vary in magnitude and reverse its direction, but also its rate of change varies.

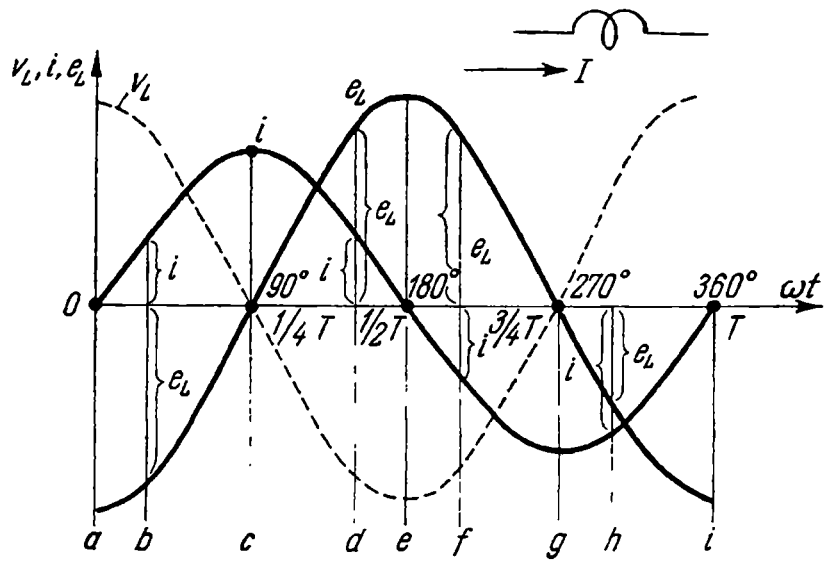


Fig. 145. Emf of self-induction in a coil connected in an a-c circuit

All these variations combine to produce an alternating magnetic field which threads the turns of the coil, inducing an emf of self-induction in them.

In our case (Fig. 145) the inductance of the coil remains constant. Therefore, the value of the emf of self-induction depends solely on the rate of change of current, being highest at the zero values of current. Therefore, it may be said that the emf of self-induction lags 90° (a quarter of a cycle) behind the current or, which is the same, behind the magnetic flux that induces it.

As has been shown earlier, two sine waves displaced 90° with respect to each other may be represented by vectors placed at right angles (Fig. 146).

Since the emf of self-induction opposes the change of current in a-c circuits, the applied voltage should counterbalance this emf. In other words, the supply voltage at any time shoud be equal and opposite in sign to the emf of self-induction. If this requirement is not met, no current will flow through

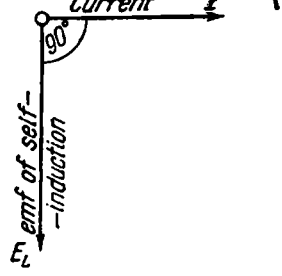


Fig. 146. Current through a coil leads the emf of self-induction by 90°

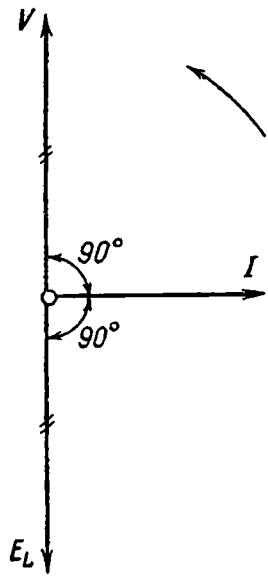


Fig. 147. Supply voltage applied to a coil leads the current by 90° and opposes the emf of self-induction

the inductor. This condition is illustrated in Fig. 147 where E_L is the emf of self-induction, V is the applied voltage, and I is the current flowing through the inductor. The applied voltage V will lead the current in the inductor by 90°. This is true of all a-c circuits containing inductance (the windings of generators, motors, transformers, and induction coils).

It can be shown that the rate of change of current is proportional to the angular frequency ω. Then the effective emf of self-induction can be found from the formula

$$E_L = \omega LI = 2\pi fLI.$$

Since $E_L = V_L$, $V_L = 2\pi fLI$. Denoting $2\pi fL$ by X_L gives $V_L = X_L I$. The quantity X_L, known as *inductive reactance*, plays a part similar to resistance in Ohm's Law. Therefore, Ohm's Law for a-c circuits containing inductive reactance will be

$$I = V_L / X_L.$$

Inductive reactance is expressed in ohms and the formula for it is

$$X_L = \omega L = 2\pi fL.$$

As follows from this formula, inductive reactance depends on the frequency of the current and the inductance of the circuit. Therefore, the same inductor connected in circuits differing in frequency will have different inductive reactances. For a coil of 0.05 henry inductance connected in a circuit operating at 50 c/s the inductive reactance will be

$$X_L = 2\pi fL = 2 \times 3.14 \times 50 \times 0.05 = 15.7 \text{ ohms.}$$

When the same coil is connected in a circuit operating at 400 c/s, its inductive reactance will be

$$X_L = 2\pi fL = 2 \times 3.14 \times 400 \times 0.05 = 125.6 \text{ ohms.}$$

The proportion of the applied voltage that goes to counterbalance the inductive reactance is called the *inductive voltage drop* or the reactive voltage:

$$V_L = X_L I.$$

It can be shown that inductance absorbs no power (Fig. 148). During the first quarter of a cycle the current and magnetic flux of the coil increase, and the coil draws some power from the supply circuit to build up the magnetic field (the power is positive, and the energy is represented by the area between the curve p and the axis ωt). The energy stored in the magnetic field during the build-up can be found from the formula

$$W_M = LI_{max}^2/2.$$

In the second quarter-cycle the current decreases. The emf of self-induction will, however, tend to oppose its decrease. The coil becomes, as it were, a generator of electric energy, returning the stored energy to the

magnetic field (now the power is negative, and the curve p in Fig. 148 is located below the axis ωt).

This chain of events repeats itself during the second half-cycle. Thus, a proportion of power is continually exchanged between the field and the inductive circuit. This is why the power in an inductance is not a "loss",

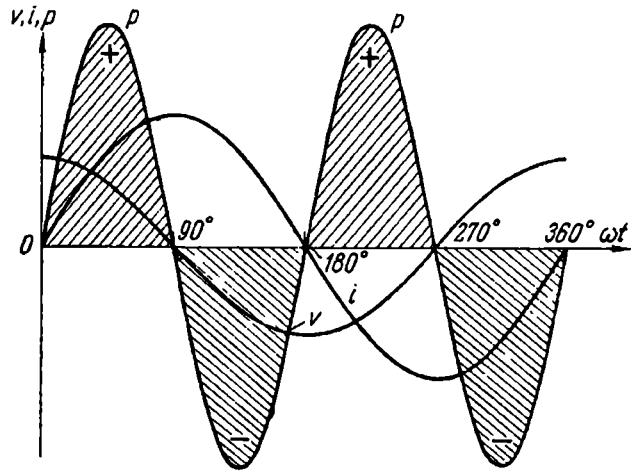


Fig. 148. Curves of instantaneous voltage, current and power in a circuit containing inductance only

although there is a potential difference between the circuit terminals and a current flows around the circuit. This conclusion is also borne out by the formula for true power:

$$P = VI \cos \varphi.$$

In our case, the current lags 90 degrees behind the voltage, and $\cos \varphi = \cos 90° = 0$. Thus, the true power in an inductance is zero, and there is no loss of power.

85. The Skin Effect

The resistance of a conductor is not the same for alternating current as it is for direct current. The reason is this. A straight conductor carrying current has a magnetic field in the form of concentric circles. This field is produced both around and inside the conductor. The current through the conductor can be pictured as several parallel streams. Those closer to the centre of the conductor link a greater magnetic flux. The inductance of a current flow and the inductive reactance are directly proportional to the magnetic flux linked with it. Therefore, the inductance of the central parts of the conductor is greater than that of the outer skin, as is the inductive reactance, and the current flows mainly at and near the surface of the conductor where the reactance is least. The useful cross-section of the conductor is less than the actual area, and the effective resistance is consequently higher. This is called the *skin effect*.

The skin effect is negligible at a standard frequency of 50 c/s, in conductors with small cross-sections and in copper wires, but it is considerable at high frequencies, in conductors of large cross-section and in iron or steel wires.

86. Alternating-current Circuit Containing Capacitance

If we connect an ideal capacitor (i.e., one with no losses) in a d-c current, it will become charged to full emf almost instantaneously. The charging current will flow only during the "building-up" period and will cease

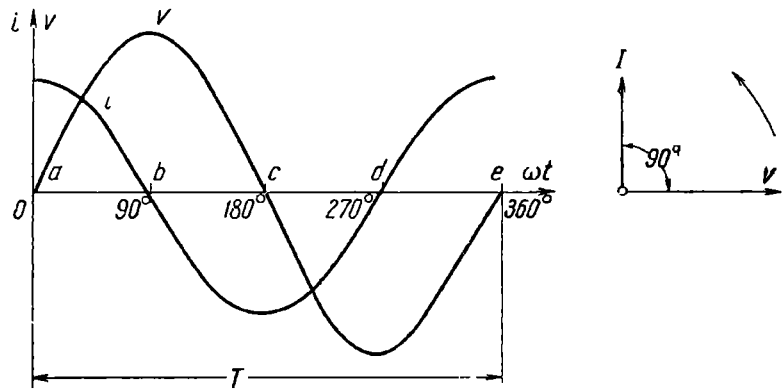


Fig. 149. Graph and vector diagram of an a-c circuit with capacitance only

flowing as soon as the capacitor has attained the steady emf of the source. This implies that for a direct current, a capacitor is a break in the circuit or an infinitely high resistance.

In Fig. 149 a sine-wave a-c voltage is applied to a capacitor. During the first quarter-cycle, the applied voltage increases to the peak value, and the capacitor is charged to that value. The current is largest at the beginning of the cycle and becomes zero at the maximum value of the voltage, so there is a phase difference of 90 degrees between voltage and current. During the first quarter-cycle the current flows in the normal direction through the circuit; hence the current is positive.

In the second quarter-cycle, the voltage applied to the capacitor decreases. The capacitor loses its charge, and its current flows against the applied voltage as the capacitor discharges into the circuit. Hence, the current is negative during this quarter-cycle and reaches a maximum value when the voltage is zero.

The third and fourth quarter-cycles repeat the events of the first and second, respectively, with the difference that the polarity of the applied voltage is reversed, and there are corresponding current changes. In other words, an alternating current flows in the circuit because of the charging and discharging of the capacitance. As shown by Fig. 149, the current

starts its cycle 90 degrees ahead of the voltage, so the current in a capacitor leads the applied voltage by 90 degrees—the opposite of the inductance current-voltage relationship.

The current in a capacitor circuit can be found from the formula

$$i = dq/dt,$$

where q is the quantity of electricity flowing around the circuit.

From electrostatics

$$q = Cv_c = Cv,$$

where

C = capacitance of capacitor;
v = supply voltage;
v_c = voltage across capacitor plates.

Finally, for the current we have

$$i = C(dv_c/dt) = C(dv/dt).$$

From this expression it follows that i has a maximum value when the ratio dv/dt is greatest (positions a, c and e in the graph). When $dv/dt = 0$ (positions b and d in Fig. 149), i is also zero.

It has been stated earlier that the rate of change of current is proportional to the angular frequency ω. Therefore from the formula

$$i = C\,dv/dt$$

it follows that the rate of change of voltage is likewise proportional to the angular frequency, and for the effective current we have

$$I = 2\pi fCV.$$

Denoting $1/2\pi fC$ as X_C, we obtain what is known as *capacitive reactance*, which likewise plays a part similar to that of resistance in Ohm's Law. Therefore, Ohm's Law for a capacitive circuit will be

$$I = V/X_C.$$

The voltage across the plates of a capacitor is

$$V_C = I_C X_C.$$

That part of the applied voltage which charges a capacitor is termed the *capacitive voltage drop* and is designated V_C.

As is the case with inductive reactance, capacitive reactance is a frequency-dependent quantity. As distinct from the former, however, it decreases with increasing a-c frequency.

Example 6. Calculate the reactance of a capacitor with a capacitance of 5 μF at 50 c/s.

$$X_C = 1/2\pi fC = 1/2 \times 3.14 \times 50 \times 5 \times 10^{-6} = 636 \text{ ohms}.$$

At 400 c/s,

$$X_C = 1/2 \times 3.14 \times 400 \times 5 \times 10^{-6} = 79.5 \text{ ohms.}$$

Although the unit of capacitive reactance is the ohm, there is no power dissipation in capacitance. The energy stored in the capacitor in one quarter of a cycle is simply returned to the circuit in the next. This is borne out by the formula for true power:

$$P = VI \cos \varphi.$$

In a capacitive circuit the current leads the applied voltage by 90 degrees. Therefore $\varphi = 90°$, and $\cos \varphi = 0$, and the true power in a capacitive circuit is also zero.

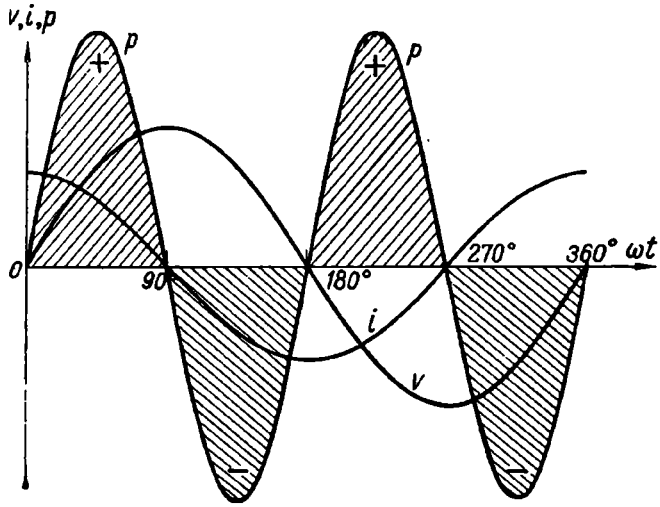


Fig. 150. Instantaneous power in a capacitive circuit

The same is shown graphically in Fig. 150: the energy taken from the supply circuit is stored in the capacitor during the first quarter-cycle and returned during the next.

The energy stored by a capacitor at maximum voltage across its plates can be determined from the formula

$$W_C = CV^2_{max}/2.$$

87. Resistance and Inductive Reactance in Series

Practical circuits often contain both resistance and reactance (inductive or capacitive). An induction coil, for example, has both resistance and inductance. Therefore, for a current to be able to flow through it, the applied voltage should be high enough to counteract both the resistive voltage drop and the emf of self-induction in the coil. This example is illustrated in Fig. 151.

In Fig. 152, I stands for the alternating current of the coil and E_L for the emf of self-induction, which lags 90° behind the coil current. V_L denotes that part of the applied voltage which goes to counterbalance E_L. It is equal but opposite in sign to the latter. V_R stands for that part of the applied voltage which counterbalances the resistive voltage drop; it is in phase with the coil current. The total applied voltage must be equal to the vectorial sum of V_R and V_L, since the two are vectors. The diagonal of the parallelogram will give the resultant (total) applied voltage.

Referring to Fig. 152, the total applied voltage V leads the current I by an angle φ:

$$\tan \varphi = V_L/V_R = IX_L/IR = X_L/R.$$

In a purely inductive circuit, the applied voltage leads the current by 90 degrees. In an inductive circuit, also containing a resistance, this angle is less than 90 degrees. When the inductance of the circuit is zero, the phase difference is also zero, and the current in the coil is in step with the applied voltage.

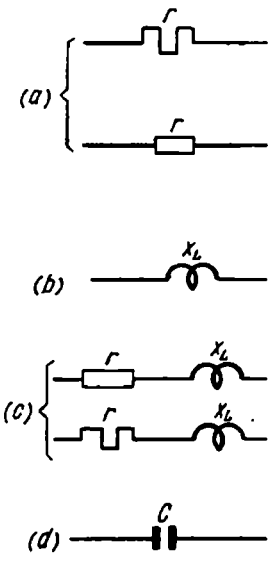


Fig. 151. Symbols for resistances in a-c circuits

Fig. 153 gives a graph of instantaneous power for a circuit containing a resistance and an inductive reactance connected in series. As distinct from a purely capacitive or a purely inductive circuit, the true power dissipated in the circuit is different from zero. During a part of the cycle, some power is used up in heating the conductor and building up a magnetic field around the coil (the power is positive). During another part of the cycle the power is returned to the circuit (the power is negative).

To sum up, the average or true power of an alternating current is directly proportional to the voltage V and the current I, and inversely proportional to the phase angle φ between them. The greater the angle, the smaller the true power.

88. The Voltage Triangle

Fig. 154 gives the triangle of Fig. 152 cross-hatched. It is a right triangle, and any side can be found by the theorem of Pythagoras (the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides). Therefore

$$V^2 = V_R^2 + V_L^2,$$

whence

$$V = \sqrt{V_R^2 + V_L^2}.$$

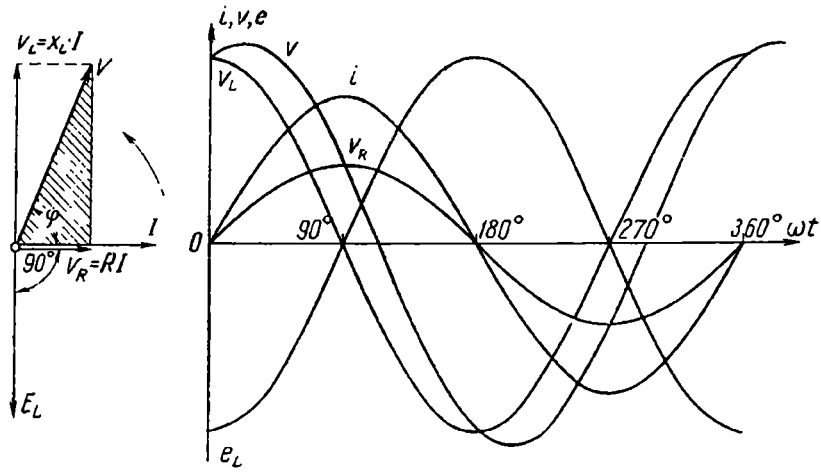


Fig. 152. Vector diagram and graph for a series *RL* circuit

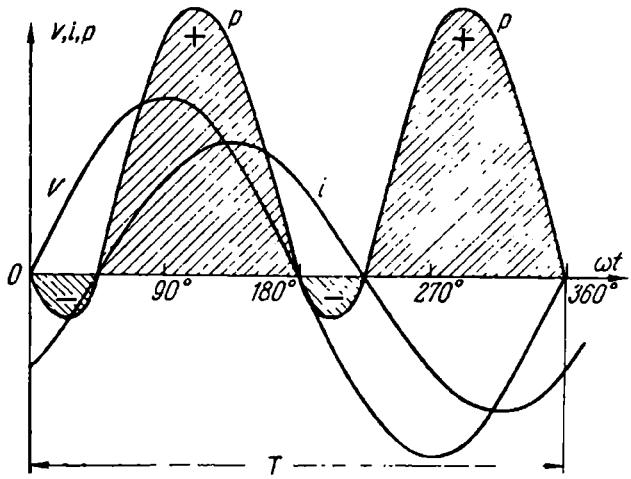


Fig. 153. Instantaneous power in a series *RL* circuit

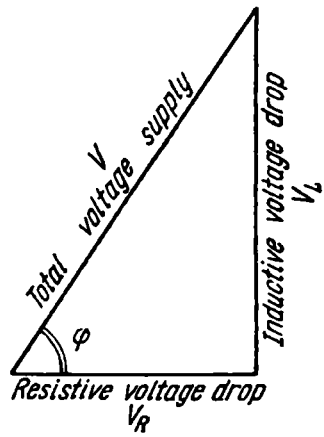


Fig. 154. Voltage triangle

Example 7. Find the voltage which must be applied to a coil in order to produce a current of 5 A, if the resistance is $R = 6$ ohms and the inductive reactance is $X_L = 8$ ohms.

The resistive voltage drop

$$V_R = IR = 5 \times 6 = 30 \text{ V}.$$

The inductive voltage drop

$$V_L = IX_L = 5 \times 8 = 40 \text{ V}.$$

The total voltage drop is equal to the applied voltage:

$$V = \sqrt{V_R^2 + V_L^2} = \sqrt{30^2 + 40^2} = \sqrt{900 + 1600} = \sqrt{2500} = 50 \text{ V}.$$

It should be noted that a voltmeter connected in the supply circuit will give readings different from the sum of V_R and V_L ($30 + 40 \neq 50$).

89. The Impedance Triangle

When a circuit contains both resistance and reactance (inductive or capacitive), their combined effect is called *impedance*, symbolized by the letter Z.

If the sides of the triangle shown in Fig. 155*a* are divided by the current I (Fig. 155*b*), the angles will remain the same, and the new triangle will be similar to the first one.

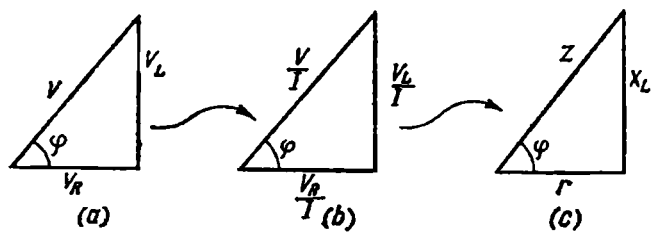


Fig. 155. Impedance triangles

In the new triangle (Fig. 156) one of the sides will represent the resistance of the circuit, the other will give the inductive reactance, and the hypotenuse will indicate the impedance.

As follows from the triangle and the theorem of Pythagoras, the impedance Z is equal to the vectorial sum of the resistance R and the inductive reactance X_L:

$$Z^2 = R^2 + X_L^2,$$

whence

$$Z = \sqrt{R^2 + X_L^2}.$$

If one of the components making up the impedance (resistance or reactance) is less, by a factor of 10 or more, than the other, it may be neglected. This can be easily proved by direct calculation.

Example 8. Find the impedance of a circuit where $R = 9$ ohms and $X_L = 12$ ohms.

$$Z = \sqrt{R^2+X_L^2} = \sqrt{81+144} = \sqrt{225} = 15 \text{ ohms.}$$

Simple addition of the resistance and reactance would give a wrong value of impedance:

$$9+12 = 21 \text{ ohms.}$$

Example 9. The impedance of an electromagnet is 25 ohms. The resistance is 15 ohms. Determine the inductive reactance.

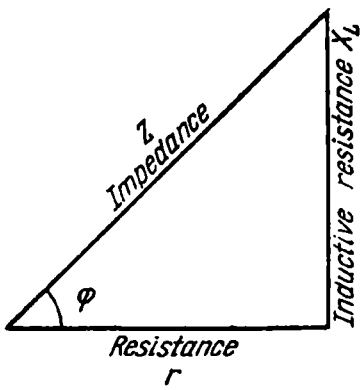


Fig. 156. Impedance triangle

$$Z^2 = R^2+X_L;$$
$$X_L^2 = Z^2-R^2;$$
$$X_L = \sqrt{Z^2-R^2}.$$

Therefore,

$$X_L = \sqrt{25^2-15^2} = \sqrt{625-225} = \sqrt{400} = 20 \text{ ohms.}$$

Example 10. The inductive reactance of a motor winding is 14 ohms. The impedance is 22 ohms. Find the resistance.

$$Z^2 = R^2+X_L^2;$$
$$R^2 = Z^2-X_L^2;$$
$$R = \sqrt{Z^2-X_L^2}.$$

Therefore,

$$R = \sqrt{22^2-14^2} = \sqrt{484-196} = \sqrt{288} = 16.97 \text{ ohms.}$$

Example 11. What will be the readings of a voltmeter connected in the circuit shown in Fig. 157?

$$V_R = IR = 5\times12 = 60 \text{ V.}$$
$$V_L = IX_L = 5\times16 = 80 \text{ V.}$$
$$V = \sqrt{V_R^2+V_L^2} = \sqrt{60^2+80^2} = \sqrt{3600+6400} = \sqrt{10{,}000} = 100 \text{ V.}$$

The impedance will be

$$Z = \sqrt{R^2+X_L^2} = \sqrt{12^2+16^2} = \sqrt{144+256} = \sqrt{400} = 20 \text{ ohms.}$$

Multiplying Z by the current I gives

$$I\times Z = 5\times20 = 100 \text{ V.}$$

Consequently, if $V_R = IR$ and $V_L = IX_L$, then $V = IZ$.

90. Ohm's Law for a Circuit Containing Resistance and Inductance

As follows from $V = IZ$,

$$I = V/Z \quad \text{or} \quad I = V/\sqrt{R^2+X_L^2}.$$

The above expression is Ohm's Law for a circuit containing resistance R and inductance L connected in series.

Example 12. Calculate the current flowing through a coil of 5 ohms inductive reactance and 1 ohm resistance if the applied voltage is 12 V.

The impedance of the coil is

$$Z = \sqrt{R^2+X_L^2} = \sqrt{1^2+5^2} = \sqrt{26} = 5.1 \text{ ohms.}$$

From Ohm's Law for a circuit containing resistance and inductance connected in series it follows that

$$I = V/Z = 12 \div 5.1 = 2.35 \text{ A.}$$

In purely resistive circuits the current will be

$$I = V/R.$$

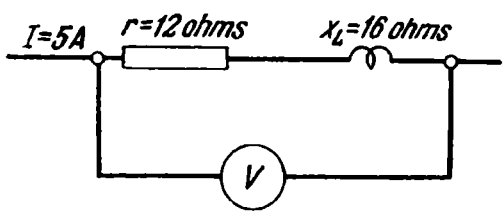


Fig. 157. See Ex. 11

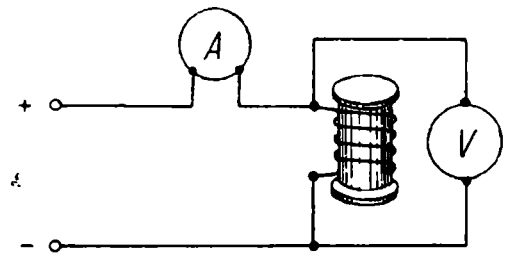


Fig. 158. Determining the resistance of an a-c load

Where the resistance is negligibly small and the circuit may be taken as purely inductive, Ohm's Law can be rewritten thus:

$$I = V/X_L.$$

The effective resistance of a circuit having a certain impedance (the winding of an electric motor, an induction coil or an electromagnet) can be determined by connecting it to a d-c source (Fig. 158) and by dividing the voltage across the circuit by the current around it, since

$$R = V/I.$$

This is only true of low frequencies where the skin effect may be neglected and the effective resistance of the circuit is equal to its ohmic (d-c) resistance.

91. Resistance and Capacitance in Series

Fig. 159 shows a circuit (and its vector diagram) containing resistance and capacitance connected in series.

The applied voltage V is the vectorial sum of the resistive voltage drop V_R which is in phase with the current, and the capacitive voltage drop V_C, which lags 90° behind the current. The applied voltage lags behind the current by an angle φ:

$$\tan \varphi = V_C/V_R = X_C I/RI = X_C/R = 1/R\omega C.$$

Resistance and capacitance connected in series are often met with in practice. By way of example, a capacitor having a real dielectric and connected in an a-c circuit may be pictured as a series combination of R and C. If the resistance of the dielectric were infinitely great (for an ideal dielectric $R = \infty$), the phase angle between the voltage and current would be 90 degrees. The conduction current due to imperfections in the dielectric, however, reduces the phase angle. The difference between 90° and the phase angle between voltage and current, designated by the letter δ (delta), is known as the *dielectric loss angle.*

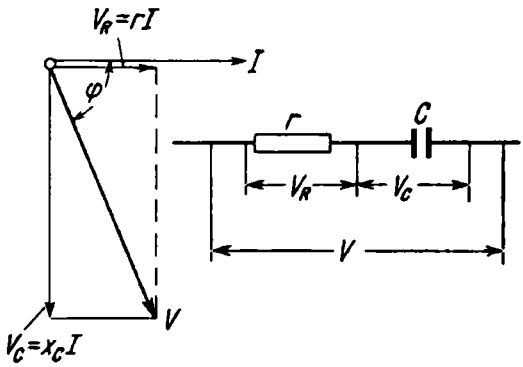


Fig. 159. A series RC circuit

The power loss in a dielectric, or dielectric loss, can be determined from the equation

$$P = V^2 2\pi f C \tan \delta \text{ (watts)},$$

where V = voltage in volts;
f = frequency in c/s;
C = capacitance in farads.

Figure 160 shows instantaneous values of the voltage and current for R and C connected in series. Fig. 161 shows their instantaneous power.

Referring to the graph, during some part of the cycle a proportion of energy goes to heat the resistance R and to build up an electric field (the power is positive). During another part of the cycle the energy stored in

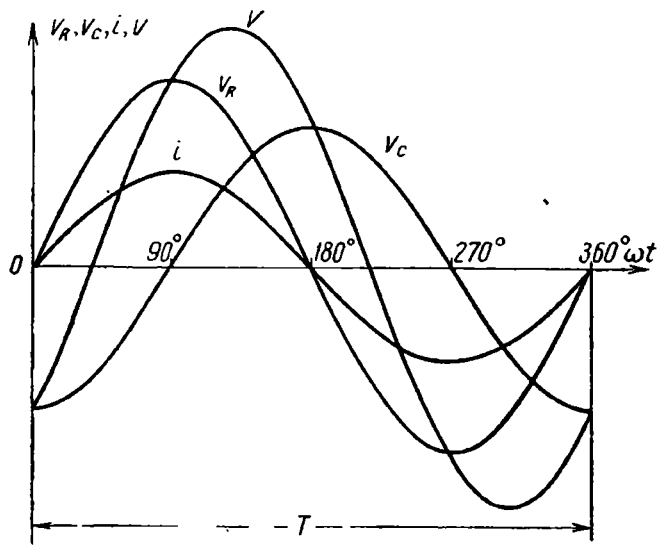


Fig. 160. Instantaneous voltage and current for a series RC circuit

the electric field of the capacitor is returned to the circuit. The true power will, as before, be

$$P = VI \cos \varphi.$$

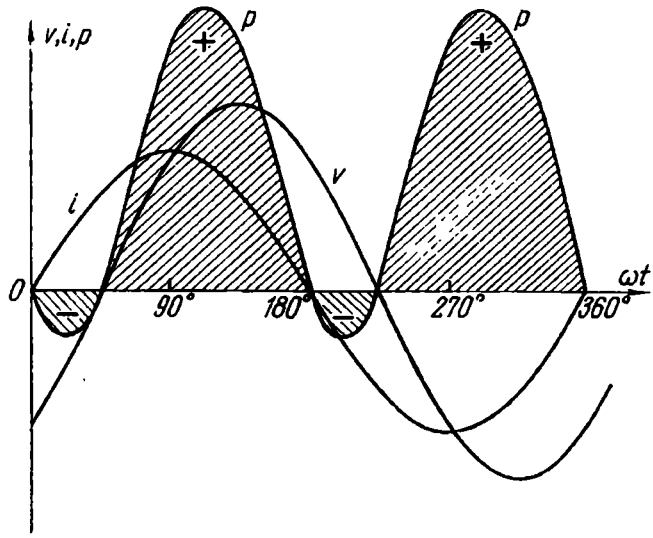


Fig. 161. Instantaneous power in a series *RC* circuit

From the vector diagram it follows that:

$$V = \sqrt{I^2R^2 + I^2X_C^2} = I\sqrt{R^2 + X_C^2}.$$

Since

$$\sqrt{R^2 + X_C^2} = Z,$$

Ohm's Law for a circuit containing resistance and capacitance in series (called an *RC* network) will be

$$I = V/Z.$$

Dividing the sides of the voltage triangle (Fig. 159) by the current gives an impedance triangle for the same circuit (Fig. 162).

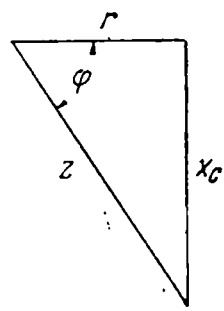


Fig. 162. Impedance triangle for a series *RC* circuit

92. Resistance, Inductance and Capacitance in Series

Fig. 163 shows a circuit containing resistance, inductance and capacitance connected in series and its vector diagram.

The applied voltage is the sum of the resistive voltage drop, the inductive voltage drop and the capacitive voltage drop.

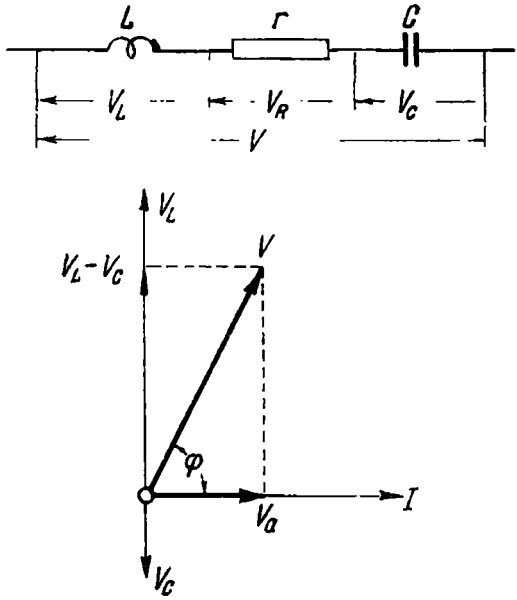


Fig. 163. *R*, *L* and *C* connected in series

V_L and V_C are 180° out of phase with each other (or reversed in phase). Therefore, when combined by the parallelogram law they cancel each other.

From the vector diagram we find that

$$V = \sqrt{I^2R^2+I^2(X_L-X_C)^2} = \\ = I\sqrt{R^2+(X_L-X_C)^2}.$$

Ohm's Law for this combination will be

$$I = V/\sqrt{R^2+(X_L-X_C)^2} = V/Z.$$

When $X_L=X_C$, the current $I= V/R$; the circuit behaves like a purely resistive circuit, and the applied voltage and the current are in phase. This is known as the *voltage*, or *series, resonance*. Fig. 164 shows a graph and a vector diagram for voltage resonance, which occurs when

$$X_L = X_C \quad \text{or} \quad \omega L = 1/\omega C.$$

The following cases of voltage resonance are possible:

(1) the inductance is constant and the capacitance varies so that

$$C = 1/\omega^2 L;$$

(2) the capacitance is constant and the inductance varies so that

$$L = 1/\omega^2 C;$$

(3) L and C vary so that

$$\omega L = 1/\omega C;$$

(4) and, finally, the angular frequency varies so that

$$\omega = 1/\sqrt{LC}.$$

Example 13. In a series LCR circuit $R = 6$ ohms; $X_L = 10$ ohms; and $X_C = 2$ ohms. The applied voltage is 120 V. Find the current under these conditions and at resonance if $X_L = X_C = 10$ ohms.

The current around the circuit is

$$I = V/\sqrt{R^2+(X_L - X_C)^2} = 120/\sqrt{6^2+(10-2)^2} = 120/10 = 12 \text{ A}.$$

The component voltage drops are:

$$V_R = IR = 12\times6 = 72 \text{ V};$$
$$V_L = IX_L = 12\times10 = 120 \text{ V};$$
$$V_C = IX_C = 12\times2 = 24 \text{ V}.$$

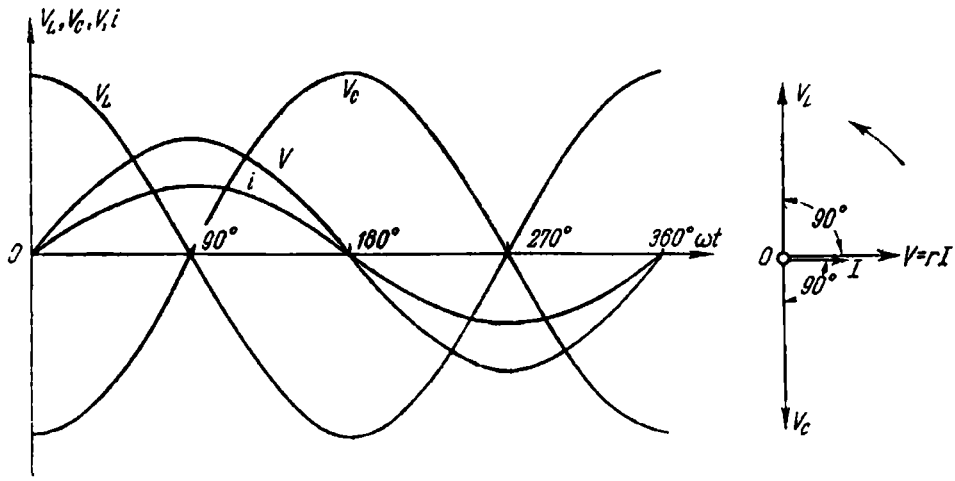


Fig. 164. Graph and vector diagram for voltage (series) resonance

The current at resonance is

$$I = V/R = 120 \div 6 = 20 \text{ A}.$$

The component voltage drops at resonance are:

$$V_R = IR = 20\times6 = 120 \text{ V};$$
$$V_L = IX_L = 20\times10 = 200 \text{ V};$$
$$V_C = IX_C = 20\times10 = 200 \text{ V}.$$

The examples given above show that voltage resonance may lead to a big rise in the component voltage drops. Under certain conditions, this may result in the rupture of insulation in inductors, apparatus and instruments, the breakdown of the dielectric in capacitors, etc.

93. Admittance

Calculation of parallel a-c circuits involves the treatment of what is known as *admittance*, the reciprocal of impedance. It is the ease with which an alternating current flows in a circuit.

Consider the vector diagram in Fig. 165. The projection of the total current I on the voltage vector V resolves the current into two components.

The component which coincides in direction with the voltage vector is *active* or *energy current*. It is designated I_a and the formula for it is

$$I_a = I \cos \varphi.$$

The other component is in quadrature with the voltage vector and is termed *reactive, quadrature,* or *wattless current*. It is designated I_q and the formula for it is

$$I_q = I \sin \varphi.$$

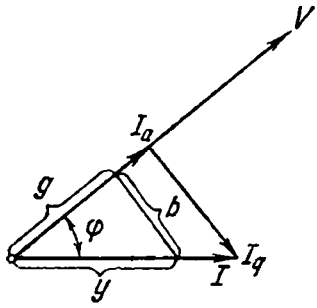


Fig. 165. Resolution of current into active and reactive components

From Ohm's Law for a-c circuits

$$I = V/Z.$$

It follows from the impedance triangle that

$$\cos \varphi = R/Z; \quad \sin \varphi = X/Z.$$

Using the three expressions, we obtain

$$I_a = I \cos \varphi = (V/Z)(R/Z) = V(R/Z^2).$$

By analogy with the formula for direct current ($I = VG$), the ratio R/Z^2 can be replaced by G. Then the expression will have the form

$$I_a = VG.$$

The quantity G is *conductance*.
Similarly

$$I_q = I \sin \varphi = (V/Z)(X/Z) = V(X/Z^2).$$

Replacing the ratio X/Z^2 by B, we obtain

$$I_q = VB.$$

The quantity B is *susceptance* (the reciprocal of reactance).
Finally

$$I = V/Z = V(1/Z).$$

Designating $1/Z$ as Y will give

$$I = VY.$$

The quantity Y is termed *admittance*.
Conductance, susceptance and admittance are all expressed in mhos (ohm^{-1}).
Fig. 165 shows a current triangle in which the sides are I, I_a and I_q.
By Pythagoras' theorem

$$I^2 = I_a^2 + I_q^2 \quad \text{or} \quad I = \sqrt{I_a^2 + I_q^2}.$$

Dividing the sides of the current triangle by V gives

$$I_a/V = G; \quad I_q/V = B; \quad I/V = Y.$$

What has been obtained is an admittance triangle with the sides G, B and Y.

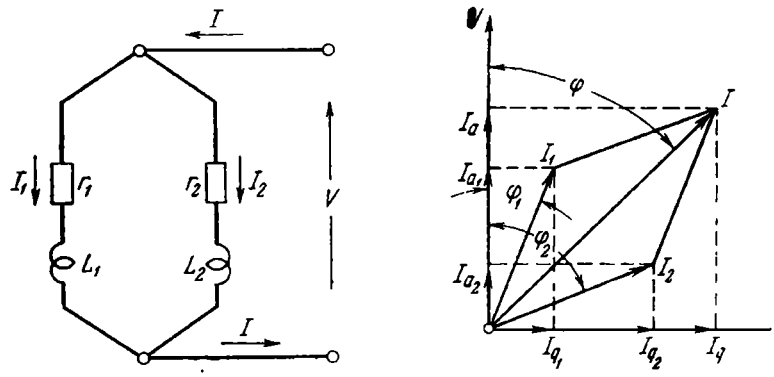


Fig. 166. *RL* circuits connected in parallel

From the admittance triangle

$$Y^2 = G^2 + B^2 \quad \text{or} \quad Y = \sqrt{G^2 + B^2}.$$

94. Parallel Alternating-current Circuits

Fig. 166 (on the left) shows a parallel combination of two series *RL* circuits (R_1L_1 and R_2L_2 respectively).

The impedances of the branches are

$$Z_1 = \sqrt{R_1^2 + (\omega L_1)^2};$$

$$Z_2 = \sqrt{R_2^2 + (\omega L_2)^2}.$$

The currents of the branches are

$$I_1 = V/Z_1 = V/\sqrt{R_1^2 + (\omega L_1)^2};$$

$$I_2 = V/Z_2 = V/\sqrt{R_2^2 + (\omega L_2)^2}.$$

The phase angles between the voltages and currents in the branches are

$$\cos \varphi_1 = R_1/Z_1 \quad \text{and} \quad \cos \varphi_2 = R_2/Z_2.$$

In Fig. 166 (on the right) is a vector diagram for the two series *RL* circuits connected in parallel. The construction of a vector diagram for *RL* circuits in parallel begins with the voltage vector, since the applied voltage is common to both branches. Due to the presence of inductance in the

branches, their currents lag behind the applied voltage by angles φ_1 and φ_2.

Next, the current vectors I_1 and I_2 are constructed. Their combination by the parallelogram law gives the total current vector I. As follows from the vector diagram

$$I_a = I_{a1} + I_{a2}.$$

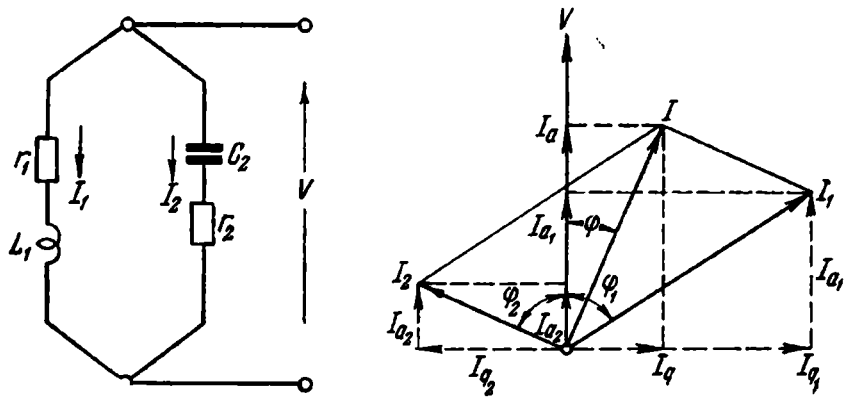


Fig. 167. *LC* circuits connected in parallel

This expression can be rewritten as follows:

$$VG = VG_1 + VG_2$$

or

$$G = G_1 + G_2.$$

Similarly,

$$I_q = I_{q1} + I_{q2};$$

$$VB = VB_1 + VB_2;$$

$$B = B_1 + B_2.$$

The total current is

$$I = \sqrt{I_a^2 + I_q^2} = \sqrt{V^2G^2 + V^2B^2} = V\sqrt{G^2 + B^2} = VY.$$

Thus, the admittance of a parallel combination of series *RL* circuits is the sum of the conductances and susceptances of the branches.

Example 14. In the circuit shown in Fig. 166, $R_1 = 4$ ohms; $L_1 = 0.01$ henry; $R_2 = 3$ ohms; and $L_2 = 0.02$ henry. The applied voltage is 127 V, 50 c/s. Calculate the currents in the branches and the total current.

Solution.

$$Z_1 = \sqrt{R_1^2 + (\omega L_1)^2} = \sqrt{4^2 + (2 \times 3.14 \times 50 \times 0.01)^2} = 5.075 \text{ ohms};$$

$$Z_2 = \sqrt{R_2^2 + (\omega L_2)^2} = \sqrt{3^2 + (2 \times 3.14 \times 50 \times 0.02)^2} = 6.95 \text{ ohms}.$$

$$\cos \varphi_1 = R_1/Z_1 = 4 \div 5.075 = 0.788; \quad \varphi_1 = 38°;$$

$$\cos \varphi_2 = R_2/Z_2 = 3 \div 6.95 = 0.432; \quad \varphi_2 = 64°20';$$

$$I_1 = V/Z_1 = 127 \div 5.075 = 25 \text{ A};$$

$$I_2 = V/Z_2 = 127 \div 6.95 = 18.3 \text{ A}.$$

In order to determine the total current we find

$$G_1+R_1/Z_1^2 = 4 \div 5.075^2 = 0.155 \text{ mho};$$
$$B_1 = X_1/Z_1^2 = 314 \times 0.01 \div 5.075^2 = 0.122 \text{ mho};$$
$$G_2 = R_2/Z_2^2 = 3 \div 6.95^2 = 0.0622 \text{ mho};$$
$$B_2 = X_2/Z_2^2 = 314 \times 0.02 \div 6.95^2 = 0.13 \text{ mho};$$
$$G = G_1+G_2 = 0.155+0.0622 = 0.2172 \text{ mho};$$
$$B = B_1+B_2 = 0.122+0.13 = 0.252 \text{ mho};$$
$$Y = \sqrt{G^2+B^2} = \sqrt{0.2172^2+0.252^2} = 0.332 \text{ mho}.$$

The total current will be

$$I = VY = 127 \times 0.332 = 42.2 \text{ A};$$
$$\tan \varphi = B/G = 0.252 \div 0.2172 = 1.16; \quad \varphi = 49°15'.$$

In Fig. 167 (left) is a parallel combination of series LC circuits. The impedances of the branches are

$$Z_1 = \sqrt{R_1^2+(\omega L_1)^2};$$
$$Z_2 = \sqrt{R_2^2+(1/\omega C_2)^2}.$$

The currents in the branches are

$$I_1 = V/Z_1 = V/\sqrt{R_1^2+(\omega L_1)^2};$$
$$I_2 = V/Z_2 = V/\sqrt{R_2^2+(1/\omega C_2)^2}.$$

The phase angles between the voltages and the currents in the branches are

$$\cos \varphi_1 = R_1/Z_1;$$
$$\cos \varphi_2 = R_2/Z.$$

The construction of the vector diagram shown on the right of Fig. 167 begins with the voltage vector V. Then the current vectors I_1 and I_2 are constructed at the angles φ_1 and φ_2. The current I_1 in the inductive branch lags behind the voltage by the angle φ_1, while the current I_2 in the capacitive branch leads the voltage by the angle φ_2. Combining the current vectors I_1 and I_2 by the parallelogram law gives the total current vector I.

As follows from the vector diagram

$$I_a = I_{a1}+I_{a2}$$

or

$$VG = VG_1+VG_2,$$

whence

$$G = G_1+G_2.$$

Thus, the conductance of a parallel combination of series LC circuits is equal to the sum of the conductances of the component branches.

The susceptance of the parallel combination will be

$$I_q = I_{q1} - I_{q2}$$

or

$$VB = VB_1 - VB_2,$$

whence

$$B = B_1 - B_2,$$

where B_2 is the susceptance of a capacitive circuit equal to $B_2 = X_C/Z_2^2 = (1/\omega C_2)/R_2^2 + (1/\omega C_2)^2$ mhos.

In other words, the susceptance of a parallel combination of series LC circuits is equal to the difference of the susceptances of the component branches.

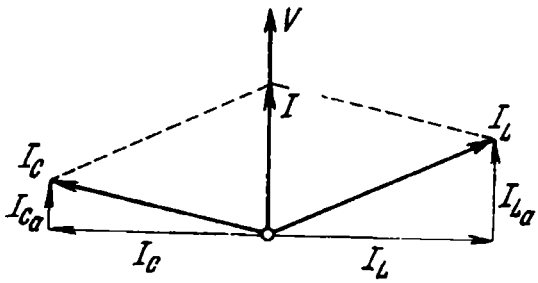


Fig. 168. Vector diagram for current (parallel) resonance

When L, C and R are connected in parallel, they can be so matched or the frequency f can be so chosen, that the total current I will be in phase with the applied voltage V. This condition is *current, or parallel, resonance.*

Fig. 168 shows that current resonance will take place when the reactive currents in the inductive branch (I_L) and in the capacitive branch (I_C) are equal, or $I_L = I_C$, since the total current in such a case will be in phase with the applied voltage V.

By properly matching the inductances, capacitances and resistances in the branches, the total current I may be made relatively small, while the currents in the inductive and capacitive branches will be very large.

Example 15. In the circuit shown in Fig. 167, $R_1 = 5$ ohms; $L_1 = 0.05$ henry; $R_2 = 5$ ohms; $C_2 = 100$ μF. The applied voltage is 220 V a-c, 50 c/s. Calculate the currents in the branches and the total current.

Solution.

$$\omega = 2\pi f = 2 \times 3.14 \times 50 = 314 \text{ sec}^{-1};$$

$$Z_1 = \sqrt{R^2 + (\omega L_1)^2} = \sqrt{5^2 + (314 \times 0.05)^2} = 16.5 \text{ ohms};$$

$$\cos \varphi_1 = R_1/Z_1 = 5 \div 16.5 = 0.303; \quad \varphi_1 = 72°20';$$

$$Z_2 = \sqrt{R^2 + (1/\omega C_2)^2} = \sqrt{5^2 + (1/314 \times 100 \times 10^{-6})^2} = 32.2 \text{ ohms};$$

$$\cos \varphi_2 = R_2/Z_2 = 5 \div 32.2 = 0.155; \quad \varphi_2 = 81°.$$

The currents in the branches are

$$I_1 = V/Z_1 = 220 \div 16.5 = 13.33 \text{ A};$$

$$I_2 = V/Z_2 = 220 \div 32.2 = 6.84 \text{ A}.$$

The conductances of the branches are

$$G_1 = R_1/Z_1^2 = 5 \div 16.5^2 = 0.0184 \text{ mho};$$
$$G_2 = R_2/Z_2^2 = 5 \div 32.2^2 = 0.0048 \text{ mho}.$$

The susceptances of the branches are

$$B_1 = X_1/Z_1^2 = 314 \times 0.05 \div 16.5^2 = 0.0579 \text{ mho};$$
$$B_2 = X_2/Z_2^2 = 1/(314 \times 100 \times 10^{-6}) \div 32.2^2 = 0.0307 \text{ mho}.$$

The total conductance and susceptance is

$$G = G_1 + G_2 = 0.0184 + 0.0048 = 0.0232 \text{ mho};$$
$$B = B_1 - B_2 = 0.0579 - 0.0307 = 0.0272 \text{ mho}.$$

The total admittance will be

$$Y = \sqrt{G^2 + B^2} = \sqrt{0.0232^2 + 0.0272^2} = 0.0358 \text{ mho}.$$

The total current is

$$I = VY = 220 \times 0.0358 = 7.9 \text{ A};$$
$$\tan \varphi = B/G = 0.0272 \div 0.0232 = 1.17; \quad \cos \varphi = 0.649; \quad \varphi = 49°30'.$$

95. The Oscillatory Circuit

When discussing circuits containing inductance and capacitance connected in parallel, we saw that the total current could be small while the currents in the branches could be fairly large. If the resistance of such a circuit is zero, the currents in the branches will be in quadrature with the applied voltage. If, in addition, the reactances are made equal ($X_L = X_C$), equal currents will flow in the branches, and the total current will be zero. Fig. 169 shows a vector diagram for such a case.

Since the total current is zero, the input wires can be disconnected from the a-c source. Yet an alternating current will continue to flow in the closed circuit made up of the inductance and the capacitance.

Consider the alternating current generated by a capacitor discharging into an induction coil.

A charged capacitor stores a quantity of electric energy. When connected across a coil, it begins to discharge, and the energy stored in it decreases. The discharge current passing through the coil sets up a magnetic field around it. Consequently, the coil accumulates magnetic energy. When the capacitor is fully discharged, its store of energy is nil. At this moment the store of magnetic energy in the coil is greatest. Now the coil becomes a current generator and begins to charge the capacitor. When the magnetic field is building up, the emf of self-induction produced in the coil tends to oppose current flow. When the magnetic field is collapsing, the emf of self-induction tends to maintain current flow. By the time the magnetic energy of the coil becomes zero, the capacitor will have been charged in the opposite sense. If the resistance of the circuit is zero, the new charge will be equal

to the initial one. Now the capacitor begins to discharge again, establishing a current in the circuit reversed in direction, and the sequence is repeated.

The alternate exchange of energy between the circuit and the magnetic field is the basis of electromagnetic oscillations. Circuits where this exchange takes place are called *oscillatory circuits.*

If there were no losses of power in the oscillatory circuit itself, this interchange of stored energy between the electric and magnetic fields would continue in the form of undamped oscillations for an indefinitely long time. However, due to the presence of resistance in the circuit, the stored energy is reduced in every cycle via heat dissipation in the resistance, and the oscillations gradually die out.

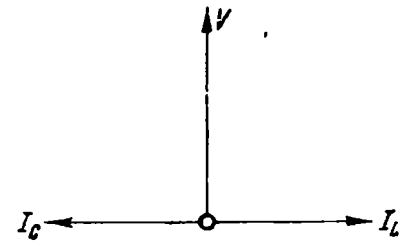


Fig. 169. Simplified vector diagram for current (parallel) resonance

The period of electromagnetic oscillations in an oscillatory circuit is given by the formula

$$T = 2\pi\sqrt{LC} \text{ sec.}$$

The quantity T is termed the *natural period* of an oscillatory circuit.

The natural frequency of an oscillatory circuit can be determined from the relationship

$$f = 1/T = 1/2\pi\sqrt{LC} \text{ sec}^{-1}.$$

Thus, the period of an oscillatory circuit can be changed by two methods: by varying either the inductance of the coil or the capacitance of the capacitor. Both methods are used in practical oscillatory circuits.

The oscillatory circuit is the basis of radio transmitters and receivers.

In simple terms, radio transmission is carried out as follows. Valve oscillators feed high-frequency electromagnetic oscillations (known as carrier waves) into an aerial. In order to transmit speech or music, the carrier wave must be modulated by the audio-frequency waves, which are produced in a microphone by voice and by music. The modulated carrier wave is then radiated by the aerial into space.

At the receiving end, another aerial picks up the modulated signal, and a weak emf is induced in it at the frequency of the incoming signal. For the listener to be able to pick up the desired signal from among the many incoming signals, the aerial is connected to an oscillatory circuit. Varying the capacitance or inductance of the oscillatory circuit, the listener tunes it to "resonance" with the desired emf induced in the aerial, and it is amplified many times. The signal, still at the high frequency of the carrier wave, is then further amplified by radio valves or semiconductor devices. The frequency of the modulated signal is beyond the limits of audibility and is therefore far too high to operate telephones or loudspeakers. It must be *demodulated.* This is called *detection* or *demodulation.* Detection is accom-

plished by crystal detectors, by radio valves, or by semiconductor devices. The result is the audio-frequency wave, which was originally transmitted. This wave is then amplified and fed to phones or a speaker.

In Russia, radio communication was pioneered by Alexander Popov (1850-1905).

96. The Power Triangle

Multiplying the sides of the voltage triangle (Fig. 170*a*) by the current I (Fig. 170*b*) will give a power triangle (Fig. 170*c*). Each side of this triangle represents power (Fig. 171).

The base of the triangle is the *true power*, P. When the current and the voltage vary according to the sine law, the power is

$$P = VI \cos \varphi.$$

where V and I are the effective values of the voltage and current, respectively, and φ is the phase angle between the two. Also the power $P = V_R I$. Since $V_R = IR$, $P = I^2R$.

True power is measured with a wattmeter and is expressed in watts or kilowatts. It gives a measure of the load on the prime mover of the generator. In a-c circuits it is mainly dissipated as heat. In a-c motors, it is mostly converted into mechanical power.

The product VI in the formula of true power is the *volt-ampere* and is often called the *apparent power*. In our triangle, it is the hypotenuse, S:

$$S = VI.$$

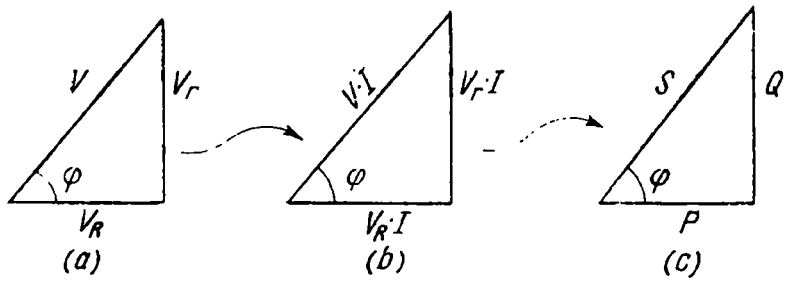


Fig. 170. Construction of a power triangle

It is measured in volt-amperes (VA) or kilovolt-amperes (kVA) as the product of voltmeter and ammeter readings. Apparent power enters into calculations of the overall dimensions of generators and transformers.

The height of our triangle is the *reactive power*, Q;

$$Q = V_r I.$$

Since $V_r = IX$ (where X is the reactance), we have

$$Q = I^2X.$$

Reactive power is not useless; it is required for the production of alternating magnetic fields essential to the action of electrical machines. From our triangle

$$Q = VI \sin \varphi.$$

It represents power which is delivered to the circuit during part of a cycle but is returned to the source during another part when the polarity

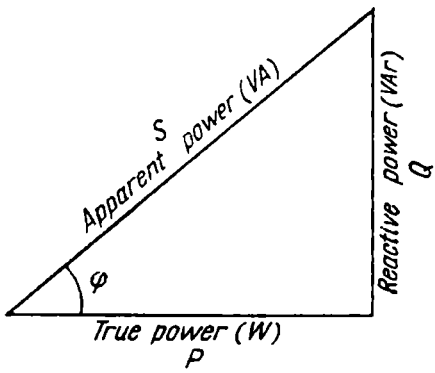


Fig. 171. Power triangle

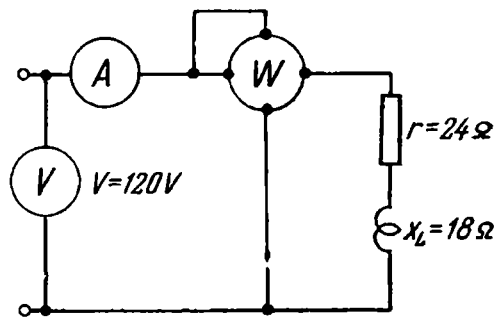


Fig. 172. An electric circuit containing resistance, inductance and measuring instruments

of voltage or current is reversed. The unit is the volt-ampere reactive (VAr) or the kilovolt-ampere reactive (kVAr).

By the theorem of Pythagoras

$$S^2 = P^2 + Q^2$$

or

$$S = \sqrt{P^2 + Q^2}.$$

As an illustration to the above discussion, let us examine the circuit shown in Fig. 172. It contains an inductive reactance, a resistance, an ammeter, a voltmeter, and a wattmeter.

1. Let the circuit be connected to a d-c source. Since with direct current the inductive reactance X_L is zero, the circuit will become purely resistive and

$$I = V/R = 120 \div 24 = 5 \text{ A}.$$

The ammeter will read 5 A.

The power delivered to the circuit is

$$P = VI = 120 \times 5 = 600 \text{ watts}$$

or

$$P = I^2R = 25 \times 24 = 600 \text{ watts}.$$

The wattmeter will therefore read 600 watts. Thus, a wattmeter connected in a d-c circuit reads the watts consumed by the circuit. Its readings are the product of the voltmeter and ammeter readings.

2. Let the same circuit be connected to an a-c source. Then

$$Z = \sqrt{R^2+X_L^2} = \sqrt{24^2+18^2} = \sqrt{900} = 30 \text{ ohms.}$$

The current in the circuit is

$$I = V/Z = 120 \div 30 = 4 \text{ A.}$$

The ammeter will read 4 A.
The power dissipated as heat will be

$$P = I^2R = 4^2 \times 24 = 384 \text{ watts.}$$

The wattmeter will read 384 watts.
The apparent power delivered by the source to the circuit is

$$S = VI = 120 \times 4 = 480 \text{ VA.}$$

Thus, of all the power (apparent power) delivered by a source to an a-c circuit containing resistance and inductive reactance, only the resistance actually consumes power (true power). The balance (reactive power) is transferred back and forth between the source and the circuit.

97. The Power Factor

The ratio of the true power to the apparent power is called the power factor of the circuit, abbreviated to PF or pf.

$$\text{PF} = \frac{\text{true power}}{\text{apparent power}}.$$

In the general case, the true power is less than the apparent power, and so the denominator in the above fraction is greater than the numerator. Therefore, the power factor is less than unity.

Only in the case of a purely resistive circuit is the power factor equal to unity (the numerator and denominator are the same).

From the power triangle (Fig. 171) we have

$$P/S = \cos \varphi.$$

The power factor, or $\cos \varphi$, is measured with an instrument called a *power-factor meter* which is described in detail elsewhere in this book.

Example 16. Calculate the power factor of the utilizing plant if the ammeter reads 10 A, the voltmeter 120 V, and the wattmeter 1 kW.

$$S = VI = 120 \times 10 = 1200 \text{ VA}$$
$$\text{PF} = P/S = 1000/1200 = 0.83.$$

Example 17. Determine the true power delivered to the circuit by a single-phase a-c generator if the voltmeter on the generator instrument-board reads 220 V, the ammeter 20 A and the power-factor meter 0.8.

$$P = VI \cos \varphi = 220 \times 20 \times 0.8 = 3.52 \text{ kW.}$$

Apparent power

$$S = VI = 220 \times 20 = 4400 \text{ VA} = 4.4 \text{ kVA}.$$

Example 18. The voltmeter on the panel of an electric motor reads 120 V, the ammeter 450 A and the wattmeter 50 kW. Calculate (1) the impedance; (2) the effective resistance; (3) the inductive reactance; (4) the apparent power; (5) the power factor: and (6) the reactive power.

$$Z = V/I = 120 \div 450 = 0.267 \text{ ohm}.$$

Since $P = I^2R$,

$$R = P/I^2 = 50{,}000 \div 450^2 = 0.247 \text{ ohm}.$$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{0.267^2 - 0.247^2} = \sqrt{0.01} = 0.1 \text{ ohm}$$

$$S = VI = 120 \times 450 = 54{,}000 \text{ VA} = 54 \text{ kVA}.$$

$$\cos \varphi = P/S = 50{,}000 \div 54{,}000 = 0.927.$$

$$Q = \sqrt{S^2 - P^2} = \sqrt{54{,}000^2 - 50{,}000^2} = \sqrt{416{,}000{,}000} = 20{,}396 \text{ VAr} = 20.396 \text{ kVAr}.$$

The impedance, voltage and power triangles constructed for one and the same circuit are similar, since their sides are proportional. Fig. 173 shows how each of them can give the power factor of the circuit and supply any other unknown parameters of the circuit.

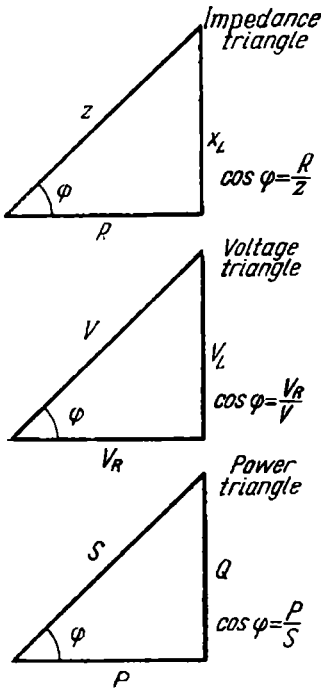


Fig. 173. Determining the power factor from resistance, voltage and power triangles

Example 19. Calculate (1) the impedance; (2) the inductive reactance; (3) the applied voltage; (4) the resistive voltage drop (or active voltage); (5) the inductive voltage drop (or reactive voltage); (6) the apparent power; (7) the true power; and (8) the reactive power, if $I = 6$ A; $R = 3$ ohms; PF = 0.8, and the current is lagging.

As follows from the impedance triangle

$$\cos \varphi = R/Z,$$

hence

$$Z = R/\cos \varphi = 3 \div 0.8 = 3.75 \text{ ohms}.$$

$$V = IZ = 6 \times 3.75 = 22.5 \text{ V}.$$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{3.75^2 - 3^2} = \sqrt{14.06 - 9} =$$

$$= \sqrt{5.06} = 2.24 \text{ ohms}.$$

$$V_R = IR = 6 \times 3 = 18 \text{ V}.$$

$$V_L = IX_L = 6 \times 2.24 = 13.45 \text{ V}.$$

$$S = VI = 22.5 \times 6 = 135 \text{ VA}.$$

$$P = I^2R = 36 \times 3 = 108 \text{ W}.$$

or

$$P = VI \cos \varphi = 22.5 \times 6 \times 0.8 = 108 \text{ W}.$$

or

$$Q = V_L I = 13.45 \times 6 = 81 \text{ VAr}$$

$$Q = \sqrt{S^2 - P^2} = \sqrt{135^2 - 108^2} = 81 \text{ VAr}$$

$$Q = I^2X_L = 6^2 \times 2.24 = 81 \text{ VAr}.$$

98. Power Factor Improvement

Let there be a single-phase a-c generator operating at 1200 V and delivering 240 kVA apparent power. The current delivered to the line will be

$$I = S/V = 240{,}000 \div 1200 = 200 \text{ A}.$$

In the case of a purely resistive load (heating appliances, incandescent lamps, etc.), all the power delivered to the line will be true power, and the power factor will be unity.

Then the true power of the line

$$P = VI \cos \varphi = 1200 \times 200 \times 1 = 240 \text{ kW}.$$

In the case of a load consisting of resistance and inductive reactance and having PF = 0.8, the true power will be $P = VI \cos \varphi = 1200 \times 200 \times 0.8 =$ $= 192$ kW.

Now the generator delivers less than its rated power, although the current flowing in its winding remains 200 A. It would seem that the rated power output could be obtained by increasing the current. This, however, cannot be done, since the generator windings would be dangerously heated.

In the case of a load having PF = 0.5 the true power will be

$$P = VI \cos \varphi = 1200 \times 200 \times 0.5 = 120 \text{ kW}.$$

Thus, the true power delivered by a source decreases with decreasing power factor, while the reactive power increases.

The consequences of a low power factor may be summarized as follows:

(1) The apparent power of generators and transformers has to be increased.

As follows from the formula $\cos \varphi = P/S$, $S = P/\cos \varphi$.

If, for example, the power rating of the motors in a shop is 80 kW, and the power factor of the shop's load is 0.8, the transformer required to power the motors will have to have the following apparent power:

$$S = P/\cos \varphi = 80 \div 0.8 = 100 \text{ kVA}.$$

Should the power factor of the shop's load drop to 0.6, then (with the same motors) the transformer will have to have a greater apparent power:

$$S = 80 \div 0.6 = 133 \text{ kVA},$$

i.e., a bigger transformer will have to be used.

(2) The efficiency of generators and transformers is reduced. In the case of a low PF load, the generator or transformer carry the full current without delivering their rated power to the circuit, and a good proportion of the energy of the prime movers is wasted.

(3) Power losses and voltage losses increase, or (to avoid them) larger cables have to be used. From $P = VI \cos \varphi$ it follows that

$$I = P/V \cos \varphi.$$

Then for $P = 1$ kW, PF $= 0.9$ and $V = 200$ V.

$$I = 1000 \div (200 \times 0.6) = 5.55 \text{ A.}$$

For PF $= 0.6$

$$I = 1000 \div (200 \times 0.6) = 8.35 \text{ A.}$$

Thus, with the same power consumption, more current is taken by electrical plant of a lower power factor. As a result, greater heat (I^2R) and voltage losses are incurred, or larger cables have to be employed.

All in all, it will cost more to supply a low PF consumer than a high PF one. It is for this reason that the tariff rates for an alternating-current supply to power consumers take into account both true and reactive power consumption as measured by wattmeters and var-meters, respectively. For the same true and reactive power, the respective energies can be determined by the equations

$$W_t = Pt \text{ kWh},$$
$$W_r = Qt \text{ kVArh}.$$

The effect of this method of charging for a power supply is that the consumer pays not only for the kWh of energy he uses, but also for the reactive kVArh which is required to produce the alternating magnetic fields in his machinery.

A decrease in the power factor of the consumer's plant may be due to several causes. The most common are the following:

(1) *Alternating-current motors operating under a low load.* In such cases the true power consumed by motors decreases in proportion to the load, while the reactive power decreases to a lesser extent.

For example, a 400-kW, 1000-rpm induction motor has a power factor of 0.83 under full load. At 75-per cent load the power factor will be 0.8, at half-load 0.7, and at a quarter load 0.5. Motors operating at no-load have a power factor ranging from 0.1 to 0.3, depending on their type, power rating and rpm.

(2) *Wrong type of motor.* High-speed, high-power motors have a larger power factor than low-speed low-power motors. Enclosed motors have a lower power factor compared with the open type.

(3) *Increase in mains voltage.* During low-load periods, lunch time, etc., the mains voltage goes up several volts. This increase raises the magnetizing current of inductive reactances, and the power factor of the electrical plant as a whole drops.

(4) *Improper maintenance and repair of motors.* In reconditioned motors, less wire is sometimes used than in factory-wound machines. In such a case, leakage of magnetic flux increases, and the power factor of the motor falls. In the case of heavily worn-out bearings, the rotor may catch at the stator. Instead of replacing the defective bearings, operators sometimes remove some metal from the rotor by turning. This is a wrong and into-

lerable practice, since an increased air gap between the rotor and stator requires a greater magnitizing current, and the power factor again falls.

From the foregoing it is clear why it is essential to keep electrical plant running at a high power factor. The improvement of the power factor can be accomplished in several ways:

1. by proper selection of motors (type, rating and rpm);
2. by running motors at their rated load;
3. by reducing no-load periods of motor operation;
4. by timely and proper maintenance and repair of motors;
5. by using capacitors.

Capacitors offer a number of advantages as a means of power-factor correction. They are small in weight, have no rotating parts and are therefore easy and safe to operate, and dissipate very little power.

As will be recalled, a capacitor connected in an a-c circuit has the property of taking leading current, and in a perfect capacitor this current is 90 degrees leading. Since the magnetizing current taken up by a perfect inductance is 90 degrees lagging, a capacitor may compensate for lagging (low) power factors—a phenomenon known as *phase compensation* and widely used in practice.

As a matter of fact, the phase angle can be reduced to zero. For reasons of economy, however, it is deemed practicable to maintain the power factor at 0.9 to 0.95.

Furthermore, the capacitor acts as a reservoir of stored energy available for interchange between its dielectric field and the magnetic fields of inductive equipment, thereby virtually limiting the magnetizing-current requirements to the circuit between the capacitor and induction motor or other inductive apparatus.

The value of a corrective capacitor can be calculated as follows.

Fig. 174*a* shows an a-c circuit containing an inductive load. A bank of capacitors is connected in parallel with the load in order to correct the power factor of the circuit. The construction of the vector diagram begins with the voltage vector V. Due to the inductive load, the current I_1 lags behind the applied voltage by an angle φ_1. It is required to reduce the phase angle between the applied voltage and the total current to an angle φ, or, which is the same, to raise the power factor from $\cos \varphi_1$ to $\cos \varphi$.

The vector Oc, which is the active or energy component of the current I_1, is

$$Oc = I_1 \cos \varphi_1 = Oa \cos \varphi_1.$$

From the formula for a-c power, $P = VI \cos \varphi$, we have

$$Oc = I_1 \cos \varphi_1 = P/V.$$

The total current I of the circuit is equal to the vectorial sum of the load current I_1 and the capacitance current I_C.

It follows from the triangles *Oac* and *Obc* that

$$ac = Oc \tan \varphi_1;$$
$$bc = Oc \tan \varphi.$$

From the diagram we have

$$ab = Od = ac - bc = Oc \tan \varphi_1 - Oc \tan \varphi = Oc (\tan \varphi_1 - \tan \varphi).$$

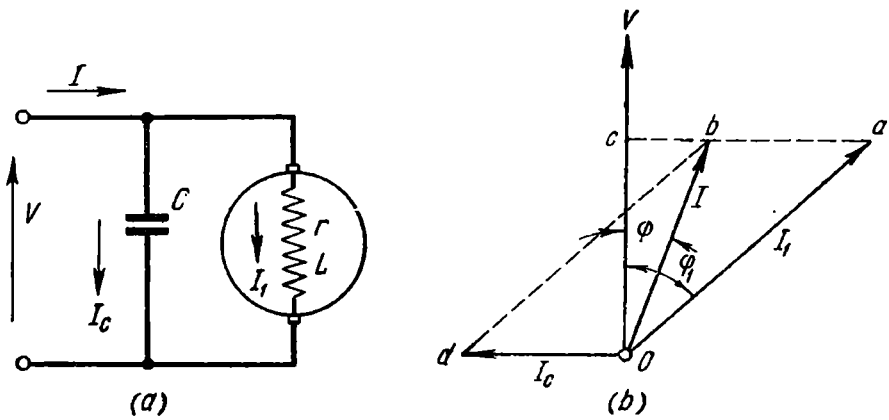


Fig. 174. Power-factor correction by means of static capacitors:
a—connection diagram; *b*—vector diagram

Since $Oc = P/V$ and $ab = I_C$,

$$I_C = P/V (\tan \varphi_1 - \tan \varphi).$$

At the same time

$$I_C = V\omega C.$$

Consequently, $C = (P/\omega V^2) (\tan \varphi_1 - \tan \varphi)$.

Example 20. The electric motors of a mine consume 2000 kW at 6 kV and PF = 0.6. Calculate the capacitance of the capacitor that should be connected across the busbars in order to raise the power factor to 0.9 at $f = 50$ c/s.

Solution.

$$\cos \varphi_1 = 0.6; \quad \varphi_1 = 53°10'; \quad \tan \varphi_1 = 1.335;$$
$$\cos \varphi = 0.9; \quad \varphi = 25°50'; \quad \tan \varphi = 0.484;$$
$$\omega = 2\pi f = 2 \times 3.14 \times 50 = 314 \text{ sec}^{-1};$$

$$C = (P/\omega V^2) (\tan \varphi_1 - \tan \varphi) = \frac{2{,}000.000}{314 \times 6000^2} (1.335 - 0.484) = 0.00015 \text{ F} = 150 \mu\text{F}.$$

Exercises

1. Determine the frequency of the current supplied by a 24-pole generator at 250 rpm.
2. A 20-pole a-c generator supplies a current at 50 c/s. Find the rpm.

3. How many poles are there in an a-c generator if its speed is 300 rpm and the frequency of the current is 50 c/s?

4. A coil is connected in a single-phase a-c circuit with a voltage of 120 V. The emf of self-induction is 100 V. What is the resistive voltage drop?

5. What will be the reading of a voltmeter connected across an electromagnet if the inductive reactance is 6 ohms, the resistance 8 ohms and the current through the magnet winding 5 A?

6. When a coil is connected across a d-c source of 120 V, the ammeter reads 4 A; when the same coil is connected across an a-c source of 380 V, the ammeter reading is 5 A. Calculate the impedance, resistance and inductive reactance of the coil.

7. Two conductors with a resistance of 6 and 5 ohms and an inductive reactance of 3 and 8 ohms, respectively, are connected in series. What voltage should be applied to obtain a current of 4 A in the circuit?

8. The current flowing through a conductor of 7 ohms resistance and 5 ohms inductive reactance is 6 A. Another conductor of 2 ohms resistance and 4 ohms inductive reactance is connected in parallel with it. Calculate the current in the second conductor.

9. A single-phase motor connected in a 120-V circuit takes a current of 2 A. The power factor is 0.85. Calculate the true power and apparent power delivered to the motor.

10. The current, voltage and power of an installation are 40 A, 110 V and 4 kW, respectively. Calculate (1) the impedance; (2) the resistance; (3) the inductive reactance; (4) the apparent power; (5) the power factor; (6) the reactive power; (7) the inductance drop; and (8) the resistance drop, if it is known that the load is inductive.

11. Calculate the current taken by a single-phase 5-kW motor connected in a 120-V circuit and having a power factor of 0.8.

12. An electric station has a 200-kVA, 100-V, single-phase a-c generator. Calculate the true power delivered by the generator at power factors of 1, 0.8, 0.6, 0.4, and 0.2.

13. An induction coil having a power factor of 0.3 carries an alternating current of 10 A. The wattmeter connected across the coils reads 250 W. Calculate (1) the voltage across the coil; (2) the impedance; (3) the resistance; and (4) the inductive reactance.

14. An induction coil is connected in a d-c circuit of 12 V and takes a current of 8 A. It takes the same current when connected in an a-c circuit of 20 V. Calculate (1) the resistance; (2) the inductive reactance; (3) the impedance; (4) the apparent power; (5) the true power; and (6) the reactive power of the coil.

15. The current, voltage and power factor of an electric installation are 20 A, 120 V and 0.8, respectively. Calculate (1) the apparent power; (2) the true power; (3) the reactive power; (4) the impedance; (5) the resistance; (6) the inductive reactance; (7) the inductance drop; and (8) the resistance drop.

16. A single-phase motor operating on 220 V at a power factor of 0.8 develops 10 hp effective and has an efficiency of 85 per cent. Calculate the power of the generator supplying the motor if the energy is transmitted over a line 50 m long made of copper wire 16 mm^2 in cross-section. The inductive reactance of the wires may be neglected.

17. The current, the voltage, and the resistance of an induction coil are 8 A, 220 V and 15 ohms, respectively. Calculate (1) the true power; (2) the apparent power; (3) the reactive power; (4) the impedance; (5) the inductive reactance; (6) the power factor; (7) the inductance drop; and (8) the resistance drop of the coil.

18. The nameplate of a single-phase motor reads: $V = 120$ V; $I = 5$ A; PF = 0.8. Calculate (1) the resistance; (2) the inductive reactance; and (3) the impedance of its winding.

19. An induction coil having a soft-iron core is connected in a single-phase circuit. The circuit also contains a voltmeter, an ammeter and a wattmeter. The respective readings are 5 A, 120 V and 200 W. The resistance of the coil is 2 ohms. Calculate (1) the power factor; (2) the emf of self-induction; (3) the power loss in the winding; and (4) the power loss in the core.

20. An induction coil having a steel core and a power factor 0.2 is connected in a 60-V circuit and takes a current of 3 A. The effective resistance of the coil is 2 ohms. Calculate (1) the emf of self-induction; (2) the power loss in the winding; and (3) the power loss in the core.

Review Questions

1. How is alternating current produced?
2. Define the period and frequency of an alternating current.
3. How can the frequency be determined if the number of poles and the rpm of an a-c generator are known?
4. What are the effects of alternating current?
5. What effect is produced by an inductance connected in an a-c circuit?
6. Define resistance and inductive reactance.
7. Define power factor.
8. How can the true and apparent power of alternating current be measured?
9. What effect is produced by a capacitance connected in an a-c circuit?
10. How can the power factor be improved?
11. What is the effective (root-mean-square) value of an alternating current?
12. What is voltage (series) resonance?
13. What is current (parallel) resonance?

Chapter Eight

THREE-PHASE ALTERNATING CURRENT

99. Polyphase Systems

A *polyphase system* is an alternating-current circuit or network to which are applied two or more emfs of the same frequency but displaced in time phase.

A polyphase system is called *symmetrical* if the applied voltages are equal and displaced in phase by angles equal to $2\pi/N$ (where N is the number of phases).

Each emf may act independently of the others in the system. Such systems are called *noninterlinked.*

Each emf of a polyphase system is referred to as a *phase.* In a noninterlinked system, the separate phases are not interconnected either electrically or magnetically, and they can be calculated from formulas for single-phase alternating current.

A serious drawback of noninterlinked systems is that they use a large number of wires (equal to 2 N). In the case of a three-phase system, six wires will be required.

Polyphase systems in which the individual phases are electrically connected are called *interlinked* and are widely used.

A polyphase system has many advantages. The same power can be transmitted over smaller wires than in the case of a single-phase alternating-current system. It can produce a revolving magnetic field in stationary coils or windings, which is a decisive advantage in many types of electrical apparatus.

100. Three-phase Systems

Of all polyphase systems, the three-phase system is used most widely.

The emfs in a three-phase system are generated as follows. If three turns are placed between the poles N and S of a uniform magnetic field (Fig. 175)

so that they are spaced 120° apart and are rotating at a constant angular velocity, the emfs induced in them will also be spaced 120° apart. Assuming that the first turn has zero-phase displacement, the induced emfs can be expressed thus:

$$e_1 = E_{max1} \sin \omega t;$$
$$e_2 = E_{max2} \sin (\omega t - 120°);$$
$$e_3 = E_{max3} \sin (\omega t - 240°).$$

On the right of Fig. 175 is the vector diagram for these emfs. The lengths of the vectors are equal to the rms values of the emfs.

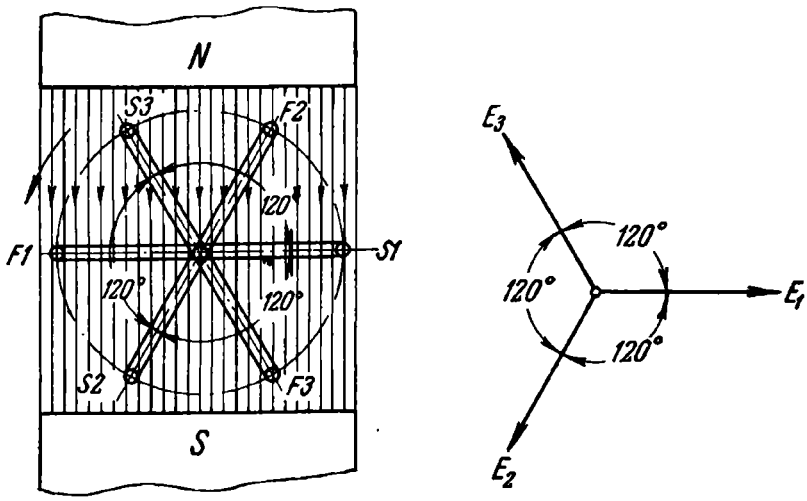


Fig. 175. Rotation of conductors in a uniform magnetic field

A three-phase generator has three armature windings spaced 120° apart. Each winding is called a *phase winding*, or simply a *phase*.

The diagram in Fig. 176 (on the left) shows a two-pole three-phase generator schematically. The stator carries three windings with an equal number of turns, spaced 120° apart. The letters *A*, *B* and *C* mark the starts and the letters *X*, *Y* and *Z* the finishes of the windings. The magnetic field is set up by the winding carried by the rotor, which is therefore called the field winding. The field winding is energized by a direct current supplied to it from a d-c generator or a rectifier through brushes and slip-rings. The rotor is driven by a suitable prime mover (which may be a steam turbine, a water wheel, an internal-combustion engine, etc.). The magnetic field cuts across the stator windings and sinusoidal emfs are induced in them.

The graph on the right of Fig. 176 shows the waveforms of the induced emfs. The position in which the rotor is found in the left-hand diagram corresponds to the dotted line ae_1 in the right-hand graph. At this instant, winding *I* is above and midway between the poles of the electromagnet;

the induced emf in it is at its highest and, according to the right-hand rule, points from the finish to the start of the winding. The emf in winding *II* lags one-third of a cycle behind that of winding *I*, while the emf in winding *III* lags behind that of winding *II* also by one-third of a cycle.

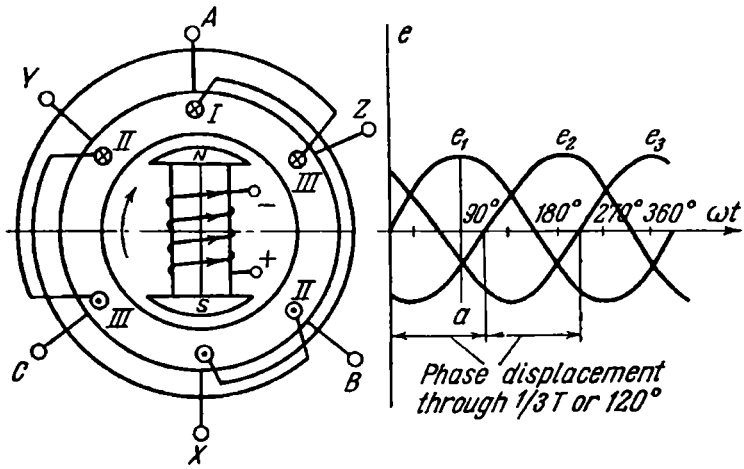


Fig. 176. Generation of a three-phase current

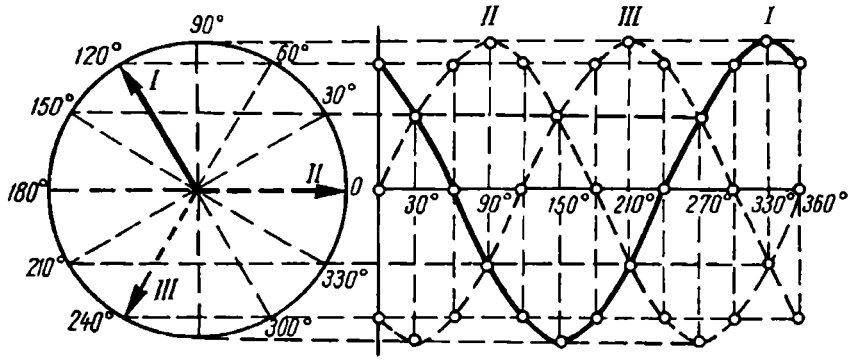


Fig. 177. Development of sine waves displaced through 120° by three rotating vectors spaced 120° apart

Windings *II* and *III* are in the proximity of the S pole, and the emfs in them point from the start to the finish of the respective windings.

The sine waves shown in Fig. 176 can also be developed by means of three rotating vectors equal in length to the maximum emfs in the armature windings of the generator, spaced 120° apart (Fig. 177).

Everything said about the emfs fully applies to the currents and voltages of a three-phase generator. By definition, this is a *symmetrical three-phase system.*

Since the phase windings of the generator have an equal number of turns and are wound with wire of the same cross-section, the induced emfs are equal in magnitude.

If each individual phase of the generator is connected to a separate circuit (Fig. 178), we have a noninterlinked three-phase six-wire system. Then, by Ohm's Law, the current in each phase will be

$$I_{ph} = V_{ph}/Z_{ph},$$

where

I_{ph} = phase current;

V_{ph} = phase voltage;

Z_{ph} = phase impedance.

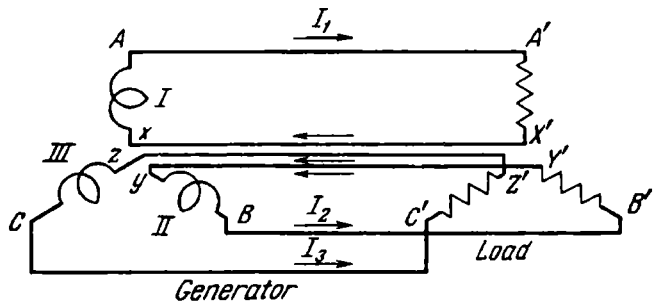


Fig. 178. Six-wire three-phase system

Noninterlinked three-phase systems have now fallen out of use; considerations of economy in transmission and distribution call for interconnection of the phase windings. The two principal methods of connecting the separate phases of a three-phase system are the star connection and the delta connection.

101. Star Connection

In the star connection, also known as the Y (wye) connection, all the finishes are connected together to form a neutral point, leaving the starts as terminals of the three-phase star system. The symbol for star connection is Y.

In Fig. 179, the neutral point, O, of the generator is connected to a neutral wire which terminates at the neutral point of the load (O'). The remaining three wires running from the generator to the load are called *line wires*, or simply lines. Thus, the three-phase star system is a three-phase four-wire system.

A comparison of a noninterlinked (Fig. 178) and a four-wire (Fig. 179) three-phase system will show that in the former case three wires of the system serve as return wires, while in the latter case the neutral wire serves as the return wire. The current in the neutral wire is the vectorial sum of the three-phase currents: I_A, I_B and I_C

$$\bar{I}_0 = \bar{I}_A + \bar{I}_B + \bar{I}_C.$$

The voltages measured between the phase beginnings and the neutral are called phase voltages and designated V_A, V_B and V_C, or, generally, V_{ph}. The phase emfs are denoted by the letters E_A, E_B and E_C, or, generally, E_{ph}. If the resistance of the generator windings is neglected, we may write

$$E_A = V_A; \quad E_B = V_B; \quad E_C = V_C; \quad E_{ph} = V_{ph}.$$

The voltages measured between the phase beginnings A and B, B and C, and C and A are called *line voltages* and are designated V_{AB}, V_{BC}, V_{CA} or, generally, V_l. The arrows in Fig. 179 indicate an arbitrary positive direction, which is from the generator to the load in the lines and from the load to the generator in the neutral.

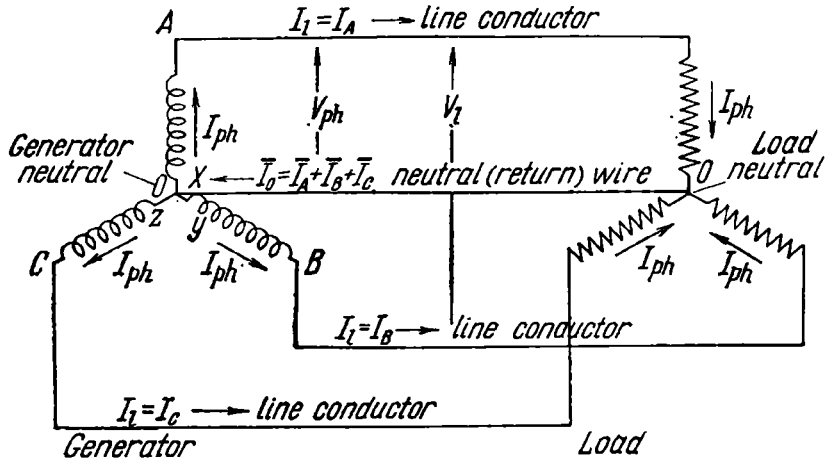


Fig. 179. Four-wire three-phase system

A voltmeter connected to the points A and B will read the line voltage V_{AB}. Since the arbitrary positive direction of the phase voltages V_A, V_B and V_C is from the beginnings towards the ends of the phase windings, the vector of the line voltage V_{AB} is equal to the vectorial difference between the phase voltages V_A and V_B:

$$\bar{V}_{AB} = \bar{V}_A - \bar{V}_B.$$

Similarly

$$\bar{V}_{BC} = \bar{V}_B - \bar{V}_C;$$
$$\bar{V}_{CA} = \bar{V}_C - \bar{V}_A.$$

To put it another way, the instantaneous value of a line voltage is equal to the difference between the instantaneous values of the respective phase voltages. In Fig. 180 vector subtraction is replaced by vector addition:

$$V_A + (-V_B); \quad V_B + (-V_C); \quad V_C + (-V_A).$$

Referring to the vector diagram, the vectors of the line voltages form a closed triangle.

Fig. 181 relates the line and phase voltages of a three-phase star system:

$$V_{BC} = 2V_B \cos 30°.$$

Since $\cos 30° = \frac{\sqrt{3}}{2}$, it follows that $V_{BC} = \sqrt{3}\,V_B$ or, generally, $V_l = \sqrt{3}\,V_{ph}$. Thus, in a three-phase star system the line voltage is $\sqrt{3}$ times the phase voltage. From now on, when discussing the voltage in three-phase circuits, the line voltage will be meant if not otherwise indicated.

The current flowing in a phase winding is called *phase current* and is generally designated by

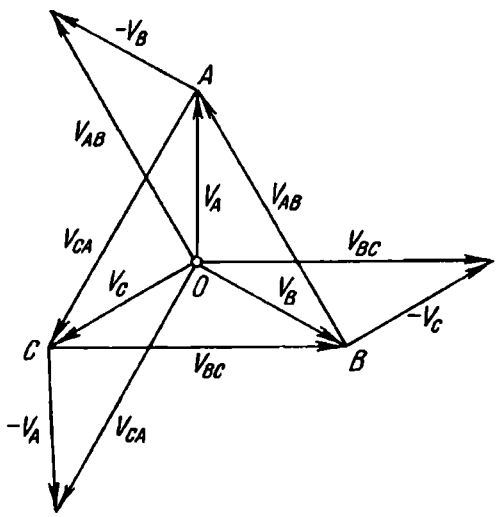


Fig. 180. Phase and line voltages in a star connection

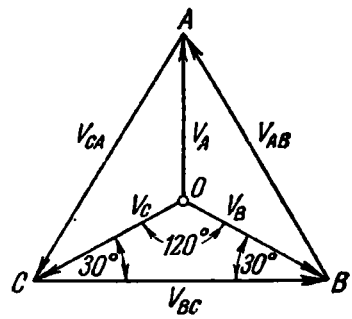


Fig. 181. Relationship between phase and line voltages in a star connection

the symbol I_{ph}. The current flowing in a line is called *line current* and is denoted by the symbol I_l.

Fig. 179 shows that in a 3-phase star system the line current is equal to the phase current, or $I_l = I_{ph}$.

When the load on the phases is equal in magnitude and character, it is called *balanced* or *symmetrical*. The condition for a balanced (or symmetrical) load is expressed thus:

$$Z_1 = Z_2 = Z_3.$$

The load will not be balanced if $Z_1 = R_1 = 5$ ohms, $Z_2 = \omega L_2 = 5$ ohms and $Z_3 = 1/\omega C_3 = 5$ ohms, since the load is the same in magnitude only and differs in character (R_1 is a resistance, ωL_2 is an inductive reactance, and $1/\omega C_3$ is a capacitive reactance).

The phase currents in a balanced 3-phase star system are

$$I_A = V_A/Z_A; \quad I_B = V_B/Z_B; \quad I_C = V_C/Z_C.$$

$$I_A = I_B = I_C.$$

Since the impedances of the phases are the same both in magnitude and character, the power factors of the phases will likewise be the same:

$$\cos \varphi_1 = R_A/Z_A; \quad \cos \varphi_2 = R_B/Z_B; \quad \cos \varphi_3 = R_C/Z_C;$$
$$\cos \varphi_1 = \cos \varphi_2 = \cos = \varphi_3.$$

If the resistance of the line wires is zero, the voltage across the load terminals will be equal to the voltage across the generator terminals. Addition

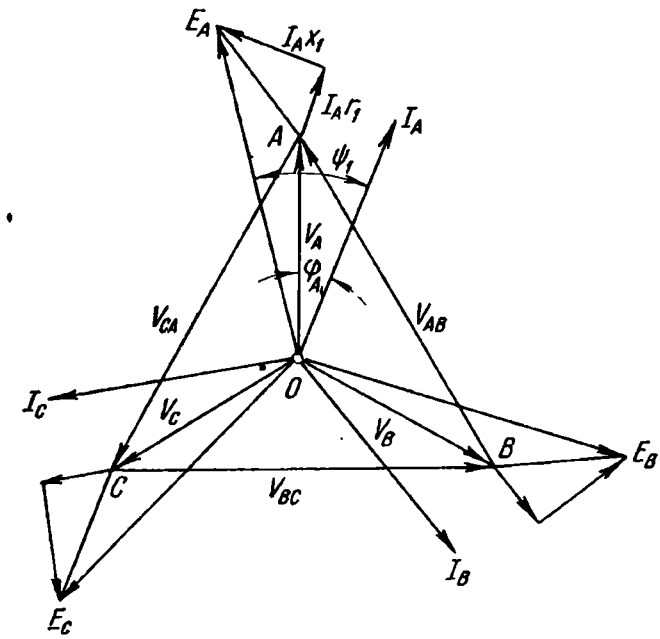


Fig. 182. Vector diagram of currents and voltages in the case of a star connection

of the voltage drop across the phase windings to the generator voltage will give the phase emfs of the generator.

Fig. 182 is a vector diagram for the currents, voltages and emfs of a 3-phase star system.

As has been stated already, the current in the neutral is equal to the vectorial sum of the three-phase currents. Fig. 183 shows the waveforms of the phase currents in a balanced 3-phase star system. Since the load is balanced, the maximum value is the same for the three sine waves.

At time *a* the current in the third phase, i_3, is zero. The instantaneous value of the current in the first phase is i_1 and in the second phase i_2; they are equal in magnitude but opposite in sign, and therefore cancel each other. So the total current in the neutral is zero. The same is true of time *c*.

At time *b* the current in the first phase has the maximum positive value i_1. At the same time, the currents in the second and third phases, i_2 and i_3,

being equal and negative, add together to a value equal to i_1. Again, the total current in the neutral is zero.

It can be shown that in the case of balanced loading the total current in the neutral will be zero at any instant. This implies that in the case of

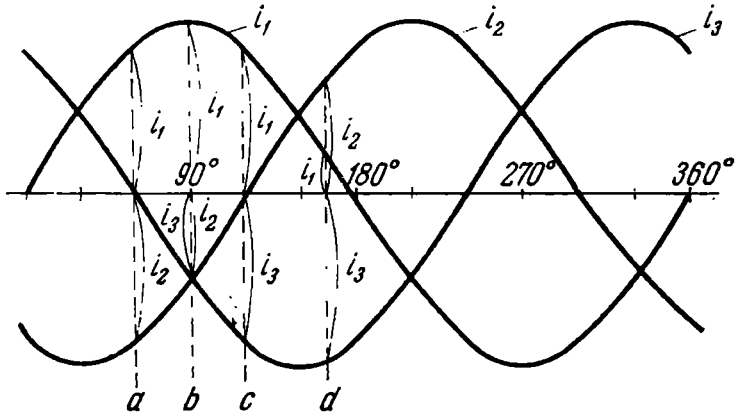


Fig. 183. Determining the current in the neutral in a balanced three-phase star system

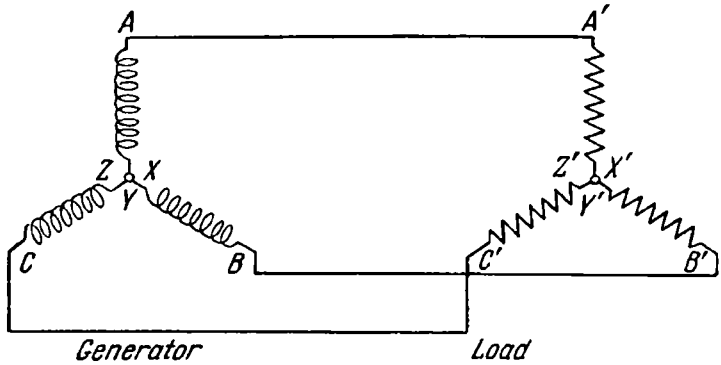


Fig. 184. Three-wire three-phase system

three-phase a-c motors, three-phase furnaces and other balanced loads, the neutral wire may well be dispensed with, giving a three-phase three-wire system (Fig. 184). On the other hand, unbalanced loads, such as lighting plant, require a four-wire system.

Example 1. In a load the phase impedances are respectively $Z_A = 5$ ohms; $Z_B = 10$ ohms; $Z_C = 20$ ohms. The windings of the load are star-connected and operate on 380 V. Calculate the current in the neutral if the phase power factors are the same and the phase impedances are identical in character.

Solution.

The phase currents are

$$I_A = V_A/Z_A = 380 \div 5 \times \sqrt{3} = 44 \text{ A};$$
$$I_B = V_B/Z_B = 220 \div 10 = 22 \text{ A};$$
$$I_C = V_C/Z_C = 220 \div 20 = 11 \text{ A}.$$

Since the currents are 120° apart, the respective current vectors may be drawn from a common point, each making an angle of 120° with the others (Fig. 185). Vectorial addition will give the current in the neutral, I_0. In our case it is 31 A.

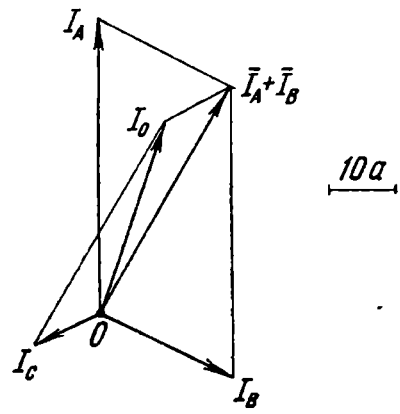


Fig. 185. Determining the current in the neutral by vectors

The importance of a separate return (neutral) wire in the case of unbalanced loads (such as lighting plant) will be clear from the following discussion.

Let there be a three-phase receiving network (Fig. 186*a*) with an unbalanced resistive load on the phases. The phase resistances are in the ratio $R_A:R_B:R_C = 1:2:3$. The system has no neutral wire.

In the case of a short-circuit in phase A (Fig. 186*b*) $R_A=0$, the neutral point O' will be A of the vector diagram (Fig. 186*d*). The voltage across the other two phases B and C will rise $\sqrt{3}$ times, since they are now connected between the lines. In the case of an open-circuit in phase A, $R_A =$ infinity, and the phase resistances R_B and R_C will be connected in series between the lines B and C. The neutral point will be at point D on the side BC of the line-voltage triangle. The point D divides BC in the proportion $R_B:R_C = 2:3$. It can be proved that with the resistance of phase A varying

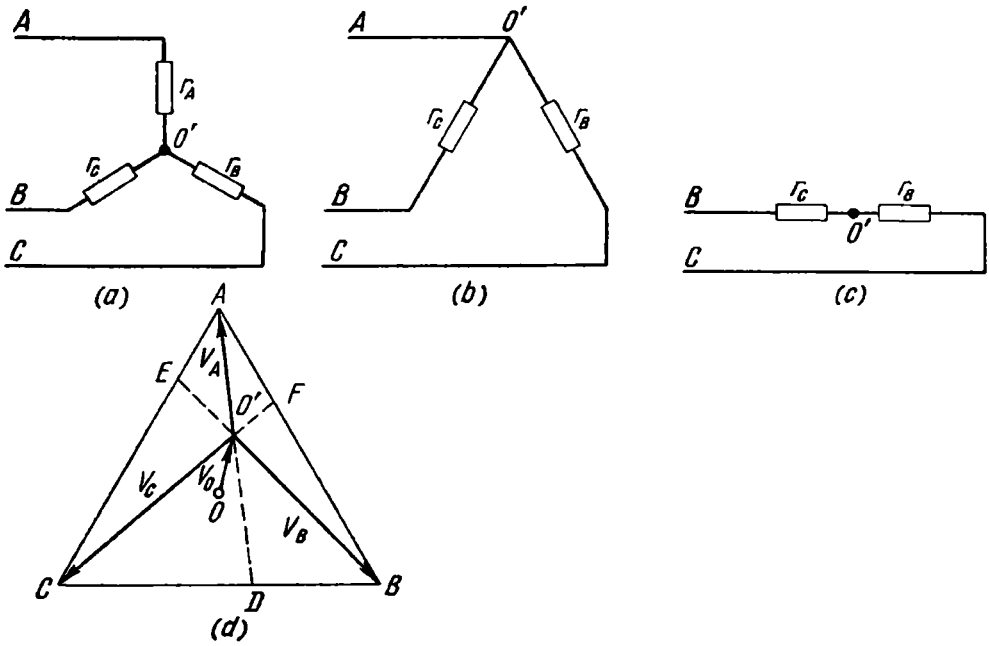


Fig. 186. Determining the neutral point of a load

from zero to infinity the neutral point of the receiving network will move along the line connecting the points A and D.

Reasoning similarly for phases B and C will give two more lines: BE for a short and an open circuit in phase B and CF for a short and an open circuit in phase C. The line BE divides the side AC of the triangle in the proportion $R_A : R_C = 1:3$. The line CF divides the side AB of the triangle in the proportion $R_A : R_B = 1:2$. The point of intersection of the lines AD, BE and CF is the neutral point O' of the receiving network for the assumed loading.

The vectors $\overline{O'A}$, $\overline{O'B}$ and $\overline{O'C}$ are the phase voltages across the consumer's terminals. As follows from the diagram, these phase voltages in the case of unbalanced loading will be different in magnitude, being proportional to the resistance of the respective phase.

The displacement of the neutral point in the receiving network due to unbalanced loading results in unsatisfactory operation of the network, especially of lighting plant. As the number of lamps connected to a phase increases, their resistance decreases, as does the phase voltage, and the lamps will give less light. Furthermore, a potential difference builds up between the neutral points of the generator and the receiving network, giving rise to a circulating current and causing appreciable heating.

These troubles are obviated when a separate neutral wire is provided.

102. Delta Connection

Fig. 187 shows a noninterlinked three-phase six-wire system. Connecting the start of phase A to the finish of phase C, the finish of phase A to the start of phase B, and the finish of phase B to the start of phase C and connecting the junctions thus obtained to the line wires makes a 3-phase delta system (Fig. 188).

Referring to Fig. 188, the line voltage in the delta connection equals the phase voltage:

$$V_l = V_{ph}.$$

By Kirchhoff's first Law, the currents for the three junctions A', B' and C' of the load will be

$$\begin{aligned} \bar{I}_A + \bar{I}_{CA} &= \bar{I}_{AB}; \\ \bar{I}_B + \bar{I}_{AB} &= \bar{I}_{BC}; \\ \bar{I}_C + \bar{I}_{BC} &= \bar{I}_{CA}; \end{aligned}$$

whence

$$\begin{aligned} \bar{I}_A &= \bar{I}_{AB} - \bar{I}_{CA}; \\ \bar{I}_B &= \bar{I}_{BC} - \bar{I}_{AB}; \\ \bar{I}_C &= \bar{I}_{CA} - \bar{I}_{BC}. \end{aligned}$$

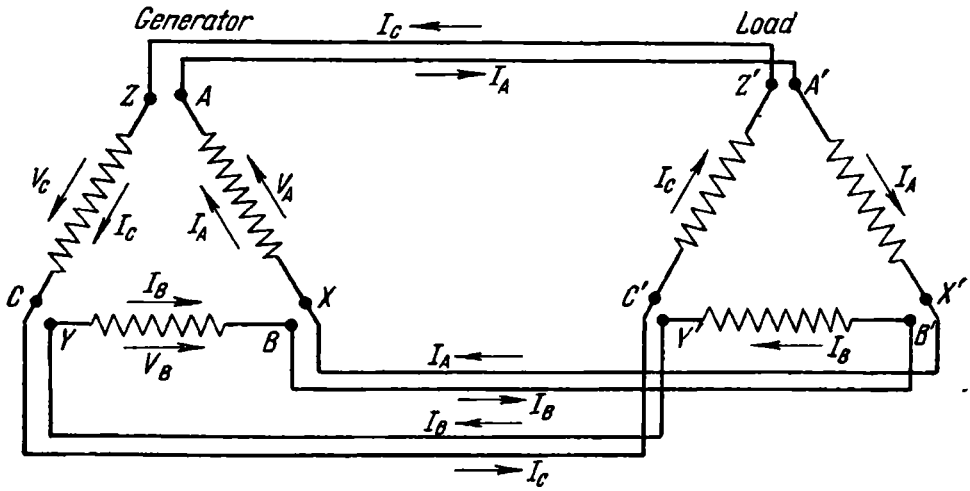


Fig. 187. Noninterlinked three-phase system

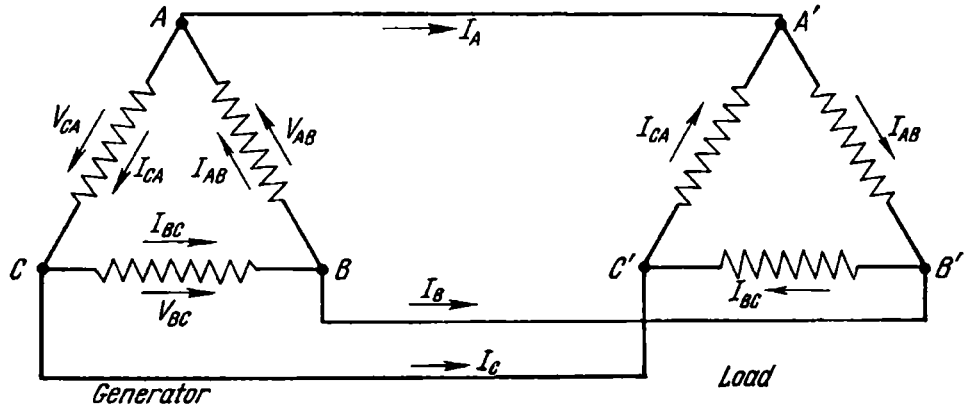


Fig. 188. Interlinked star-connected three-phase system

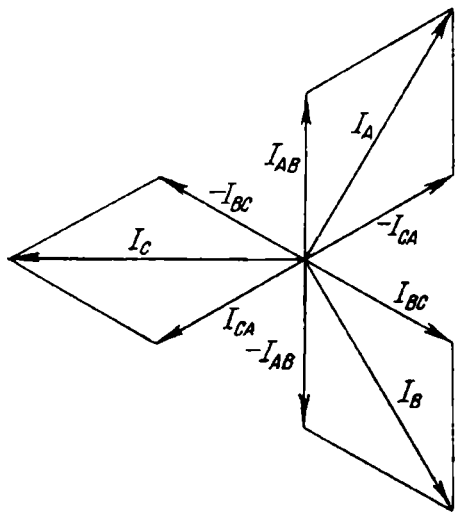


Fig. 189. Phase and line currents in a star-connected system

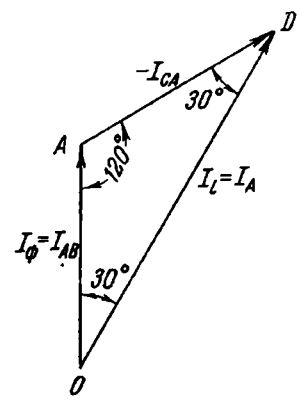


Fig. 190. Relation between phase and line currents in a star-connected system

In other words, the line currents are equal to the vectorial difference of the phase currents. In the case of balanced loading, the phase currents are equal in magnitude and spaced 120° apart. Vectorial subtraction of the phase currents, according to the above equation, gives the line currents (Fig. 189). Fig. 190 shows the relation between the phase and line currents in the delta connection:

$$I_l = 2I_{ph} \cos 30°.$$

Since $\cos 30° = \sqrt{3}/2$, it follows that $I_l = 2I_{ph}(\sqrt{3}/2) = \sqrt{3}I_{ph}$. Fig. 191 is the vectorial diagram for currents and voltages in a delta-connected resistive-inductive load. The diagram is constructed as follows. First, an equilateral line-voltage triangle is constructed for V_{AB}, V_{BC} and V_{AC}, which are equal to the phase voltages of the load. The vectors of the phase currents are then drawn so that they make the angles φ_{AB}, φ_{BC} and φ_{CA} with the line voltages. Next, the line currents I_A, I_B and I_C are determined as explained above.

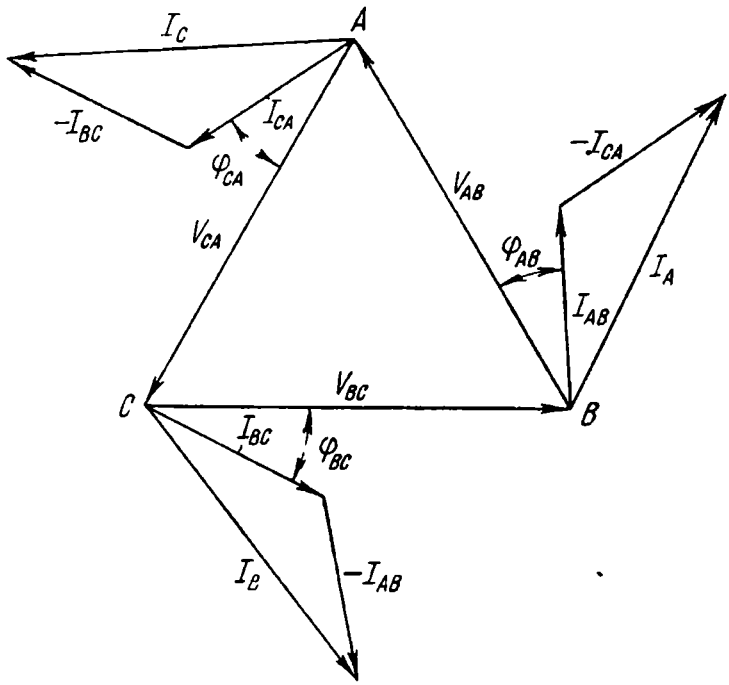


Fig. 191. Vector diagram of currents and voltages in a balanced star-connected system

Example 2. The line voltage applied to a three-phase motor is 220 V. The impedance of the motor winding is 10 ohms. Calculate the currents in the line wires and the motor windings if the latter are delta-connected (Fig. 192*a*).

By Ohm's Law

$$I_{ph} = V_{ph}/Z.$$

Since in the delta connection $V_l = V_{ph}$,

$$I_{ph} = 220 \div 10 = 22 \text{ A}.$$

The insulation of the motor phase winding is designed for 220 V, while the cross-section of the phase-winding wire will stand up to heating due to a current of 22 A.

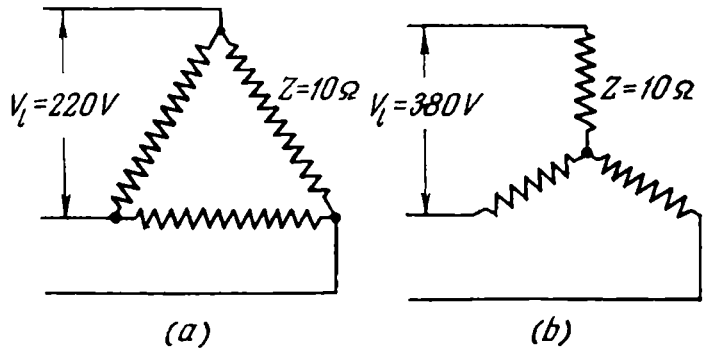


Fig. 192. See Ex. 2

In the delta connection, $I_l = I_{ph}\sqrt{3}$. Therefore,

$$I_l = 22 \times \sqrt{3} = 22 \times 1.73 = 38 \text{ A}.$$

This motor can also be connected to receive a line voltage of 380 V by reconnecting its windings into a star (Fig. 192*b*).

In the star connection, the line voltage is 1.73 times the line voltage, or $V_l = V_{ph}\sqrt{3}$. Therefore, the phase voltage of the motor will be

$$V_{ph} = V_l/\sqrt{3} = 380 \div 1.73 = 220 \text{ V},$$

i.e., the phase voltage of the motor will remain 220 V as before.

The phase current will also remain unchanged:

$$I_{ph} = 220 \div 10 = 22 \text{ A}.$$

In most cases, three-phase motors and other equipment have all the six ends of their windings brought outside, so that they can be connected either in a star or in a delta. These terminals are usually taken out to a terminal board of some insulating material.

Fig. 193 shows the connection of the terminals on a terminal board to the windings of a three-phase machine. Copper straps between the terminals facilitate reconnection.

If the nameplate of a motor reads "127/220 V", this means that the motor may be run on both 127 V and 220 V. If the line voltage is 127 V, the motor windings should be delta-connected (Fig. 193*b*), in which case the

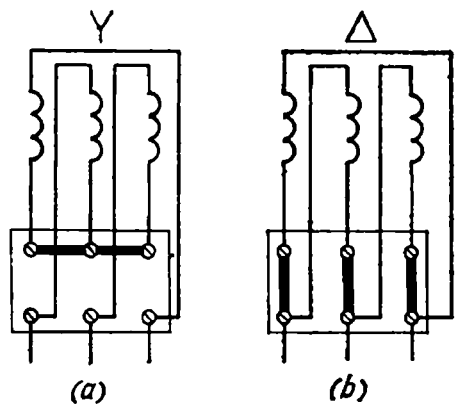


Fig. 193. Connection of leads of winding to the terminals of a three-phase machine

voltage across each phase (or coil) of the motor will be 127 V. When the voltage is 220 V, the motor windings should be connected in a star (Fig. 193*a*); then the voltage across each coil will be again 127 V.

103. Three-phase Power

The true power of a single-phase alternating current is given by

$$P = I_{ph} V_{ph} \cos \varphi \text{ (watts)},$$

where I_{ph} and V_{ph} are the phase current and phase voltage, respectively, and $\cos \varphi$ is the phase angle between them.

In a balanced three-phase system, the power delivered to and taken by each phase is the same. Therefore, the power of the three phases is

$$P = 3I_{ph}V_{ph} \cos \varphi \text{ (watts)}.$$

In the case of the star connection

$$I_l = I_{ph}; \quad V_l = \sqrt{3}\, V_{ph}.$$

Substituting these relationships in the above formula gives

$$P = 3I_l(V_l/\sqrt{3}) \cos \varphi$$

or

$$P = 3I_l V_l \cos \varphi \text{ (watts)}.$$

The apparent power will be

$$S = \sqrt{3} I_l V_l \text{ (volt-amperes)}.$$

For the delta connection

$$V_l = V_{ph}; \quad I_l = I_{ph}\sqrt{3}.$$

Substituting these relationships in the power formula gives

$$P = 3(I_l/\sqrt{3})V_l \cos \varphi$$

or

$$P = \sqrt{3} I_l V_l \cos \varphi \text{ (watts)}.$$

The apparent power will be

$$S = \sqrt{3} I_l V_l \text{ (volt-amperes)}.$$

Thus, true and apparent power are the same for both the star and delta connections, and they can be found for a three-phase system from the formulas

$$P = \sqrt{3}\, VI \cos \varphi \text{ (watts) and } S = \sqrt{3}\, VI \text{ (volt-amperes)}.$$

These formulas hold only for balanced loading.

Example 3. A three-phase symmetrical load has a resistance of 6 ohms and an inductive reactance of 8 ohms in each phase. The line voltage is 220 V. Calculate the power of the load if it is star-connected.

$$V_{ph} = V_l/\sqrt{3} = 220 \div \sqrt{3} = 127 \text{ V};$$

$$Z = \sqrt{R^2+X_L^2} = \sqrt{6^2+8^2} = \sqrt{100} = 10 \text{ ohms};$$

$$I_{ph} = V_{ph}/Z = 127 \div 10 = 12.7 \text{ A};$$

$$\cos\varphi = R/Z = 6 \div 10 = 0.6;$$

$$P_{ph} = I_{ph}V_{ph}\cos\varphi = 12.7 \times 127 \times 0.6 = 967.74 \text{ watts}.$$

The power of the three phases will be

$$P = 967.74 \times 3 = 2903.22 \text{ watts} = \text{approx. } 2.9 \text{ kW},$$

or

$$P = \sqrt{3}\,VI\cos\varphi = \sqrt{3} \times 220 \times 12.7 \times 0.6 = \text{approx. } 2.9 \text{ kW}.$$

In the case of an unsymmetrical or unbalanced load the total power is found by summing up the power in the separate phases.

Example 4. A line voltage of 380 V supplies a delta-connected three-phase load (Fig. 194).

Phase *I* has a resistance of 8 ohms and an inductive reactance of 4 ohms; phase *II* has a resistance of 2 ohms and an inductive reactance of 6 ohms; phase *III* has a resistance of 3 ohms and an inductive reactance of 5 ohms. Calculate the total power of the three phases.

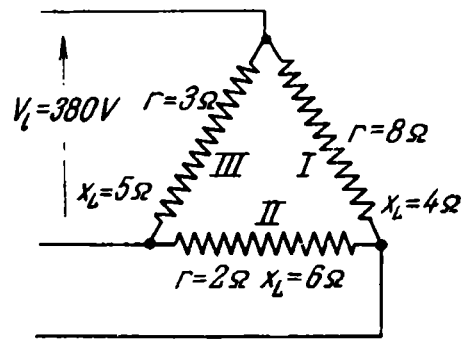


Fig. 194. See Ex. 4

$$Z_1 = \sqrt{R_1^2+X_{L1}^2} = \sqrt{8^2+4^2} = \sqrt{80} = 8.9 \text{ ohms};$$

$$I_{ph1} = V_{ph}/Z_1 = 380 \div 8.9 = 42.6 \text{ A};$$

$$\cos\varphi_1 = R_1/Z_1 = 8 \div 8.9 = 0.9;$$

$$P_1 = V_{ph1}I_{ph1}\cos\varphi_1 = 380 \times 42.6 \times 0.9 = 14{,}569 \text{ watts}.$$

$$Z_2 = \sqrt{2^2+6^2} = 6.3 \text{ ohms};$$

$$I_{ph2} = 380 \div 6.3 = 60.3 \text{ A};$$

$$\cos\varphi_2 = 2 + 6.3 = 0.32;$$

$$P_2 = 380 \times 60.3 \times 0.32 = 7332 \text{ watts}.$$

$$Z_3 = \sqrt{3^2+5^2} = 5.8 \text{ ohms};$$

$$I_{ph3} = 380 \div 5.8 = 65.5 \text{ A};$$

$$\cos\varphi_3 = 3 \div 5.8 = 0.51;$$

$$P_3 = 380 \times 65.5 \times 0.51 = 12{,}694 \text{ watts}.$$

The total power

$$P = P_1+P_2+P_3 = 14{,}569+7332+12{,}694 = \text{approx. } 34.6 \text{ kW}.$$

104. The Revolving Magnetic Field

A very remarkable property of a three-phase (and, indeed, of any polyphase) current is the ability to produce a revolving magnetic field.

To obtain a revolving magnetic field, three coils are wound on a steel ring so that they are spaced 120° apart, and a three-phase current is applied

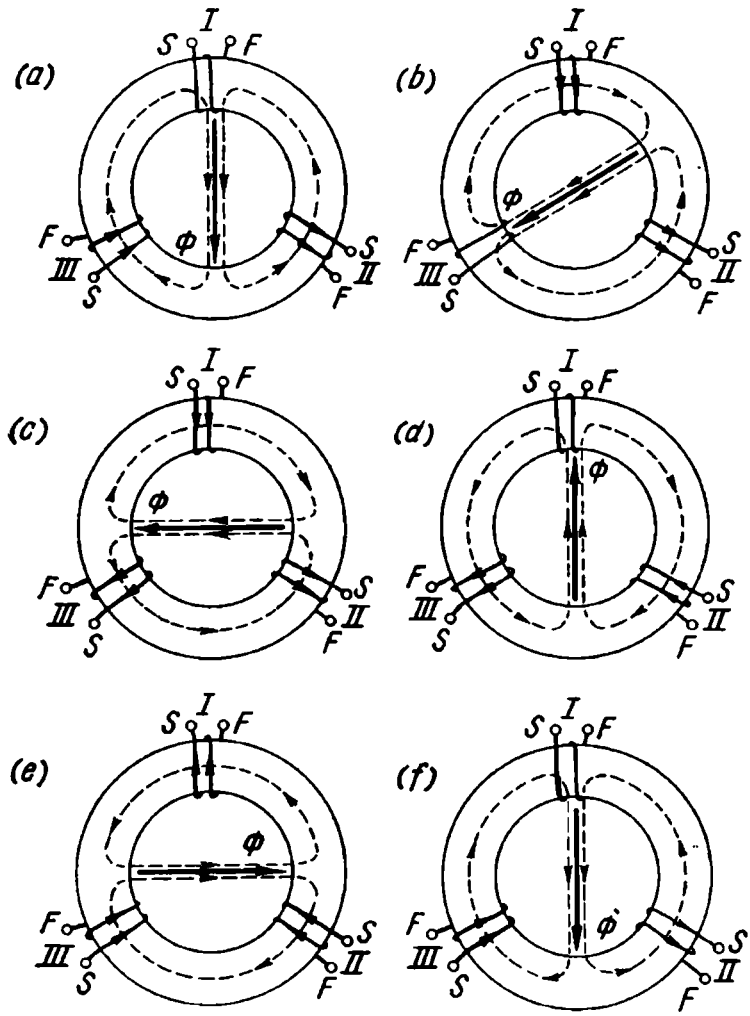


Fig. 195. Revolving magnetic field produced by a three-phase current

to them (Fig. 195). The arbitrary positive direction of the current is such that it flows from the starts to the ends of the coils, while the negative direction is from the ends to the starts of the coils.

Fig. 196 shows the waveforms of a three-phase alternating current. At time *a* the current in the first phase, i_1, is zero. The current in the third

phase, i_3, is positive and in the second phase, i_2, negative. The direction of the coil currents for the same position *a* is shown in Fig. 195. The direction of the magnetic field set up by each coil can be determined by the cork-screw rule. The direction of the resultant magnetic field is shown by the vector Φ.

As we move through consecutive positions *a* to *e* (Figs. 195 and 196), we find that the vector Φ remains constant in magnitude. Its position in space, however, varies all the time: it is revolving. Thus, a revolving magnetic field is established inside the ring.

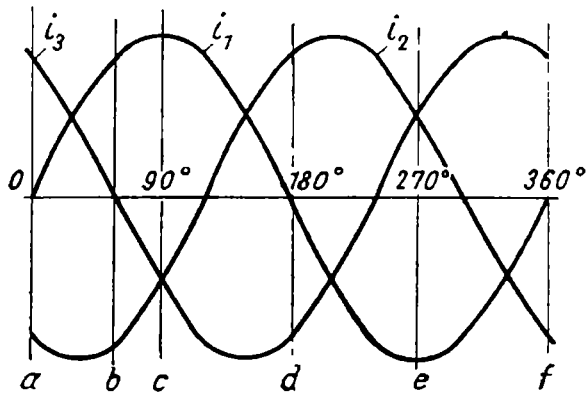


Fig. 196. Graphs of a three-phase current

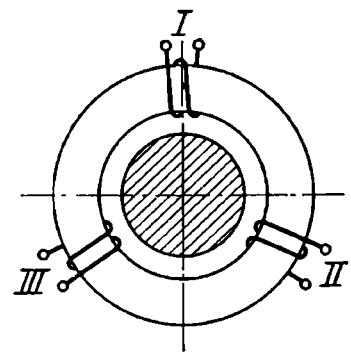


Fig. 197. A method of producing a revolving magnetic field

The sense of rotation of the magnetic field depends on the sequence of phases (*I*, *II*, *III*), i.e., it can be varied at will by reconnecting the phase wires running to the coils.

If a steel drum is placed within the steel ring carrying three coils (Fig. 197), the revolving magnetic field will cut across the drum to induce an emf in it. The induced emf will interact with the revolving magnetic field so that the drum will rotate too.

The revolving magnetic field has found many practical uses, especially in synchronous and induction motors, whose construction will be described elsewhere in the book.

A revolving magnetic field can also be produced by two-, four- and six-phase currents. In effect, it can be obtained with a single-phase alternating current by spacing two coils, say, 90° apart.

Exercises

1. The line voltage of a three-phase four-wire system is 220 V. The system powers ninety 150-W incandescent lamps, each phase containing an equal number. Calculate the voltage of the lamps and the line currents.

2. Three groups of ten lamps each are connected between the lines of a three-phase system. Each lamp takes a current of 0.5 A. Calculate the line currents.

3. A three-phase transformer has a rating of 50 kVA. Determine the true power which the transformer can deliver at a power factor of 1, 0.8, 0.6, and 0.2 in the load.

4. What current is taken by a 5.4-kW three-phase motor if the mains voltage is 220 V and the motor has a power factor of 0.8?

5. Three induction coils are star-connected. The resistance of each coil is 6 ohms and the inductive reactance is 5 ohms. Calculate the true and apparent power taken by the three coils if the mains voltage is 220 V.

6. A steel-cored induction coil has a resistance of 4 ohms and an inductive reactance of 6 ohms. What will be the readings of a three-phase wattmeter if the coils are connected star and delta? The mains voltage is 220 V.

7. A three-phase generator generates a voltage of 220 V. The wattmeter reads 13.2 kW. The generator supplies a delta-connected load consisting of incandescent lamps. Each lamp takes a current of 0.25 A. How many lamps are connected to the generator?

8. A 3-hp three-phase motor is connected to a 220-V line. The power factor of the motor is 0.8. Calculate the current the motor takes from the line.

9. A three-phase motor is connected in a 220-V line and takes a current of 10 A. The power factor of the motor is 0.85 and the efficiency is 80 per cent. Calculate the effective horsepower of the motor.

10. Three coils arranged in a delta are connected in a 120-V line. Their resistances and inductive reactances respectively are 3 ohms and 20 ohms; 2 ohms and 15 ohms; 6 ohms and 30 ohms. Calculate the true power taken by the three coils.

11. The nameplate of a three-phase generator reads: voltage 127 V; current 40 A. What is the maximum number of incandescent lamps that can be connected to the generator if the lamps are delta-connected and each lamp takes a current of 0.25 A?

Review Questions

1. What is a three-phase alternating current?
2. How is a three-phase current generated?
3. What is a six-wire three-phase system? a four-wire three-phase system? a three-wire three-phase system?
4. What is phase voltage? line voltage?
5. What is the relation between the phase and line voltages and currents in the delta and star connections?
6. How does a three-phase current produce a revolving magnetic field?

Chapter Nine

TRANSFORMERS

105. General

In Russia, the use of transformers for supplying current to arc lamps was suggested for the first time in 1876 by Yablochkov, the inventor of the arc lamp. A good deal of work on transformer design was done by another Russian, I. F. Usaguin, who suggested the transformer as a source of power for loads other than arc lamps.

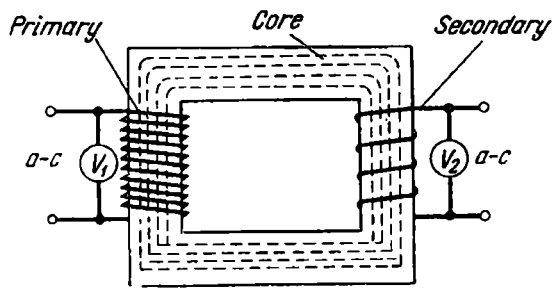


Fig. 198. Diagram of a transformer

As has been stated earlier, a transformer is a device which depends for its action on mutual induction and which serves to convert electric power in an a-c system at one voltage or current into electric power at some other voltage or current without change in the frequency.

An elementary transformer (Fig. 198) consists of a steel core which carries two windings insulated from both the core and each other. The winding connected to the supply circuit is called the *primary* and the winding connected to the receiver circuit is the *secondary*.

The alternating current traversing the primary sets up an alternating magnetic field which interlinks the secondary and induces an emf in it.

Since the magnetic field is an alternating field, the induced emf in the secondary is likewise alternating at a frequency equal to that of the current in the primary.

The alternating magnetic field cuts across both the secondary and the primary, and so an emf is also induced in the primary.

The magnitudes of the two emfs depend on the frequency of the magnetizing current, the number of turns in the respective windings, and the magnetic flux in the core, or $E = 4.44fT\Phi_m$. For a given frequency and a constant magnetic flux, the emf of each winding is dependent solely on the number of turns in it. This relationship can be expressed thus:

$$E_1/E_2 = T_1/T_2,$$

where E_1 and E_2 are the emfs in the primary and secondary, and T_1 and T_2 are the respective number of turns.

Voltmeters (V_1 and V_2 in Fig. 198) connected across the windings will indicate their respective terminal voltages, V_1 and V_2.

The difference between the emfs and the terminal voltages is so small that the terminal voltages of the primary and secondary are practically proportional to the number of turns

$$V_1/V_2 \approx T_1/T_2.$$

The difference between the emf and the terminal voltage of the primary is especially small when the secondary is open and the current in it is zero. The current flowing in the primary when the secondary is open-circuited is very small (3.5 to 10 per cent of the load current), and is known as the exciting (or magnetizing, or no-load) current of a transformer. Also, the secondary terminal voltage is equal to the emf induced in it.

In Soviet terminology the ratio V_1/V_2 is referred to as the *transformation ratio.* It is designated by the letter k:

$$k = V_1/V_2.$$

The nameplate of a transformer gives the rated voltages (both primary and secondary) for no-load operation. Using these rated voltages, it is easy to determine the transformation ratio. Alternatively, this ratio can be found experimentally by connecting voltmeters to the terminals of the primary and secondary, opening the secondary circuit, and then dividing the reading of the voltmeter connected across the primary by that of the voltmeter connected across the secondary.

Example 1. The primary of a transformer is connected to receive a voltage of 6600 V, while the secondary terminal voltage is 230 V. Determine the transformation ratio.

$$k = V_1/V_2 = 6600 \div 230 = 28.7.$$

Example 2. The primary of a transformer carries a voltage of 10,000 V. The secondary terminal voltage is 100 V. Calculate the transformation ratio and the number of turns in the secondary if the primary has 21,000 turns.

The transformation ratio will be

$$k = V_1/V_2 = 10{,}000 \div 100 = 100.$$

From $V_1/V_2 = T_1/T_2$ the number of turns in the secondary is

$$T_2 = \frac{V_2 V_1}{V_1} = \frac{100 \times 21{,}000}{10{,}000} = 210 \text{ turns.}$$

The same answer can be obtained from the turns ratio $k = T_1/T_2$:

$$T_2 = T_1/k = 21{,}000 \div 100 = 210 \text{ turns.}$$

In the above examples the transformers reduce the applied voltage and are therefore called *step-down transformers*. Their ratio is greater than unity.

Example 3. If the secondary of the transformer in the above example (with 210 turns) is connected to a supply circuit with a voltage of, say, 100 V, it will become a primary. Then, the former primary will become a secondary and its terminal voltage will be 100 times greater, since it has 100 times as many turns as the former secondary. The ratio will then be

$$k = V_1/V_2 = 100 \div 10{,}000 = 0.01.$$

The transformer used in the above example raises the applied voltage. Such transformers are known as *step-up transformers*. Their transformation ratio is less than unity.

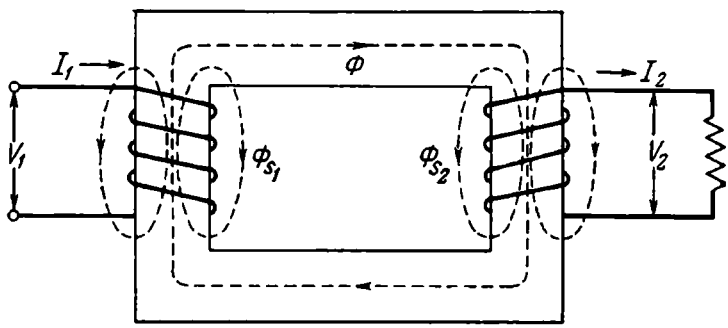


Fig. 199. Magnetic fluxes in a transformer

Not all of the magnetic flux produced by the primary links the secondary; similarly, part of the magnetic flux produced by the secondary does not link the primary. These leakage (or stray) fluxes, Φ_{s1} and Φ_{s2}, pass through the air (Fig. 199).

The stray fluxes induce emfs in the respective windings:

$$E_{s1} = 2\pi f L_{s1} I_1$$

$$E_{s2} = 2\pi f L_{s2} I_2$$

where L_{s1} and L_{s2} are the reactances produced in the windings by the leakage fluxes.

Since $$X_{L1} = 2\pi f L_1,$$
$$X_{L2} = 2\pi f L_2$$

we get $$E_{s1} = X_{L1} I_1,$$
$$E_{s2} = X_{L2} I_2,$$

where X_{L1} and X_{L2} are the inductive reactances of the windings due to the leakage (or stray) fluxes.

106. Performance of a Transformer at No-load

When the primary of a transformer is connected to an a-c source and the secondary is open, the transformer is said to be *at no-load* (there is no load on the secondary).

Fig. 200 shows the vector diagram for no-load voltage of an ideal (no-loss) single-phase transformer.

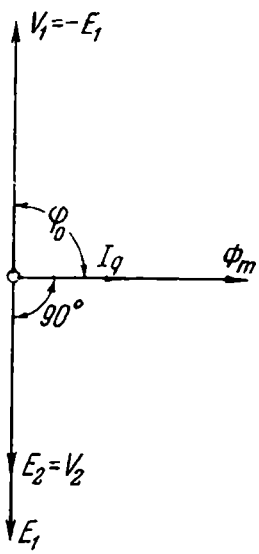


Fig. 200. Vector diagram of the no-load current in an ideal transformer

The magnetic flux Φ_m induces an emf E_1 in the primary and an emf E_2 in the secondary. As will be recalled, any emf induced by a sine-wave magnetic flux lags 90° behind the flux (see Sec. 84). Therefore, the vectors E_1 and E_2 will be at 90° and behind the flux Φ_m.

E_1 and the applied voltage V_1 are equal in magnitude and opposite in sign and so cancel each other. The no-load current I_0 of the transformer (see Fig. 201) is made up of two components: one, the current required to magnetize the core, $I_q = I_0 \sin \varphi_0$, which is in phase with the magnetic flux Φ_m; and the other, the small current required to supply the core loss, $I_a = I_0 \cos \varphi_0$. The former is the reactive current and the latter the active current. The active current is so small that the no-load current of a transformer may be taken as fully reactive. As can be seen from the vector diagram, the reactive current I_q lags 90° behind the applied voltage V_1.

The great difference between the primary and secondary voltages makes it difficult to construct the vector diagrams of the latter. It is customary, therefore, to refer the secondary to the primary, i.e., to regard the secondary as having as many turns as the primary, and to change the terminal voltage, current and impedance of the secondary so that the power rating and phase angles of the transformer remain unaffected.

The referred parameters of the secondary are designated E_2', I_2', R_2', etc. In order to obtain E_2', E_2 should be multiplied by the turns ratio

$k = T_1/T_2$. Therefore,

$$E_2' = E_2(T_1/T_2) = E_1.$$

The referred secondary current I_2' should be such that the apparent power of the secondary remains unaffected, or

$$E_2'I_2' = E_2I_2,$$

whence

$$I_2' = (E_2/E_2')I_2 = \frac{1}{k} I_2.$$

Fig. 202 is the vector diagram for the no-load voltage of a real transformer. The applied primary voltage V_1 should counterbalance the emf $-E_1$, the resistive voltage drop I_0R_1, which is in phase with the current I_0, and the inductive reactance I_0X_{s1}, which leads the current I_0 by 90°. The vectorial sum of $-E_1$, I_0R_1 and I_0X_{s1} gives the voltage V_1.

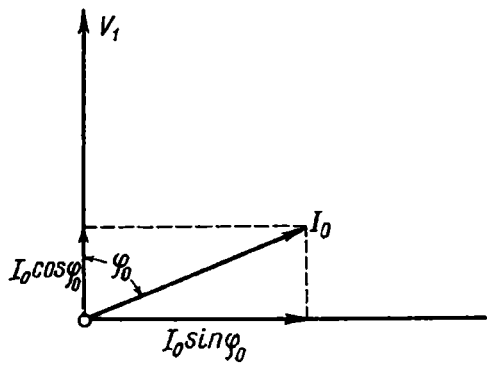


Fig. 201. Diagram of no-load current

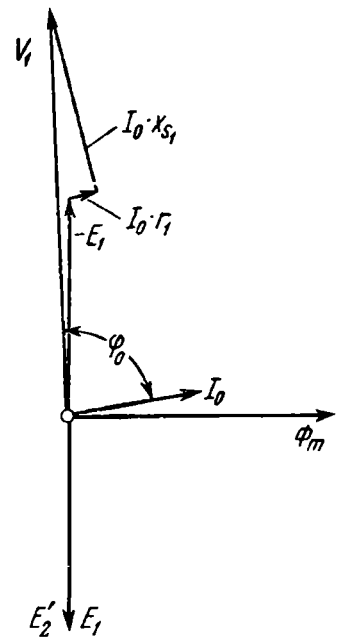


Fig. 202. Vector diagram of the no-load current of a real transformer

107. Performance of a Loaded Transformer

When the secondary circuit of a transformer is completed through an impedance, or load, the transformer is said to be loaded, and current flows through the secondary and the load. The secondary current I_2 produces a magnetic flux of its own, Φ_2, in addition to the flux produced by the primary current.

Fig. 203 is the vector diagram for a loaded (ideal) transformer. The magnetic flux Φ_m and the magnetizing current I_q are in phase. The emfs E_1 and E_2' lag 90° behind the magnetic flux Φ_m. Since the load is resistive and the transformer is free from losses, the current I_2' is also in phase with E_2'. The vector sum of the ampere-turns of the primary and secondary under load will be nearly equal to the ampere-turns of the primary at no-load:

$$\bar{I}_1T_1 + \bar{I}_2T_2 = \bar{I}_0T_1.$$

By Lenz's Law, the ampere-turns of the secondary, I_2T_2, will tend to reduce the primary magnetic flux. Therefore, the added ampere-turns produced by the secondary load must be balanced by equal and opposite primary ampere-turns. In other words, the primary ampere-turns must be made up of a component supplying the magnetic flux Φ_m, or I_0T_1, and a component making up for the added secondary ampere-turns, I_2T_2:

$$I_1T_1 = I_0T_1 - I_2T_2$$

or

$$\bar{I}_1 = -\bar{I}_2(T_2/T_1) + \bar{I}_0 = -\bar{I}_2' + \bar{I}_0.$$

So the primary current of a loaded transformer must be equal to the vector sum of the no-load current $\bar{I}_0$ and the equivalent secondary current $\bar{I}_2'$ (taken with the minus sign).

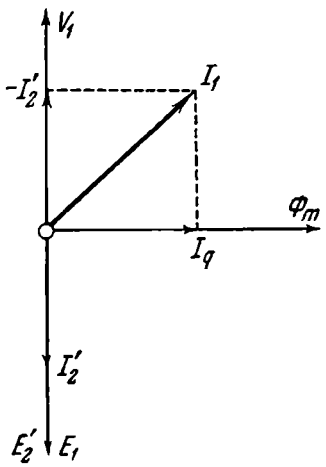


Fig. 203. Vector diagram of an ideal transformer in the case of a resistive load

Fig. 204 shows the vector diagram of a real transformer loaded with a resistance. Under load, the secondary terminal voltage V_2' is less than the secondary emf $\bar{E}_2'$ by the voltage drop across the winding. Therefore, V_2' (vector OK) can be obtained by subtracting the inductive voltage drop $I_2'X_{s2}'$ and the resistive voltage drop $I_2'R_2'$ vectorially from E_2' (vector OP). Since the load is resistive, I_2' is in phase with V_2'. The primary current $\bar{I}_1$ is obtained from the vector sum of the no-load current I_0 and the referred load voltage $\bar{V}_2'$ taken with opposite sign $-I_2'$. The applied voltage V_1 must balance the primary emf E_1 and the primary voltage drop. Therefore, the required value of V_1 will be obtained from the vector sum of $-E_1'$, which balances E_1 (vector OD), the resistive voltage drop in the primary I_1R_1 (vector DM), which is in phase with the current, and the inductive voltage drop across the primary L_1X_{s1} (vector MF). The vector OF gives both the magnitude and phase of the voltage V_1 that must be applied to the primary.

Fig. 205 shows the vector diagram of a transformer with a resistive-inductive load, and Fig. 206, that of a transformer with resistive-capacitive load.

As can be inferred from the vector diagrams for real transformers, an increase in the load current I_2 results in (1) an increased voltage drop across the secondary; (2) a decreased secondary terminal voltage V_2 (in the case of a resistive load the secondary terminal voltage may rise); (3) an increased primary current I_1; (4) an increased voltage drop across the primary; (5) a decreased E_1 for a constant primary voltage V_1; (6) a decreased E_1 due to a reduced magnetic flux Φ_m.

To sum up, an increase in the load current I_2 not only increases the current taken by a transformer from the mains (I_1), but also decreases the mag-

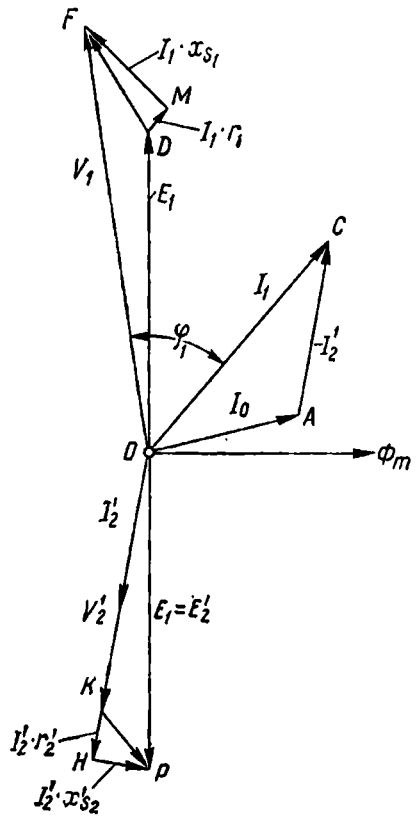


Fig. 204. Vector diagram of a real transformer in the case of a resistive load

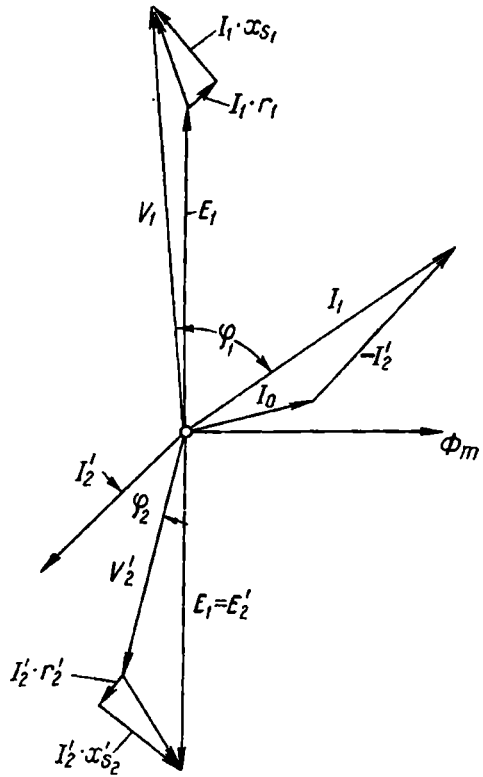


Fig. 205. Vector diagram of a real transformer in the case of a resistive-inductive load

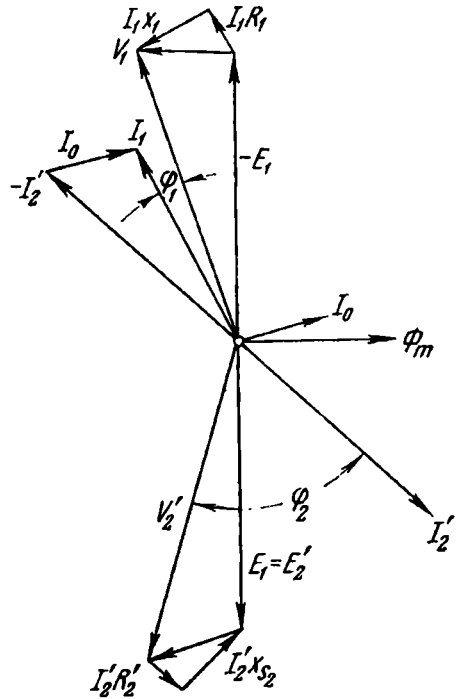


Fig. 206. Vector diagram of a real transformer in the case of a resistive-capacitive load

netic flux in the transformer core. However, changes in the magnetic flux Φ_m from no-load to full load are small. Indeed, in approximate calculations the flux is often assumed to be constant.

108. Transformer Testing

Although transformers, of all electrical apparatus, most closely approach the ideal, they still have losses. They include the *no-load loss* and the *load loss.*

The no-load loss consists of both hysteresis and eddy-current losses in the core (hence, it is referred to as the iron loss or the core loss). The load loss consists of the *copper* and *impedance losses* which, in turn, include I^2R losses and eddy-current losses in the windings and leads.

Due to these losses, the efficiency of any transformer is less than 100 per cent. In order to determine its actual performance, every transformer is subjected to a programme of tests, the principal of which are the *short-circuit test* and the *open-circuit test.*

In a short-circuit test, the secondary winding is short-circuited and a small voltage is impressed on the primary, producing the full-load current in the latter. The applied voltage is usually expressed as a percentage of the primary rated voltage. For Soviet-made power transformers, this voltage is 5 to 10 per cent (in some cases it is as high as 17 per cent).

A short-circuit test involves the use of an ammeter, a wattmeter and a voltmeter connected in the primary circuit, and an ammeter connected in the secondary winding. With the current adjusted as closely as possible to the rated value, simultaneous readings are taken on the ammeters, voltmeter and wattmeter. These readings are plotted to give the short-circuit-test characteristic of the transformer.

On account of the extremely low voltage and the consequent low magnetization of the core , the loss in the core may be neglected, and the wattage input measured in a short-circuit test will give the copper and impedance loss on full load of the complete transformer. Hence, another name for the test—the *copper-loss* and *impedance test*, and the name for the voltage used—the *impedance voltage.* A short-circuit characteristic of a transformer is shown in Fig. 208.

In an open-circuit test, the secondary winding is left open, normal voltage is applied to the primary, and wattage input and current are measured. An open-circuit test involves the use of an ammeter, a voltmeter and a wattmeter connected in the primary circuit. The voltage applied to the primary is gradually increased, and simultaneous readings are taken on the instruments. These readings are then plotted to give the no-load characteristic of the transformer (Fig. 207).

Since no current flows in the open-circuited secondary, the current in the primary will be merely that necessary to magnetize the core at normal voltage. Moreover, this magnetizing current is a very small fraction of the full-

load current and may be neglected as far as the copper loss is concerned. Consequently, the test gives the core loss alone very accurately. Hence another name for the open-circuit test—the *core-loss test.*

The open-circuit test supplies enough data to compute the resistance, power factor, and no-load current of a transformer.

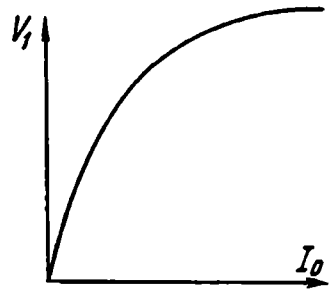


Fig. 207. No-load characteristic of a transformer

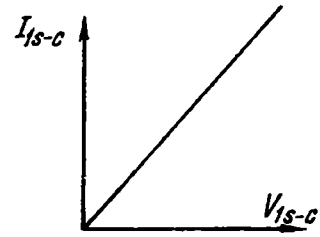


Fig. 208. Short-circuit characteristic of a transformer

On the basis of the short-circuit and open-circuit tests it is possible to compute the efficiency of a transformer. The efficiency, η, of a transformer is the ratio of the kW output to the kW input and is usually expressed as a percentage. Since the output and input kW differ only by the losses, a more convenient form of expressing efficiency is the ratio of the output kW to the output kW+kW losses:

$$\eta = P_2/P_1 = \beta P_r \cos \varphi_2/(\beta P_r \cos \varphi_2 + P_0 + \beta^2 P_c),$$

where P_0 = core loss;
$\beta^2 P_c$ = copper loss proportional to the square of the secondary overcurrent;
P_c = copper loss at rated load;
$\beta = I_2/I_{2r}$ = secondary overcurrent.

109. The Use of Transformers in Power Transmission

Example 4. An electric power station uses a 200-kW, 230-V d-c generator. It is required to calculate the cross-sectional area of the wires necessary to transmit power to a consumer at a distance of 10 km from the station.

The current flowing in the generator winding and line will be

$$I = P/V = 200{,}000 \div 230 = 870 \text{ A}.$$

Let the heat losses in the line (which are inevitable) be 10 per cent of the transmitted power, i.e., 20 kW. Then the resistance of one conductor should be

$$R = P/2I^2 = 20{,}000 \div (2 \times 870^2) = 0.013 \text{ ohm}.$$

From $R = \varrho l/S$ it follows that

$$S = \varrho l/R = (0.0175 \times 10{,}000) \div 0{,}013 = 13{,}461 \text{ mm}^2$$

The diameter of such a wire would be 13.4 cm and one metre would have a weight of 119.8 kg. For the entire line the copper conductor would weigh 2396 tons.

Obviously, low-voltage direct current is not an attractive proposition for power transmission even over relatively short distances.

Now suppose that the station uses an a-c generator rated for 200 kW at 230 V. The load is assumed to be resistive. The station also uses a step-up transformer with a transformation ratio of 1 : 43.4 (Fig. 209).

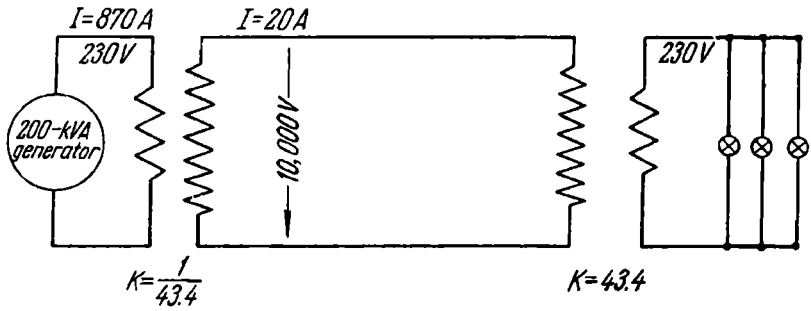


Fig. 209. The transformer in a power-transmission line

The line voltage will be $230 \times 43.4 = 10{,}000$ V. Since the losses in the transformer are negligible, the transformer input power may be taken to be equal to the transformer output power. Then, if the input power is 200 kW, the output power will also be 200 kW. Hence the secondary current will be

$$I = P/V = 200{,}000 \div 10{,}000 = 20 \text{ A}.$$

From $P=2I^2R$ it follows that, given the same heat losses in the transmission line, the current in the line is reduced by a factor of 43.4. Hence, the resistance of the conductors may be 43.4^2 times greater and their cross-sectional area may be smaller by a factor of 43.4^2. Then

$$S = 13{,}461 \div 43.4^2 = 7.1 \text{ mm}^2.$$

A standard cross-section of 10 mm² may be taken.

The above example shows how important are transformers in power transmission lines: an increase in the line voltage due to a step-up transformer reduces the line current and, consequently, the cross-section of the conductors. The longer the line, the higher the line voltage.

Ordinary loads, however, cannot be connected directly to a high-voltage line. Therefore, another, step-down, transformer has to be provided at the receiving end of the line to convert the high voltage to low voltage for the consumer. In our example (Fig. 209), use is made of a step-down transformer with a transformation ratio $k=43.4$. Due to the voltage drop across the transformer, the voltage across the consumer's terminals will be somewhat lower than 230 V.

At present, there are power transmission lines operating at 220, 287, 400 and 500 kV. Where large distances are involved, however, a d-c voltage of up to 1 MV (1 million volts) appears to be more advantageous, since the cost of the line and associated equipment is almost half that of a similar a-c line. A long-distance d-c transmission line operates as follows.

The sending station generates a three-phase voltage of about 15 kV. A step-up transformer raises this to several hundred kilovolts. The stepped-up voltage is then rectified and fed into the line. At the receiving end of the line an *inverted converter* (or simply an *inverter*) changes the d-c voltage back to a-c voltage, which is then reduced by a step-down transformer to a value convenient for consumers.

110. Transformer Construction and Classification

The core, or magnetic circuit, of a transformer is made up of silicon-steel stampings or laminations 0.35 to 0.5 mm thick. In the Soviet Union, Grade Э4-2 steel is commonly used for this purpose. It contains 4 to 4.8 per cent silicon, which improves the magnetic properties of the steel and increases its resistivity to eddy currents. The laminations are coated with a thin

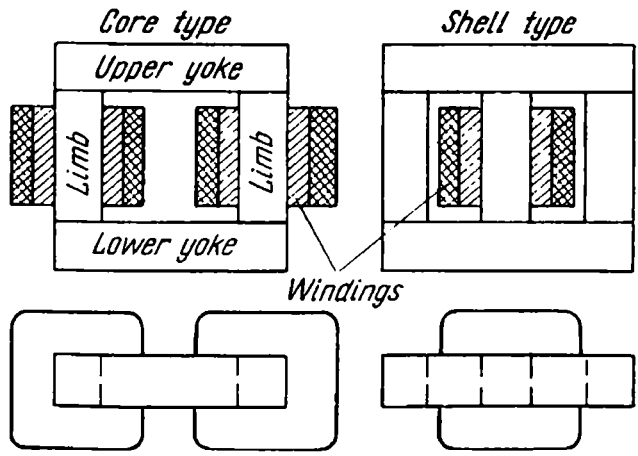


Fig. 210. Types of transformer magnetic circuits

coat of varnish in order to reduce eddy-current losses still further and are firmly clamped together by bolts passed through insulating sleeves. Those parts of the magnetic circuit which carry the transformer windings are called *limbs* or *legs*. The limbs are held together by a top and a bottom *yoke*.

Two types of magnetic circuit are in common use (Fig. 210): the *core type* (or the two-limb type) and the *shell type* (or the three-limb type). In the core type, the magnetic circuit takes the form of a single ring encircled by the transformer windings. In the shell type, the primary and secondary windings are on a common ring which is encircled by the magnetic circuit like a shell. Of the two types, the core type is easier to inspect and repair, and for this reason it is more common than the shell type.

The windings are wound with either round or strip wire on a bakelite-impregnated sleeve which is put on the limb. The low-voltage winding is placed next to the magnetic material since it is easier to insulate from the

steel core than the high-voltage winding. Another bakelite-impregnated sleeve is put on the low-voltage winding, and the high-voltage winding is wound over it. This arrangement is convenient in that it permits easy removal or repair of the high-voltage winding, which is more liable to faults than the low-voltage winding.

The leads from the two windings are taken out through bushing insulators located on the transformer cover.

Small transformers (5 to 10 kVA) are air cooled, but as the size and voltage increases, oil-cooled units become more economical, with the result that almost all power transformers are oil cooled.

In the case of oil-natural (O.N.) cooled transformers (also classed as oil-immersed self-cooled transformers), the active core, comprising the magnetic circuit and windings, is placed in a rectangular or oval tank made of sheet steel. The tank is filled with transformer oil of high dielectric strength. The oil in the tank serves the twofold purpose of insulating the turns of the coils from one another and of cooling the active core. For better heat abstraction, the tank surface may be made finned or corrugated, but a far better method is to employ a tubular tank in which external tubes are welded to the tank sides near the top and bottom. For very large transformers it becomes necessary to employ banks of tubes arranged as external *radiators*.

On taking heat from the windings, the oil expands. On cooling, it contracts and, if suitable measures are not taken, moisture-laden air detrimental to transformer operation may find its way into the space above the oil level. Ingress of air is usually prevented by rubber gaskets placed between the tank and the cover, which is bolted to the tank shell.

Heated oil may expand to a point where it can break through the joint between tank and cover. To take care of this excess volume of oil, an *expansion tank* (also called a *conservator*) is mounted on top of the tank. The conservator is connected to the tank by a tube. On heating, the oil expands and overflows into the conservator; on cooling, the oil contracts and flows back into the tank. The conservator is fitted with an oil gauge (a glass tube) which provides a convenient check on the oil level in the transformer.

Still better cooling conditions are obtained in the case of oil-natural air-blast cooling (oil-immersed forced-air-cooled transformers in another classification). In this system of cooling, an air blast is applied to an ordinary (self-cooled) tubular tank. In still another system, called forced-oil and water cooling (oil-immersed forced-oil-cooled transformers), the oil is generally drawn out of the main tank by means of a pump and forced through cooling coils, which are cooled by running water. The cooled oil is fed back into the transformer tank at the bottom.

Fig. 211 shows an oil-natural cooled (oil-immersed self-cooled) transformer, while the one in Fig. 212 is an oil-natural air-blast (oil-immersed forced-air-cooled) transformer.

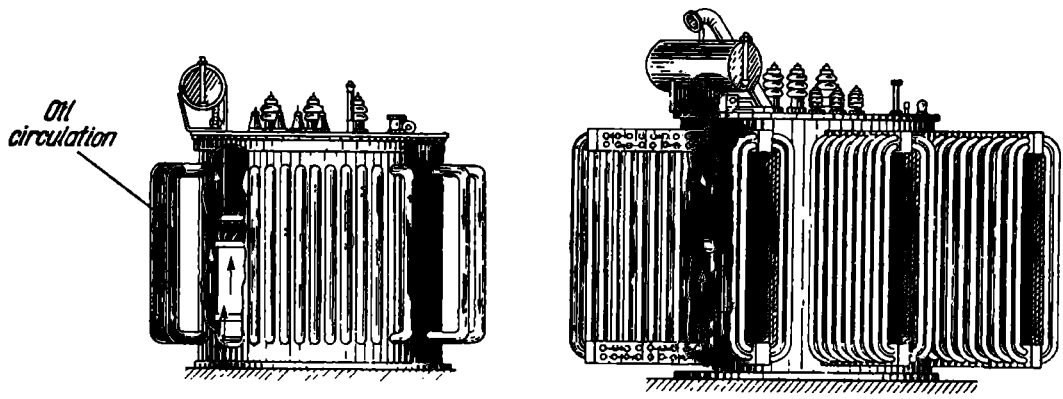


Fig. 211. Oil-natural cooled transformers:
a—Type TM-320/6 (with a tubular tank); *b*—Type TM-5600/35 (with a radiator)

The temperature of a transformer can be measured by two methods: (a) by thermometer in the cooling medium and (b) by the resistance of the transformer windings. The latter method gives a true indication whereas the former is easier to carry out.

So far we have dealt with transformers with two windings per phase. Such units are called *two-winding* transformers. There are transformers which have one primary and two secondary windings. The primary is a H.V. coil (for high voltage); one of the secondaries is a M.V. coil (for medium voltage) and the other is a L. V. coil (for low voltage). Such units are called *three-winding* transformers. The rated voltages of a three-winding transformer are given by a triple fraction; such as 220/115/10.5 kV or 110/35.8/11 kV.

Fig. 213 shows a Soviet-made single-phase three-winding step-down power transformer, designated "ОДТГ = = 90000/400". The H.V. winding carries a voltage of 400 kV, the M.V. winding a voltage of 110 kV, and the L.V. winding a voltage of 11 kV. The transformer uses natural-oil air-blast cooling, which means that its oil is contained in a plain tubular tank and air is blown over with fans. Rated at

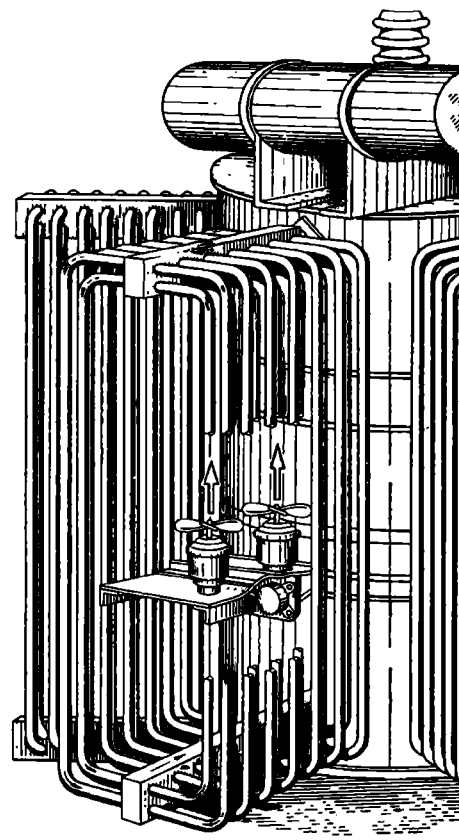


Fig. 212. Oil-natural air-blast transformer

90,000 kVA, the transformer measures 11.27 m in length, 7.61 m in width and 12 m in height and weighs 335 tons. Transformers of this type are installed at the step-down transformer substations on the 600-mile, 400-kV, three-phase Kuibyshev-Moscow transmission line. A bank of three such transformers can handle 270, MVA of power.

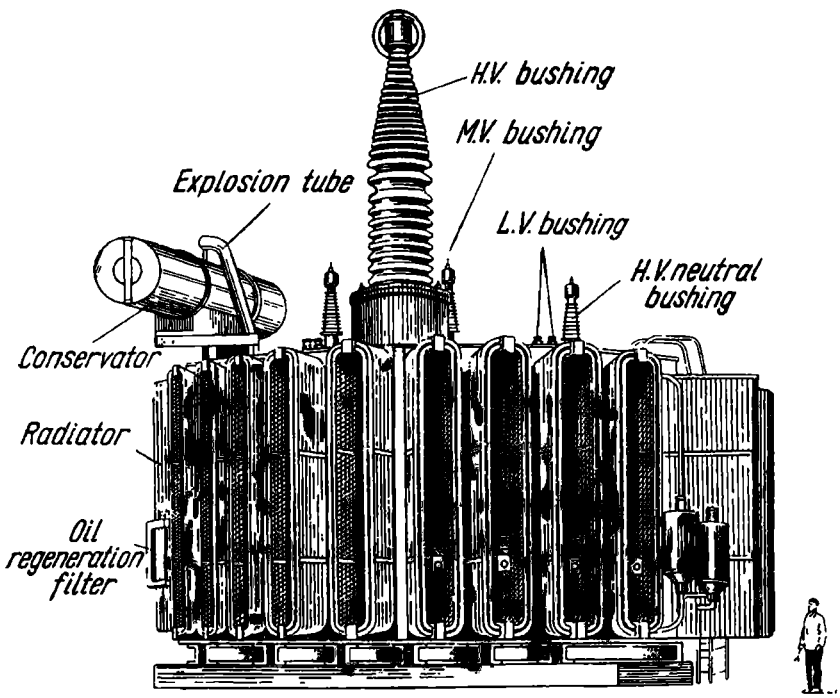


Fig. 213. Single-phase three-winding transformer

Soviet manufacturers also make 400-kV transformers for 50, 67, 83 and 90 MVA per phase.

The classification symbols of Soviet-made transformers are given below.

Code Designation of Soviet-made Power Transformers

О—single-phase;
Т—three-phase;
С—dry self-cooled (or air-natural);
М—oil-immersed self-cooled (or oil-natural);
Д—oil-immersed forced-air-cooled (or oil-natural air-blast);
Т—three-winding (one primary and two secondaries);
Г—lightning-proof;
У—sealed.

The numerals following the letter designation are arranged as a fraction, the numerator giving the kVA rating of the transformer and the denominator, its rated primary voltage in kV.

Examples.

TC-320/10: a three-phase, dry-air-cooled transformer rated for 320 kVA at 10 kV.
TM-5600/35: a three-phase oil-natural transformer rated for 5600 kVA at 35 kV.
ТМУ-50/6: a three-phase oil-immersed sealed transformer for 50 kVA at 6 kV.
ОДТГ-90,000/400: an oil-natural air-blast, single-phase, three-winding lightning-proof transformer for 90,000 kVA at 400 kV.

111. Three-phase Transformers

Three-phase transformation may well be carried out by single-phase transformers, as shown in Fig. 214. However, a three-phase transformer used for the same purpose requires one-third as much iron for the magnetic circuit as a single-phase transformer. Fig. 215 shows an oil-natural-cooled three-phase power transformer.

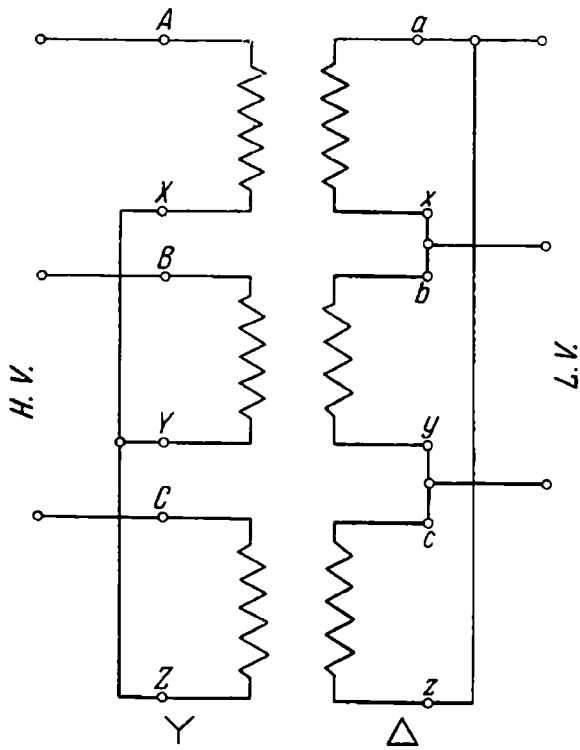


Fig. 214. Single-phase transformers in three-phase circuits

The windings of three-phase transformers can be connected as shown in Fig. 216. The symbol Y stands for "star connection"; Δ for "delta connection"; and Y_0 for "star connection with the neutral brought out". The capital letters *A*, *B* and *C* designate the beginnings of the H.V. coils, while the small letters *a*, *b*, and *c* designate the beginnings of the L.V. coils.

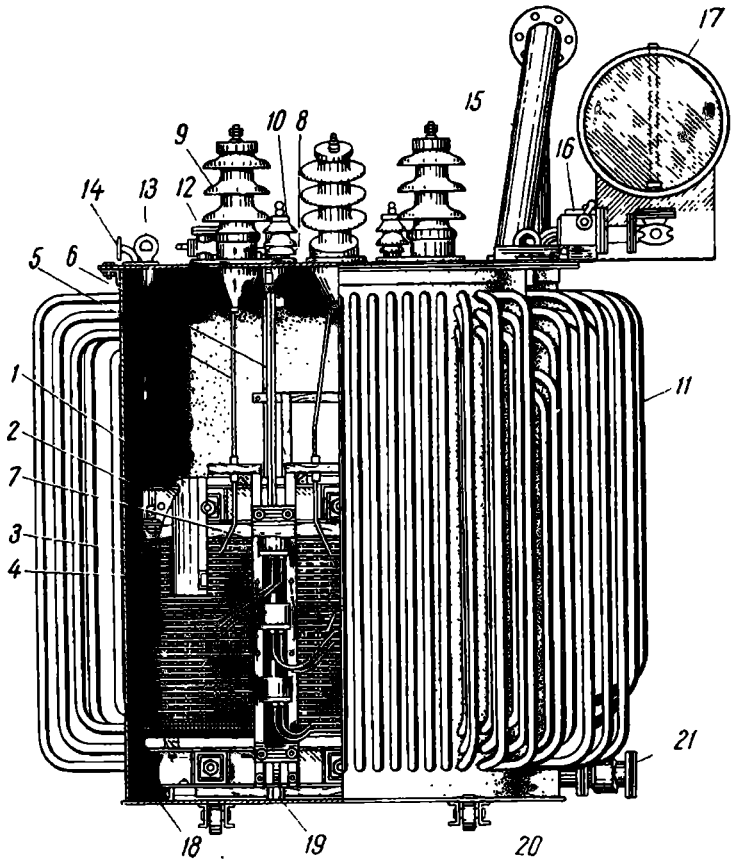


Fig. 215. Tubular-tank transformer:

1—laminated magnetic circuit; *2*—end girder; *3*—L.V. winding; *4*—H.V. winding; *5*—H.V. taps; *6*—L.V. taps; *7*—H.V. tap changer; *8*—tap-changer operating mechanism; *9*—H.V. bushing ; *10*—L.V. bushing; *11*—tubular tank; *12*—oil filler; *13*— lifting ring of the active core; *14*—vacuum-pump port; *15*—explosion tube; *16*—Buchholz relay; *17*—conservator; *18*—support angle; *19*—clamping stud of end structures; *20*—castors; *21*—oil drain

	Winding connections		Vector diagrams		Symbols
	H.V.	L.V.	H.V.	L.V.	
(a)	A B C X Y Z	o a b c x y z	B A C	b a c	Y/Y_0-12
(b)	A B C X Y Z	a b c x y z	B A C	b c a	$Y/\Delta-11$
(c)	0 A B C X Y Z	a b c x y z	B A C	b c a	$Y_0/\Delta-11$

Fig. 216. Winding connections of three-phase transformers:

a—star with the neutral brought out; *b*—star/delta; *c*—star with the neutral brought out/delta

The letters *X*, *Y*, *Z* and *x*, *y*, *z* designate the ends of the respective coils. Fig. 217 shows the terminal arrangement on the transformer cover.

Two groups of standard connections for three-phase transformers are shown in Fig. 218. In the designation "Y/Δ-11" the first symbol, Y, indicates that the H.V. winding is star-connected. The second symbol, Δ, shows that the L.V. winding is delta-connected. The numeral "11" gives the angular displacement (clockwise) between the vectors of the primary and secondary line voltages, with 30° taken as unity. For an angular displacement of 330°, it is $330 \div 30 = 11$.

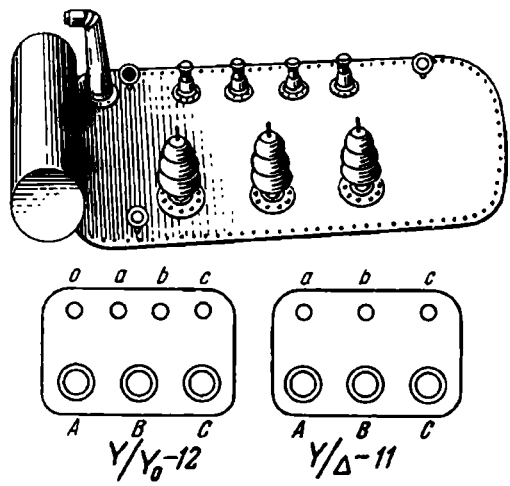


Fig. 217. Winding leads brought to the terminal panel of transformer (top, a general view of the terminal panel; bottom, terminal diagram)

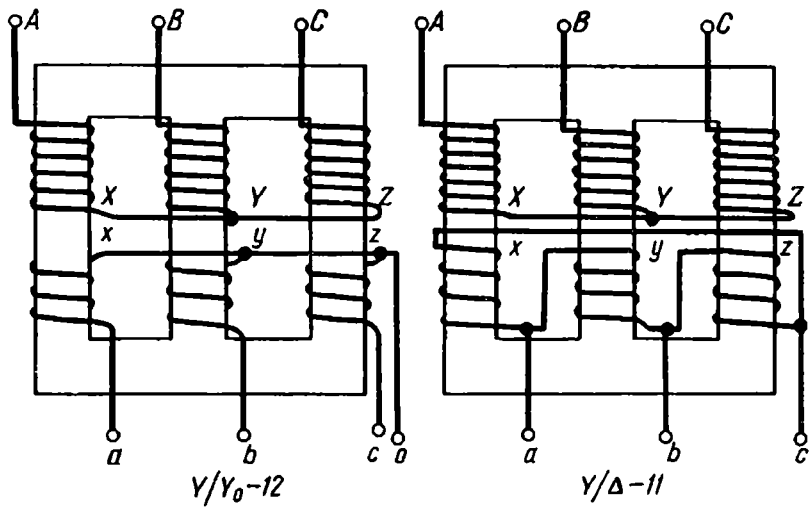


Fig. 218. Arrangement of windings in three-phase transformers

112. Parallel Operation of Transformers

Where a single transformer is not sufficient for handling all the power available, several units may be connected for parallel operation.

Parallel operation of single-phase transformers. For several single-phase transformers to operate in parallel, the following conditions must be met:

1. The transformers to be paralleled must have identical primary and secondary voltages. Then their transformation ratios will also be identical. This condition may be expressed thus:

$$V_{1I} = V_{1II};$$
$$V_{2I} = V_{2II};$$
$$k_I = k_{II}.$$

If the ratios of the transformers are not identical, their secondary emfs will be different, and different voltages will appear across the secondary terminals. The difference in emf E will give rise to a current flowing in the closed circuit of the secondary windings. This is called a *circulating current.* If the ratios differ markedly, a fairly large circulating current will flow in the transformer windings, and the parallel operation of the transformers may prove impossible.

However, it is sometimes necessary to parallel transformers with different ratios. Under the relevant U.S.S.R State Standard this difference must not exceed 0.5 per cent.

The point to be borne in mind when using in parallel transformers with different ratios is that the maximum load current should not exceed the rated load of the transformer with the lower ratio. This is because the current flowing in the transformer windings is equal to the vector sum of the load and circulating currents, and it is the transformer with the smaller ratio that will carry a bigger circulating current. To avoid overheating, the external load has to be reduced. This leaves the transformer with the greater ratio underloaded.

2. Transformers used in parallel must have identical impedance voltages. Calculation shows that in the case of identical ratios and power ratings but different impedance voltages, the load current (or apparent power) will be distributed between transformers in parallel in inverse proportion to the impedance voltage, i.e., the transformer with the larger impedance voltage will handle less power. In practice, the difference in the impedance voltage of the transformers to be paralleled should not exceed ± 10 per cent of the average, with power ratings differing by no more than three times.

3. Transformers in parallel have identical polarity. The polarity is basically the direction of the primary and secondary windings and the position of the transformer line leads relative to the beginning and end of the windings.

The latter requirement is checked as follows. A single-phase transformer (at *I* in Fig. 219) is connected in parallel with another single-phase transformer (at *II* in the same figure). Transformer *II* is connected on the H.V. side and one of its H.V. terminals is connected to any L.V. busbar. Then a voltmeter is connected between the other H.V. terminal of the incoming transformer and the other L.V. busbar. The voltmeter should be rated for twice the voltage on the L.V. side. If the voltmeter reads zero, the transformers may be connected in parallel.

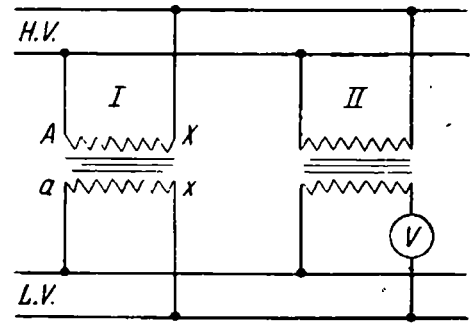


Fig. 219. Single-phase transformers connected for parallel operation

Parallel operation of three-phase transformers. In addition to the requirements listed for single-phase units, transformers for parallel operation must have identical angular displacement (or bear the same group reference numbers), and have their phase windings similarly connected. For example, a Y/Y_0-12 transformer may be connected in parallel only with a Y/Y_0-12 transformer.

113. Autotransformers

An autotransformer has only one coil, a certain portion of which is used for both the high-voltage and the low-voltage windings (Fig. 220). The emf E of an autotransformer is almost equal to the applied voltage V_1. Therefore, the emf per turn is E/T_1 or approximately V_1/T_1.

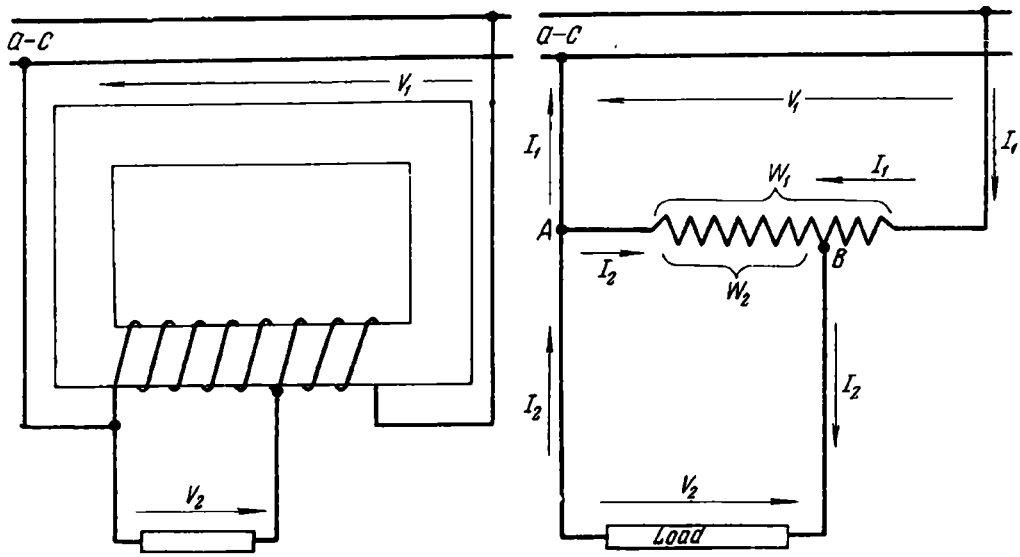


Fig. 220. Diagram of an autotransformer

Putting it another way, the voltage per turn is uniform throughout the winding. Suppose the applied voltage of the autotransformer shown in Fig. 220 is $V_1 = 800$ V. The total number of turns used as the primary (i.e., between points A and C) is 200. Then the emf or voltage per turn will be

$$V_1/T_1 = 800 \div 200 = 4 \text{ V}.$$

To obtain a secondary voltage of 600 V it is necessary merely to select any two points on the continuous winding such that $600 \div 4 = 150$ turns are included between them. If one point is at A (the beginning of the winding), the other point will be at B.

It may be noted that $V_1/V_2 = T_1/T_2$, or the voltage and turns ratios are the same as for an ordinary transformer, and the transformation ratio will be the same:

$$k = V_1/V_2.$$

Example 5. An autotransformer having 1200 turns is connected across a voltage of 500 V. What secondary voltage will be obtained if a tap is taken at the 900th turn?

$$V_1/V_2 = T_1/T_2;$$

$$V_2 = (V_1T_2)/T_1 = (500 \times 900) \div 1200 = 375 \text{ V}.$$

The applied voltage V_1 gives rise to a primary current I_1 (Fig. 220). The secondary voltage is V_2. When the secondary circuit is completed through a load, a secondary current I_2 will flow like any induced current, I_2 will tend to reduce I_1 (by Lenz's Law). Consequently, the current flowing between A and B will be the vector difference between I_1 and I_2. In approximate calculations, the vector difference is replaced by the arithmetic difference:

$$I = I_2 - I_1.$$

The efficiency of autotransformers is extremely high, and for practical purposes, the primary (input) power may be considered equal to the secondary (output) power.

Example 6. The voltage applied to an autotransformer is 400 V. The primary current is $I_1 = 30$ A. The secondary voltage is $V_2 = 300$ V. Calculate the secondary current I_2 (Fig. 220).

First determine the input (apparent) power P_1:

$$S_1 = V_1I_1 = 400 \times 30 = 12{,}000 \text{ VA} = 12 \text{ kVA}.$$

Since the losses in an autotransformer are negligible, the output (apparent) power may also be taken equal to 12 kVA. If the secondary circuit does not contain an inductance, the secondary current I_2 will be

$$I_2 = S_2/V_2 = 12{,}000 \div 300 = 40 \text{ A}.$$

The current flowing between A and B will be the difference between I_2 and I_1:

$$I = I_2 - I_1 = 40 - 30 = 10 \text{ A}.$$

This current implies that the portion of the winding between A and B may be wound with a finer wire than the remaining part. An ordinary transformer required to lower the voltage from 400 to 300 V would have two windings; i.e., it would take more copper and would cost more.

If, instead of 300 V, the secondary voltage is 350 V, the secondary current will be

$$I_2 = S_2/V_2 = 12{,}000 \div 350 = 34.3 \text{ A.}$$

The current flowing between A and B will then be still smaller:

$$I = I_2 - I_1 = 34.3 - 30 = 4.3 \text{ A}$$

and a still finer wire may be used for the winding between the two points. It follows that an autotransformer grows increasingly more advantageous, in comparison with an ordinary unit, as the secondary voltage decreases.

Let the secondary voltage be 200 V (instead of 300 V). The secondary voltage is now half the primary voltage. The secondary current will be

$$I_2 = S_2/V_2 = 12{,}000 \div 200 = 60 \text{ A.}$$

The current between A and B will be

$$I = I_2 - I_1 = 60 - 30 = 30 \text{ A,}$$

or equal to the primary current. Then the winding should be wound with wire of uniform cross-section throughout.

Now let the secondary voltage be 100 V. The secondary current will be

$$I_2 = S_2/V_2 = 12{,}000 \div 100 = 120 \text{ A.}$$

The current between A and B will be

$$I = I_2 - I_1 = 120 - 30 = 90 \text{ A.}$$

To carry such a current, the winding between A and B must be of heavy wire, and so it may prove more advantageous to use an ordinary transformer.

Example 7. An autotransformer with 1500 turns is connected to a source of 500 V. The secondary circuit has 1200 turns and is completed through a noninductive impedance of 100 ohms. Calculate the current taken by the autotransformer if the efficiency is 95 per cent. Determine the currents for portions A-B and B-C (Fig. 220).

From the formula $V_1/V_2 = T_1/T_2$, the secondary voltage V_2 is

$$V_2 = (V_1 T_2)/T_1 = 500 \times 1200 \div 1500 = 400 \text{ V.}$$

The secondary current will be

$$I_2 = V_2/R = 400 \div 100 = 4 \text{ A.}$$

At PF = 1, the output power will be

$$P_2 = V_2 I_2 = 400 \times 4 = 1600 \text{ watts.}$$

For the efficiency given, the input power will be

$$P_1 = P_2/\text{efficiency} = 1600 \div 0.95 = \text{approx. } 1685 \text{ W.}$$

The current taken by the autotransformer (the primary current) is

$$I_1 = P_1/V_1 = 1685 \div 500 = 3.37 \text{ A.}$$

The current between B and C is 3.37 A and between A and B

$$I = I_2 - I_1 = 4 - 3.37 = 0.63 \text{ A.}$$

In addition to step-down autotransformers, there are step-up units (Fig. 221). In the latter the secondary voltage exceeds the primary voltage and the current in the common portion is approximately $I = I_1 - I_2$.

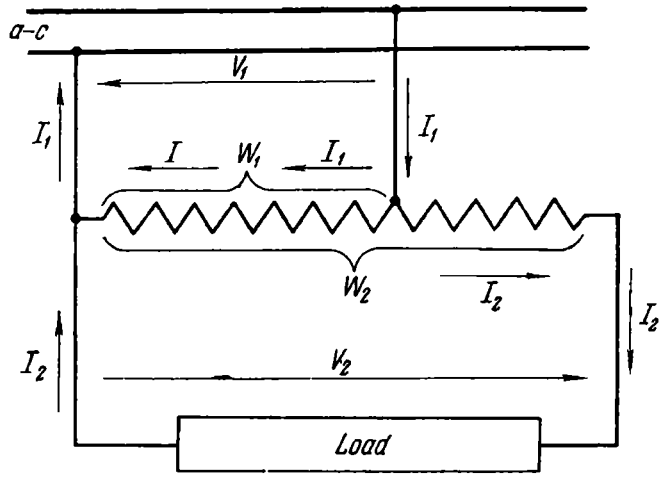


Fig. 221. Step-up autotransformer

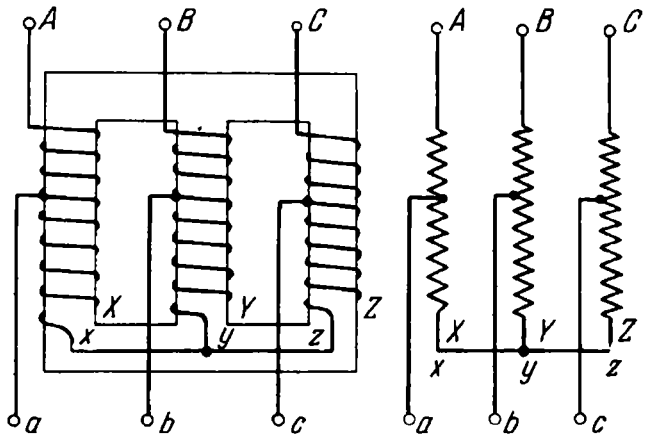


Fig. 222. Three-phase autotransformer: left, construction; right, circuit diagram

The transformation ratio of a step-up autotransformer is always less than unity:

$$k = V_1/V_2 < 1.$$

Three-phase systems use three-phase autotransformers (Fig. 222).

Autotransformers have advantages where the primary-to-secondary ratio is not large—close to unity (or, in any case, not greater than 2 : 1). Then they are cheaper to make, weigh less and are more economical to run than the ordinary transformer.

Exercises

1. A single-phase transformer is connected to receive a voltage of 220 V. The primary has 800 turns and the secondary 46 turns. Calculate the primary-to-secondary ratio and the secondary voltage.

2. The secondary of the transformer given in Ex. 1 supplies a current of 8 A for incandescent lamps. Calculate the current the transformer takes from the supply mains if its efficiency is 90 per cent.

3 A coil having a resistance of 3 ohms and an inductive reactance of 20 ohms is connected across 24-V taps of a transformer whose primary is connected to a 120-V mains. The efficiency of the transformer is 92 per cent. Calculate the current taken by the transformer from the supply circuit.

4. The output power of a three-phase transformer is 30 kVA. The efficiency of the transformer is 95 per cent and the applied voltage is 3000 V. Calculate the primary current.

5. The winding of a single-phase step-down autotransformer has 800 turns and is connected for a voltage of 450 V. Where should a tap be made to obtain a voltage of 300 V?

6. The effective power of the autotransformer given in the preceding exercise is 2 kW. The efficiency is 93 per cent. Calculate the primary current and the current flowing in the common portion of the winding.

7. An autotransformer has an efficiency of 96 per cent and is connected to receive 120 V. The current through its secondary circuit, which has a noninductive impedance, is 8 A at 150 V. Calculate the primary current and the current in the common portion of the winding.

Review Questions

1. What is the operating principle of the transformer?
2. How is a transformer constructed?
3. What are transformers used for?
4. What is the transformation ratio of a transformer and how is it determined?
5. How are transformers classed according to the construction of the magnetic circuit? According to the cooling system used?
6. How are transformer windings connected?
7. Why does an increase in transformer load bring about an increase in primary current?
8. Describe the construction, operation and purpose of autotransformers.

Chapter Ten

SYNCHRONOUS MACHINES

114. Synchronous Generators

The synchronous generator is the only type of a-c generator now in general use. It consists essentially of a member which supplies a magnetic field (the *field structure)* and a member in which an emf is induced (the *armature)*. Since it is immaterial for the generation of an induced emf whether a conductor moves across a magnetic field or vice versa, synchronous generators may be constructed with either the armature or the field structure as the revolving member.

Small generators are commonly made with a revolving armature. In this case the required magnetic field is produced by d-c electromagnets placed on the stationary member (the *stator)* and the current generated is collected by means of brushes and slip-rings on the revolving member.

Practically all large synchronous generators are made with a revolving field. There are several reasons for this. First, brushes and slip-rings cannot be safely used for collecting current from the revolving armature at the high voltages involved. Also, there must be three slip-rings for a revolving armature while only two are necessary for the low-voltage revolving field. At high power ratings, the three slip-rings for the revolving armature would be prohibitively large and heavy. An elementary diagram of a two-pole revolving-field synchronous generator is shown in Fig. 176.

The prime mover of a synchronous generator may be a steam turbine, a water wheel, an internal-combustion engine (most often a Diesel engine), an electric motor, etc.

115. Construction of the Synchronous Generator

1. The armature. The armature of a revolving-field synchronous generator consists of silicon-steel punchings assembled into an armature packet. The armature has slots which receive the armature winding. The whole structure is held in a frame which may be of cast iron or welded steel plate.

The slot insulation, which must be especially thorough on account of the high voltages involved, uses micanite plate or micanite tape. A general view of the armature and frame of a synchronous generator is shown in Fig. 223.

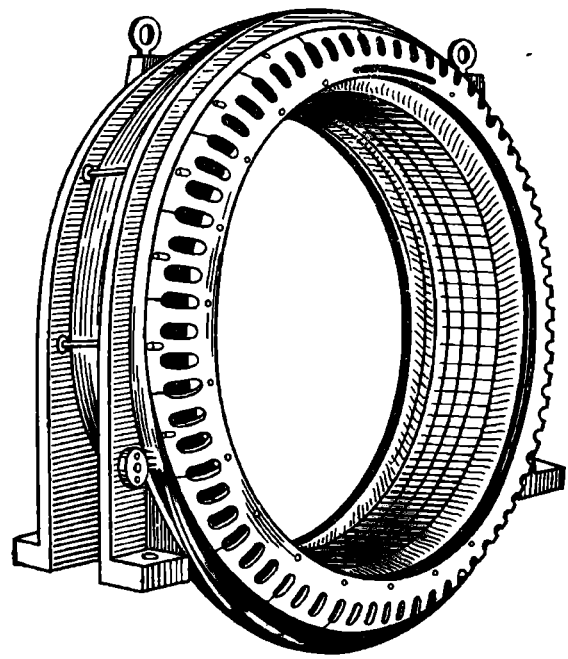


Fig. 223. Stator of a synchronous generator

2. The field structure. There are two general types of field construction: the *salient-pole type* and the *cylindrical* or *nonsalient-pole type*. Both types are shown in Fig. 224. The salient-pole type is used almost entirely for slow and moderate-speed generators, since this construction is the least expensive and permits ample space for the field ampere-turns.

It is not practicable to employ salient poles in high-speed generators because of the very high peripheral speed (100 to 170 m/sec) and the difficulty of obtaining sufficient mechanical strength.

The salient-pole type consists of a cylindrical steel forging whose rim carries the pole cores. The field coils are placed on the pole pieces and connected in series. The ends of the field winding are taken to two rings mounted on the shaft (or hub) of the field structure. The slip-rings carry brushes which are connected to a d-c source. A general view of a salient-pole field structure is shown in Fig. 225. The direct current required to excite the field coils is usually supplied by a d-c generator mounted on a common shaft (or directly coupled) with the field structure. This d-c generator is called the *exciter*. The power rating of the exciter is ordinarily 0.25 to 1 per cent of the power rating of the synchronous generator. The rated voltage of the exciter can be anywhere between 60 and 350 V. The excitation circuit of a synchronous machine is shown in Fig. 226.

There are also self-excited synchronous generators. The direct current required to excite the field structure is supplied by selenium rectifiers connected across the armature winding. Initially, the residual magnetism of

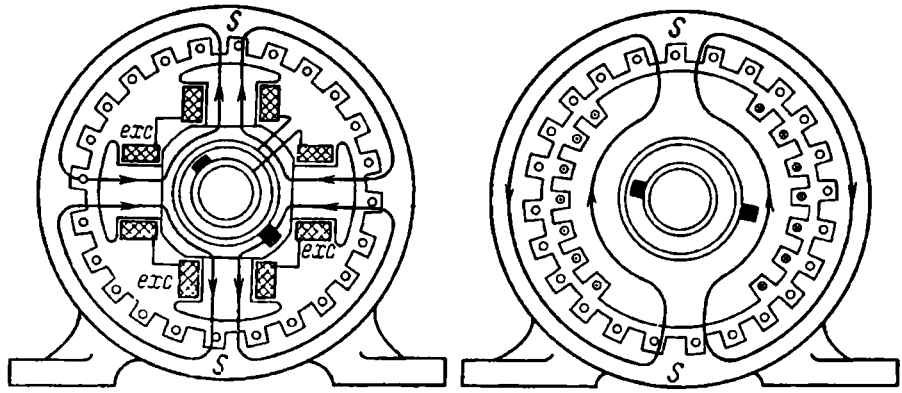


Fig. 224. Salient-pole and nonsalient-pole (cylindrical) field structures of synchronous generators

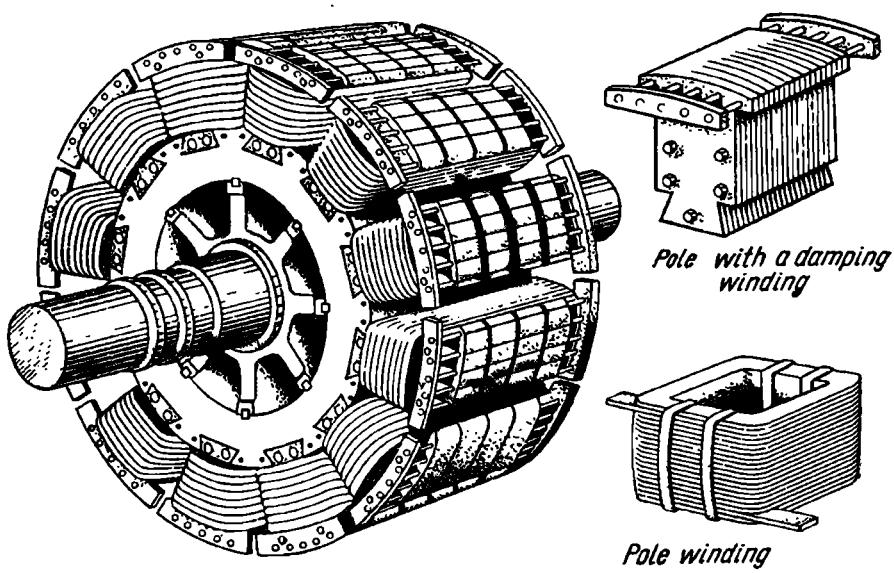


Fig. 225. Salient-pole field structure

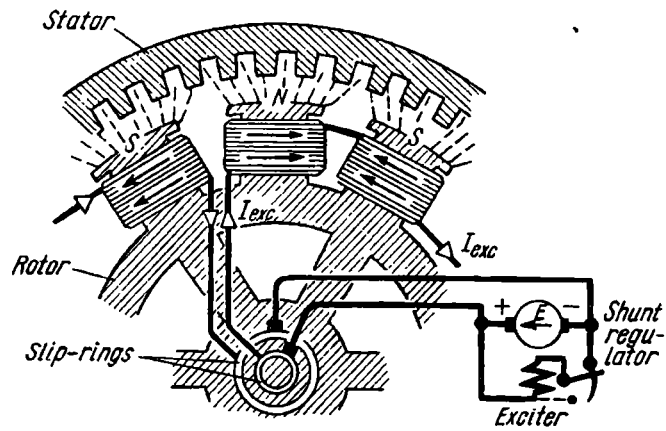


Fig. 226. Diagram showing excitation of a synchronous machine

the revolving field structure induces a small alternating emf in the armature. The selenium rectifiers turn it into a direct current, which builds up the field of the rotating member, and the voltage of the generator rises.

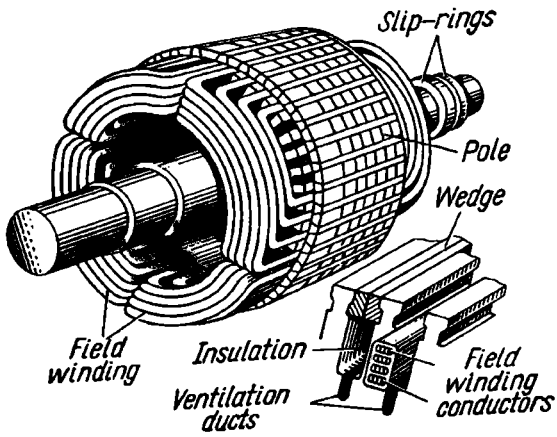


Fig. 227. Nonsalient-pole (cylindrical) field structure

The cylindrical type of field structure consists of a cylindrical steel forging which is suitably worked mechanically and treated thermally. By way of example, the field structure of a 100-MW, 3000-rpm turboalternator manufactured by the Elektrosila Works (U.S.S.R.) is 6.35 metres long and has an outside diameter of about a metre. The peripheral speed is 155 m/sec. The forging weighs 46.5 tons machined.

The forging has radial slots in which the field copper, usually in strip form, is placed. The coils are held in place by steel or bronze wedges and

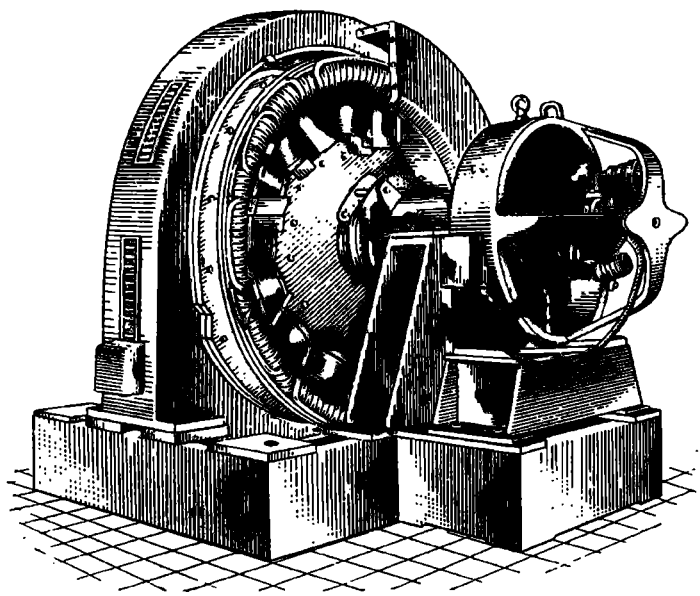


Fig. 228. Synchronous generator and exciter

the coil ends are fastened by metal rings. A general view of an assembled cylindrical field structure is shown in Fig. 227.

Synchronous generators may be cooled either by air or by hydrogen. In the former case, air is forced through the machine by fans mounted on both

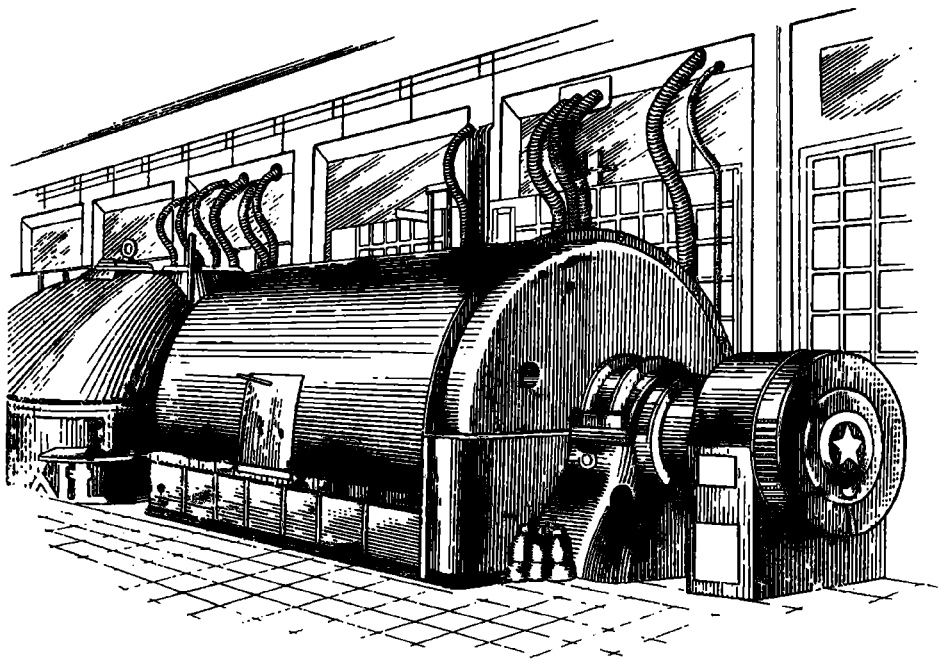


Fig. 229. 50-MW synchronous generator

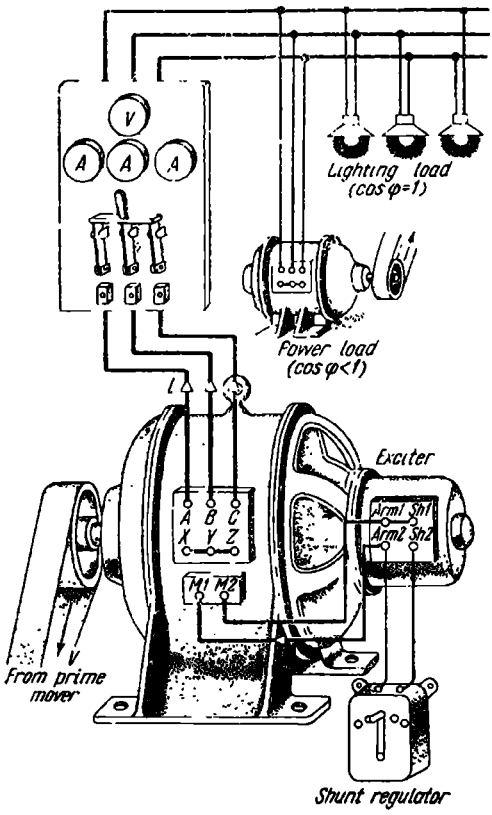


Fig. 230. Diagram of a synchronous generator and exciter

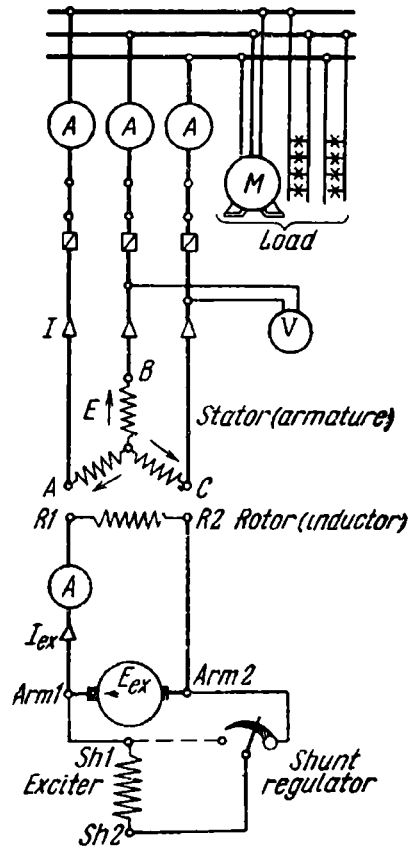


Fig. 231. Connection of a synchronous generator and load

ends of the shaft of the field structure (for machines rated at 1.5 to 50 MW) or placed in a pit beneath the machine (for machines rated up to 100 MW).

In the case of open-circuit ventilation, the incoming air is first passed through filters to remove any dust and is discharged into the atmosphere after it has moved round the machine. Where closed-circuit ventilation is employed, the air is repeatedly circulated through the machine. After the ventilating air has been heated by the various parts of the generator, it is cooled by passing it through a fin-tube cooler in which cooling water is circulated.

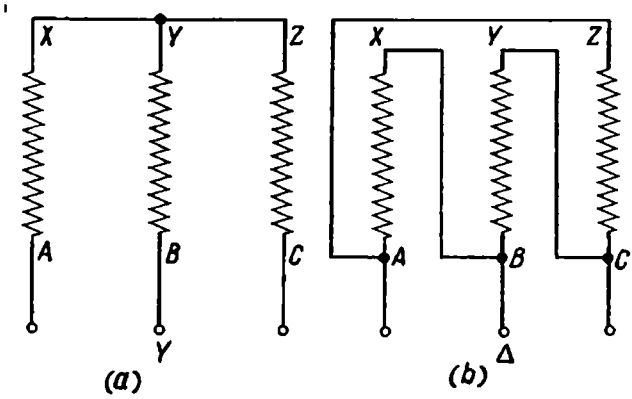


Fig. 232. Phase windings of a three-phase generator in star and in delta

To assist ventilation and cooling, ducts are provided in the stator laminations and other generator parts.

Hydrogen cooling is a far better method of generator ventilation. The thermal conductivity of hydrogen is 7.4 times that of air, and so more heat is abstracted. Also, hydrogen has a density 1/14 that of air; consequently, windage losses which account for half the total losses of a synchronous generator, are about one-tenth of their value in air. A further advantage of hydrogen as a cooling medium is that it does not oxidize the insulation and paintwork.

The armature winding of a three-phase synchronous generator has three coils (i.e., as many as there are phases) placed in the slots of the armature core and spaced 120 degrees apart. The six ends of the coils are taken to a terminal board where they can be connected either in a star (Fig. 232*a*) or in a delta (Fig. 232*b*).

116. Armature Reaction

When the armature circuit of a synchronous generator is completed through a load, an electric current traverses the armature winding, inducing a magnetic flux of its own. The armature mmf interacts with the main field to produce a resultant magnetic flux. This effect is called *armature*

reaction. There is also *armature leakage reactance.* In the subsequent discussion, armature leakage reactance will be neglected.

Armature reaction has an important bearing upon the operation of a synchronous generator. Three typical cases are discussed below.

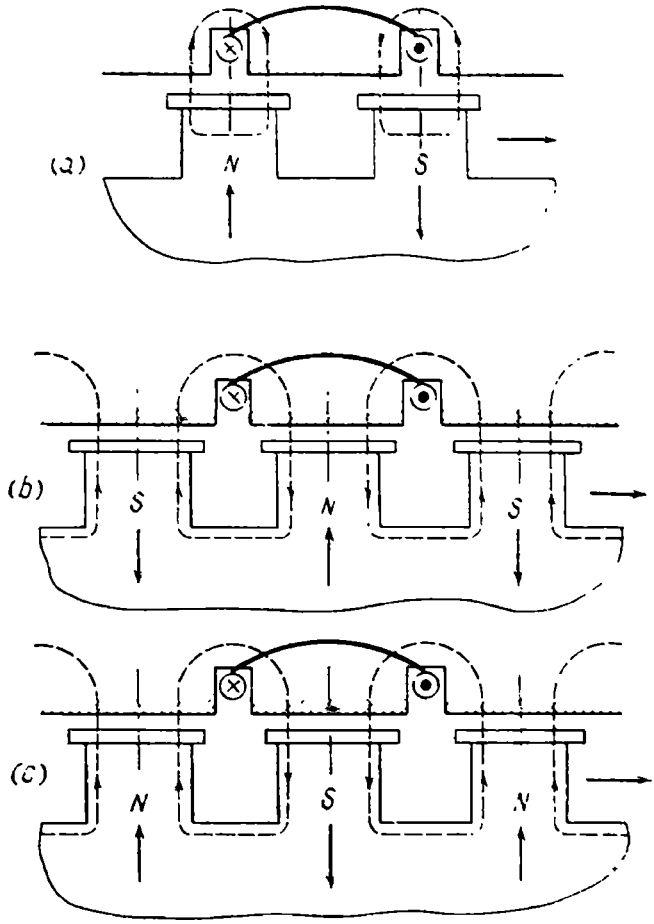


Fig. 233. Armature reaction in a synchronous generator

1. The load is resistive; therefore the armature current I is in phase with the armature emf. At time *a* (Fig. 233), the two sides of a phase coil are above the middle of the poles. Now the armature emf is at its highest, and so is the coil current. The direction of the lines of magnetic induction around the turns of the armature winding can be determined by the corkscrew rule. Referring to the diagram, the armature field strengthens the trailing pole tip and weakens the leading pole tip. This case is known as cross or quadrature-axis magnetization due to armature reaction.

2. The load is purely inductive; therefore the current lags 90° behind the emf. At time *b* (Fig. 233), the poles have moved a half pole pitch past the

coil sides. The armature mmf opposes the pole flux along their axis. This case is known as direct-axis demagnetization due to armature reaction.

3. The load of the generator is purely capacitive; therefore the current leads the emf by 90°. At time *c* (Fig. 233), the poles are within a half pole pitch of the respective coil sides, and the armature mmf is directed with the pole flux along their axis. This case is known as direct-axis magnetization due to armature reaction. At times *a*, *b* and *c* shown in Fig. 233, the largest current flows in the phase coil. At any other instant the current will be smaller, and the armature reaction of a given phase coil will be less noticeable. Ordinarily, both cross magnetization and one of the other components act simultaneously.

Thus, the armature reaction of a synchronous generator depends on the nature of the load or, which is the same, on the phase angle between the induced emf and the armature current.

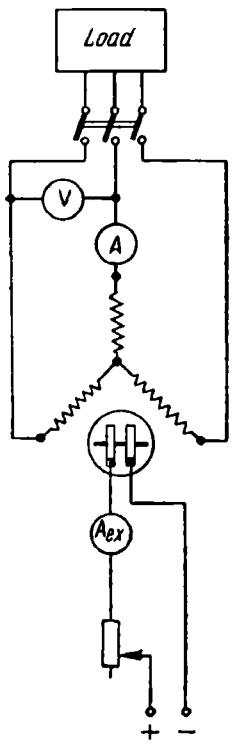


Fig. 234. Circuit for plotting the characteristics of a synchronous generator

117. Characteristics of Synchronous Generators

The performance of a synchronous generator under various operating conditions can be assessed on the basis of its characteristic curves. These curves are plotted by using the circuit shown in Fig. 234.

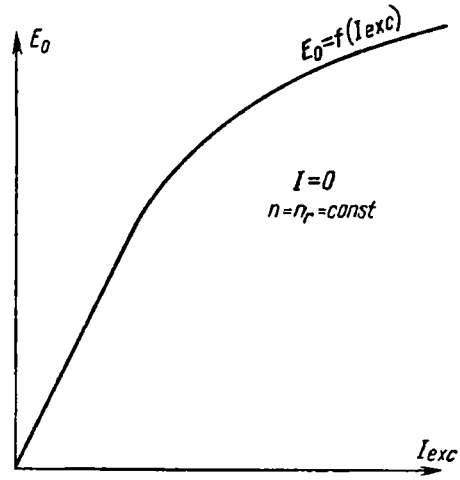


Fig. 235. No-load saturation characteristic of a synchronous generator

One of the characteristic curves plotted for a synchronous generator is that of its induced emf as a function of the excitation current at no-load (i.e., with the external circuit of the machine left open-circuited). This is

the no-load saturation characteristic:

$$E_0 = f(I_{ex}) \quad \text{at} \quad n = n_r \quad \text{and} \quad I = 0.$$

The condition $n = n_r$ indicates that the generator is running at synchronous speed*. The excitation current is gradually raised. An ammeter measures this current and a voltmeter connected across the armature winding measures the induced emf. The two readings are then plotted as shown in Fig. 235. The linear portion of the curve shows that initially the induced emf is directly proportional to the excitation current. Then the magnetic

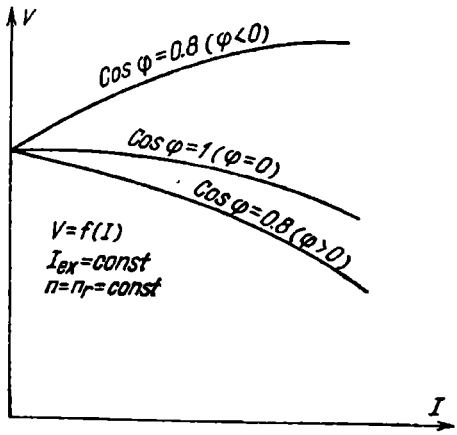


Fig. 236. External characteristics of a synchronous generator

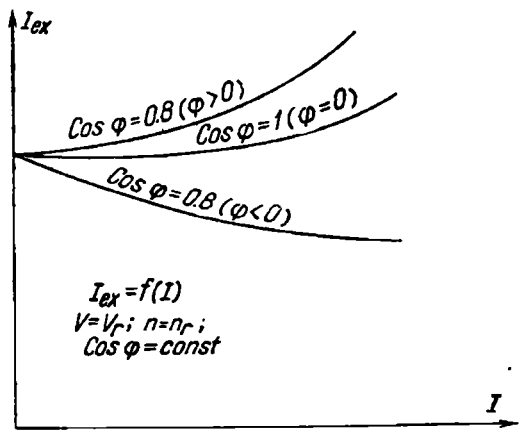


Fig. 237. Excitation characteristics of a synchronous generator

circuit gradually becomes saturated, and even a considerable increase in current gives only a negligible increase in emf. Normally, synchronous generators operate past the bend in the curve.

Another important characteristic curve is that of the terminal voltage of the generator as a function of the loadat constant excitation current, power factor and speed. This is the voltage-load (or external) characteristic:

$$V = f(I) \quad \text{at} \quad I_{ex} = \text{const}; \quad \text{PF} = \text{const};$$
$$n = n_r = \text{const}.$$

Readings are taken on the ammeter and voltmeter in the armature circuit and plotted as shown in Fig. 236. The changes in the voltage when the load is changed are due to the armature reaction and the voltage drop across the armature winding. In the case of an inductive load the reactive current demagnetizes the machine and the voltage decreases when the load

* Synchronous speed is the speed at which the magnetic field of a generator revolves. It depends on the number of poles and the frequency of the supply voltage: rpm = 60 f/p, where p = number of pole pairs.

is increased. When the load is capacitive, the generator voltage rises due to longitudinal magnetization.

A third important characteristic curve of a synchronous generator is the excitation characteristic, which relates the excitation current to the load current at rated terminal voltage and speed, and constant power factor:

$$I_{ex} = f(I) \quad \text{at} \quad V = V_r,\ n = n_r \quad \text{and} \quad \text{PF} = \text{const.}$$

The excitation curves shown in Fig. 237 show how the excitation current should be varied with load in order to make up for the voltage drop across the armature winding and the effect of the armature reaction.

118. Parallel Operation of Synchronous Generators

For two synchronous generators to be connected in parallel, they must:

(a) run at exactly the same frequency;
(b) run at exactly the same voltage;
(c) have the same phase position;
(d) have the same direction of phase.

Before the paralleling switch may be closed, the generators must therefore be "synchronized". This is done as follows. The prime mover of the incoming generator is started and brought up close to rated speed. The

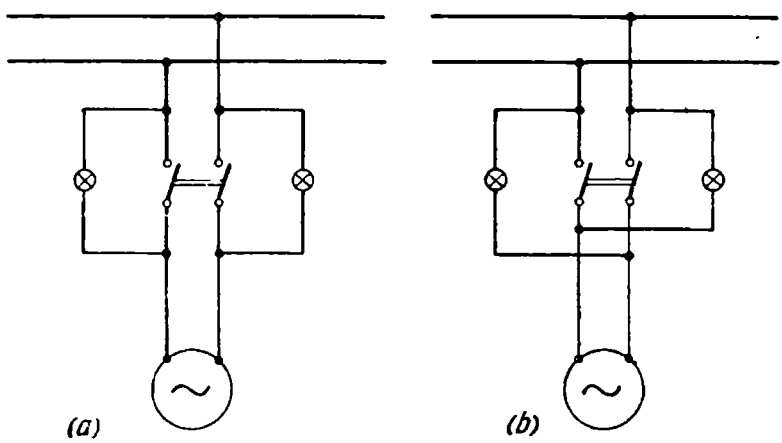


Fig. 238. Connection of synchronizing lamps:
a—synchronizing dark; *b*—synchronizing light

generator is then excited and its voltage raised to that of the running generator or the line. Next the rpm of the incoming generator is adjusted to the frequency of the line.

The third condition (known as phasing-out) is checked by means of synchronizing lamps which also provide a more accurate check on the difference in frequency, if there is any. The lamps may be connected for

either "*synchronizing dark*" or "*synchronizing light*". The connections for the dark-lamp method and the light-lamp method are shown at *a* and *b* in Fig. 238, respectively. When the lamps are dark (in the case of the connection *a*) or light (in the case of the connection *b*), the generators are in synchronism and may be thrown together by closing the paralleling switch.

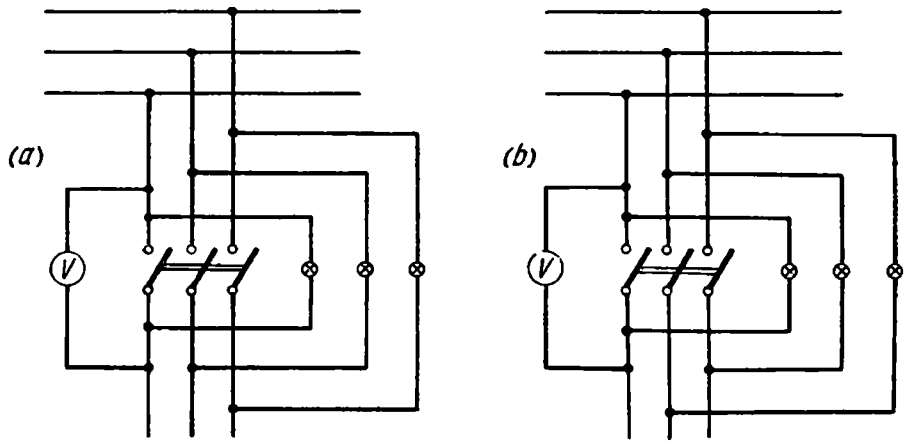


Fig. 239. Connection of synchronizing lamps for a three-phase generator:
a—synchronizing dark; *b*—synchronizing light

The same two methods of synchronizing, but with slight modifications, are also employed for three-phase synchronous generators. In the dark-lamp method (at *a* in Fig. 239), when the lamps are "dark", the machines are in phase, and the switch should be closed when the pulsation of lamp brightness is slowest or ceases altogether, i.e., at or just before the middle of the longest dark period.

In "synchronizing light" (at *b* in Fig. 239), the lamps are so arranged that they will flicker forming a rotating wheel of light when the machines are in synchronism. The synchronizing switch should be closed when two lamps are at full brilliance and the third lamp is dark.

If the machines are out of synchronism, the lamps will "rotate" in the "dark-lamp" method and become bright or dark together in the "synchronizing-light" method. In such a case, any two of the main leads on one side of the switch should be interchanged, leaving the lamps connected to the same switch terminals.

For more accurate synchronizing, use may also be made of a null-indicating centre-zero (or synchronizing) voltmeter. The lamps on high-voltage generators are connected via instrument voltage transformers (described in Chapter 14).

The difference in phase and frequency between two generators can also be indicated by a *synchroscope*, which is a small induction motor. The deflection of its vane indicates the direction of phase rotation.

In recent years, so-called self-synchronizing of generators has come into wide use. The method is basically as follows. The prime mover of the incoming generator is started, and the generator is brought up to approximately synchronous speed. The field winding is connected across a resistor which has 3 to 5 times the resistance of the field winding, and the switch is closed to put the incoming generator on the line. Then the field winding is reconnected from the resistor to the d-c source, and the generator pulls into synchronism by itself.

An idea of the performance of paralleled generators is given by what is known as the V-curve (it is V-shaped) shown in Fig. 240 which relates armature and field current. To plot it, an ammeter, wattmeter and power-factor meter are connected in the armature circuit and an ammeter in the excitation circuit. The generator under test is then brought into parallel operation and given a resistive load. The excitation current is gradually raised by means of a field rheostat, and simultaneous readings are taken on the instruments. It will be noted that the true power delivered by the generator into the line remains practically constant, and so do the wattmeter readings. If the resistive load is fixed, the armature current will be least at some value of the excitation current. This condition corresponds to the purely active load current (PF = 1). For every value of resistive load there is a definite excitation current at which the power factor is unity. Any increase in excitation current above this value will give rise to a lagging reactive current. The power-factor meter will read a decreased value, and the generator will deliver lagging reactive power into the line. Conversely, a decrease in excitation current below this value will give rise to a leading reactive current. The power factor will decrease again, and the generator will draw lagging reactive power from the line in order to supply its revolving field.

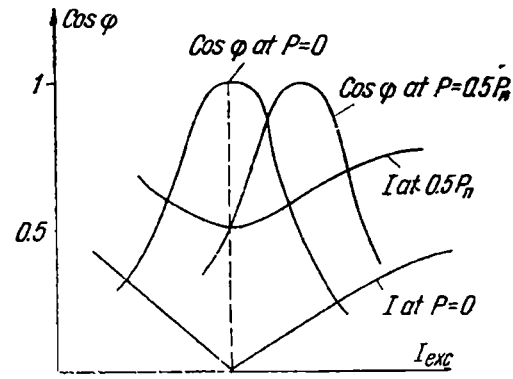


Fig. 240. V-curve of a synchronous generator and variation of power factor with excitation

119. Construction and Operation of the Synchronous Motor

A synchronous generator can be readily converted into a motor by interchanging its line and supply leads. Such a motor will be called *synchronous*. Three-phase synchronous motors are most common. The armature

winding is connected to a 3-phase a-c source and the winding of the revolving field structure is energized by direct current.

As in the synchronous generator, the speed of the revolving magnetic field set up by the armature depends on the frequency of the supply voltage and the number of poles for which the armature is wound. At 50 c/s and two poles the field will rotate at 3000 rpm, with four poles the speed will be 1500 rpm and with six poles, 1000 rpm.

The general design of a synchronous motor is the same as that of a synchronous generator with the addition of an auxiliary winding. The purpose of the auxiliary winding is to produce starting torque. This is necessary because the revolving field of the armature is unable to impart starting torque to a stationary rotor, which usually has considerable mass and inertia.

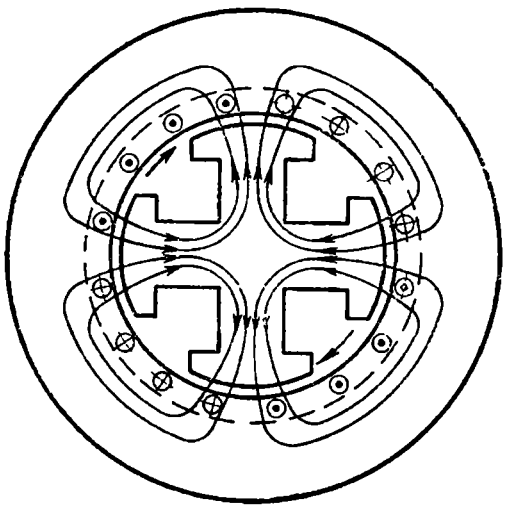


Fig. 241. Diagram of a reluctance motor

A very valuable property of synchronous motors is that they draw reactive current from the line at low excitation current (the power factor is lagging). Therefore, the excitation current can be adjusted so that no reactive current will be taken by the armature winding, and the power factor will be unity.

It is likewise possible to adjust the excitation current so that the armature winding will deliver reactive current into the line. Then the synchronous motor, while continuing to carry a mechanical load, will at the same time operate as a generator of reactive current or reactive power. In effect, it becomes a rotary capacitor and operates with a leading power factor.

It has been noted elsewhere that inductive loads (such as transformers, induction motors, etc.) require reactive power for the production of their magnetic fields. This reactive power (or reactive current) is usually drawn from a power station over wires. This arrangement can be dispensed with if such loads are linked with an overexcited synchronous motor. The latter will supply the required reactive power, thereby improving the power factor of the plant and reducing the current in the lines from the power station.

A synchronous motor intended to improve the power factor in the above way is called a *synchronous condenser.*

There is a large variety of special-purpose synchronous motors. One of them is the reluctance type widely employed in various physical instru-

ments, sound motion picture equipment, radio-phonographs, remote-control equipment, etc., where constant speed is essential. Reluctance motors are available in ratings from several tens to several hundreds of watts (Fig. 241).

The stator of a reluctance motor carries a winding energized by an alternating current (single- or three-phase). The revolving member is of the salient-pole type but has no field coils. The magnetic lines of the revolving magnetic field follow the path of least reluctance. Therefore, the rotor will tend to rotate so that the axes of its poles are aligned with the magnetic lines of the revolving field. Consequently, the rotor will rotate in synchronism with the stator field.

Small reluctance motors can be started by hand.

Compared with other a-c motors, reluctance motors have a low power factor and low efficiency.

120. Advantages and Disadvantages of Synchronous Motors

The advantages of synchronous motors are: (1) they run at constant speed irrespective of the load; (2) they have a high power factor; (3) the power factor is under the control of the operator; and (4) the electromagnetic power (and electromagnetic torque) varies linearly with voltage.

Some drawbacks of synchronous motors are: (1) a limited choice of speeds and impossibility of speed adjustment; (2) falling out of synchronism due to overloading; (3) a tendency to hunting.

For the above reasons, synchronous motors are mostly used to drive continuously operating and constant-speed equipment (piston and centrifugal pumps, centrifugal fans, compressors, motor-generator sets, power transmission, etc.).

Review Questions

1. What types of synchronous generator are there and how are they constructed?
2. How can the voltage of a synchronous generator be varied?
3. What are the effects of armature reaction in synchronous generators?
4. How should synchronous generators be connected for parallel operation?
5. Describe the construction and operating principle of the synchronous motor.
6. What are the advantages, disadvantages and uses of synchronous motors?

Chapter Eleven

INDUCTION MOTORS

121. General

The induction motor differs from other types of motor in that there is no electrical connection from the rotor winding to any source of electrical energy. The necessary voltage and current in the rotor circuit are produced by *induction* from the stator winding. Hence, its name—the induction motor.

Like other electrical machines, the induction motor can operate as both a motor and a generator.

The three-phase induction motor was invented by Dobrowolsky of Russia in 1890. Since then it has been improved and is in use in all countries.

Normally an induction motor consists of an *annular core* (the *stator*), which carries the primary winding in slots on its inner periphery. The primary winding is commonly arranged for three-phase power supply. The stator encircles a cylindrical rotor carrying the secondary winding in slots on its outer periphery.

Both the stator and rotor are assembled of silicon steel punchings 0.35 mm and 0.5 mm thick, respectively. The individual punchings are given a coat of varnish in order to reduce eddy-current losses. The air gap between the stator and rotor is made as small as practicable (0.3 to 0.35 mm in small machines and 1 to 1.5 mm in high-power machines).

The rotor winding may be one of two types: *squirrel cage* or *wound rotor* (*slip-ring*) in British terminology). In the case of a squirrel-cage machine, the rotor winding consists simply of copper or aluminium bars embedded in slots in the iron core of the rotor and connected at each end by means of copper or aluminium rings; the rotor winding thus forms a complete closed circuit in itself, resembling a squirrel cage. The rotor winding of the

wound-rotor (or slip-ring) machine is similar to the armature winding of the revolving armature type of a-c generator. The free ends of the winding are brought out to slip-rings. The rotor circuit is completed only when the slip-rings are connected either directly together or through some resistance external to the machine.

There are induction motors in which the relative positions of the primary and secondary windings are reversed relative to the ordinary induction motor. In addition to the primary winding, the rotor carries a second winding, which is connected to a commutator in the same manner as in a d-c motor. Hence the name of this type of induction motor—the *commutator type*. The two ends of each phase of the secondary winding (which is stationary) are connected to two brushes bearing upon the commutator. The position of the brushes can be varied at will, and so can the speed of the machine. In this type the speed can be varied within broad limits in both single-phase and three-phase operation. Commutator-type machines, however, are expensive to build, are complicated in construction and have to withstand adverse operating conditions. This is why they are seldom used and will not be considered in this book.

122. Theory of the Induction Motor

When the primary (stator) winding of an induction motor is connected to a three-phase power supply, a revolving magnetic field is produced within the stator. The field cuts across the secondary (rotor) winding and induces an emf in it. The frequency, f_2, of the induced emf E_2 is equal to the frequency, f_1, of the current in the stator.

The emf induced in one phase of the rotor can be determined from the formula

$$E_{2s} = 4.44 k_2 f_2 T_2 \Phi_m,$$

where $k_2 < 1$ = winding factor, or ratio of effective to total number of turns;
f_2 = frequency of the rotor current;
T_2 = number of turns in one phase of the rotor;
Φ_m = maximum magnetic flux in the stator.

The emf E_{2s} will establish in the rotor winding a current I_2, whose magnitude can be found from the formula

$$I_2 = E_{2s}/Z_{s2} = E_{2s}/\sqrt{R_2^2 + X_{s2}^2},$$

where R_2 = resistance of one phase of the rotor winding;
X_{s2} = leakage reactance of one phase of the rotor winding.

The leakage reactance X_{s2} varies with the frequency of the current and can be found from the equation

$$X_{s2} = 2\pi f_2 L_{s2},$$

where L_{s2} is the inductance due to leakage in a phase of the rotor.

When an induction motor is started up, the revolving field cuts the rotor winding at synchronous speed and induces in it line-frequency currents ($f_2 = f_1$), and so the secondary reactance X_{s2} is at its highest. Since the secondary emf E_2 at starting is at its peak, the secondary current I_2 is 2 to 7 times the rated current of the motor. The interaction of the secondary current and the revolving magnetic field of the stator will produce a force upon the rotor, tending to turn it in the same direction as that in which the magnetic field is revolving.

If the rotor winding were revolved at the same speed as the magnetic field, there would be no relative motion between the field and the rotor winding, and no current would be produced in the latter. Consequently, there would be no interaction between field and current and no torque produced, tending to keep the machine revolving. As the machine slows down, however, the magnetic field will revolve at a higher speed than the rotor, producing relative motion between the magnetic field and the rotor winding. This relative motion is responsible for the interaction of the field and current and the turning effort on the rotor. Thus, the operation of an induction motor depends upon a difference in speed between the revolving magnetic field and the actual speed of the machine or rotor. Since the speed at which the magnetic field revolves is called synchronous, another name for the induction motor is the *asynchronous machine.*

The difference in speed between the field and the rotor is called the *slip of the motor.* If the speed of the field is n_1 and that of the rotor n_2, the slip of the machine will be

$$s = n_1 - n_2 \text{ (rpm)}$$

$$\text{Per cent slip} = \frac{n_1 - n_2}{n_1} 100.$$

The per cent slip usually enters into motor calculations and is referred to simply as the slip.

From $s = (n_1 - n_2)/n_1$ it follows that

$$n_2 = n_1(1 - s).$$

If, for example, the speed of the field is 1500 rpm and that of the rotor 1450 rpm, the slip will be

$$s = \frac{n_1 - n_2}{n_1} 100 = \frac{1500 - 1450}{1500} 100 = 3.3 \text{ per cent.}$$

At starting, when the rotor speed is $n_2 = 0$, the slip is

$$s = (n_1/n_1) \times 100 = 100 \text{ per cent.}$$

At no load, n_2 is approximately equal to n_1, and so the slip is

$$s = \frac{n_1 - n_2}{n_1} \times 100 \approx 0 \text{ per cent.}$$

The slip of an induction motor varies but little with load and is usually smaller for high-power machines. It is 3 to 6 per cent for low-power units and 1 to 3 per cent for high-power units.

Example. Determine the slip of a six-pole induction motor if the speed of its rotor is 960 rpm.

$$n_1 = f_1 60/p.$$

If $f_1 = 50$ c/s

$$n_1 = (50 \times 60) \div 3 = 1000 \text{ rpm.}$$

The slip will be

$$s = \frac{1000-960}{1000} = 0.04 \text{ or } 4 \text{ per cent.}$$

Thus, as the speed of an induction motor varies from zero (at starting) to synchronous speed (no-load operation), the slip varies from 100 per cent to 0.

The frequency of both the emf and rotor current varies with the speed of the motor:

$$f_2 = \frac{p(n_1-n_2)}{60} = \frac{pn_1}{60} \times \frac{n_1-n_2}{n_1} = f_1 s.$$

At starting

$$s = 100 \text{ per cent}; \quad f_2 = f_1.$$

At no-load

$$s = 0 \text{ (approx.)}; \quad f_2 \approx 0.$$

If $f_1 = 50$ c/s, at starting f_2 will be 50 c/s. With $s = 2$ per cent, the frequency of the secondary current will be

$$f_2 = f_1 s = 50 \times 0.02 = 1 \text{ c/s.}$$

As has been noted, the emf induced in the rotor winding is

$$E_{2s} = 4.44 f_2 T_2 k_2 \Phi_m.$$

Since $f_2 = f_1 s$, then the emf E_{2s}, will at a certain slip be

$$E_{2s} = 4.44 f_1 s T_2 k_2 \Phi_m = E_2 s,$$

where E_2 is the emf of the rotor at starting (when $f_2 = f_1$).

For a given slip, the inductive reactance, which is frequency dependent, can be expressed thus:

$$X_{s2} = 2\pi f_2 L_{s2} = 2\pi f_1 s L_{s2},$$

where $2\pi f_1 s L_{s2}$ is the inductive reactance of the rotor winding at starting.

From the foregoing it may be concluded that the induction motor and the transformer have many features in common. Like the transformer, the induction motor has two windings—a primary and a secondary—magnetically coupled. At starting (or when the rotor is locked), the two windings are stationary and the frequency of their currents is the same. Changes in

the load of the transformer affect the state of its primary; a similar situation occurs in the induction motor. More specifically, an increase in load on the motor shaft increases the secondary current of the motor. Due to the magnetic coupling between the primary and secondary windings, the increase in the secondary current brings about an increase in the primary current. Consequently, an ammeter connected in the stator (primary) circuit will give a measure of the load upon the motor.

A major difference between the induction motor and the transformer lies in the construction of the magnetic circuit. In the induction motor it is separated (by an air gap) into two relatively movable portions; this is not the case in an ordinary transformer. The frequency of the secondary emf and current varies with load in the induction motor, but not in the transformer.

123. The Torque of the Induction Motor

The torque of an induction motor is greatly affected by the angular displacement between the current I_2 and the emf E_{2s} of the rotor winding.

Consider the case where the inductance of the rotor winding is small and the angular displacement may be neglected (at *a* in Fig. 242). Instead of

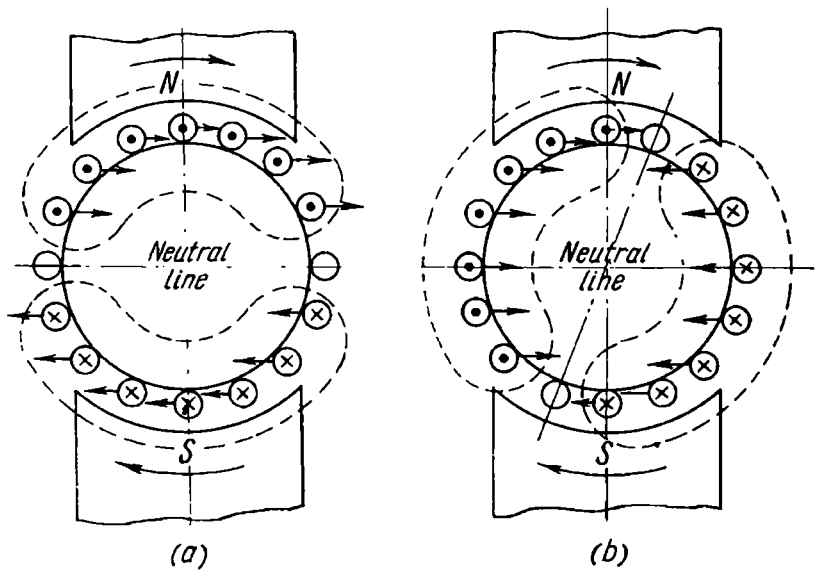


Fig. 242. The effect on the torque of the phase displacement between current and emf of the rotor

the revolving magnetic field of the stator, use is made of the field produced by the poles N and S revolving clockwise, say. The direction of the emfs and currents in the rotor winding will be given by the right-hand rule. The secondary currents will interact with the revolving magnetic field and

produce a torque. The direction of the forces acting upon a current-carrying conductor is given by the left-hand rule. Reference to the diagram shows that the torque will tend to turn the rotor in the direction of the field, i.e., clockwise.

Now consider the case where the inductance of the rotor winding is high, and so is the phase difference between the secondary current I_2 and the secondary emf E_{2s}. In Fig. 242*b* the field is shown revolving clockwise as before. The emf induced in the rotor winding has the same direction as in case *a*. Due to the lagging current, however, the axis of the magnetic field of the rotor is no longer aligned with the neutral line of the stator: it makes an angle with it opposing the rotation of the magnetic field, and will produce an additional torque opposing the main torque. Thus, the resultant torque in the case of a phase difference between the secondary current and emf is smaller than when they are in phase. It may be proved that the torque of an induction motor is supplied only by the active component of the secondary current and can be determined from the formula

$$M_r = c\Phi_m I_2 \cos \varphi_2,$$

where $I_2 = E_{2s}/Z_{s2}$;
$\cos \varphi_2 = R_2/Z_{s2}$;
Φ_m = stator magnetic flux (approximately equal to the resultant magnetic flux);
φ_2 = phase angle between the emf and current in a phase of the rotor winding;
c = constant.

After substitution

$$M_r = c\Phi_m(E_{2s}/Z_{s2})(R_2/Z_{s2}) = c_1\Phi_m(E_{2s}R_2/Z_{s2}^2).$$

Since

$$Z_{s2}^2 = R_2^2+(2\pi f_2 L_{s2})^2 = R_2^2+(2\pi s f_1 L_{s2})^2,$$

the formula of torque may be finally rewritten thus:

$$M_r = c_2 \frac{sR_2}{R_2^2+(2\pi f_1 s L_{s2})^2}.$$

This expression shows that the torque of an induction motor is slip-dependent.

Curve *A* in Fig. 243 relates the torque of an induction motor to its slip. As can be seen, at starting, when $s = 1$ and $n = 0$, the torque is negligible. It has been explained elsewhere that at this instant the current in the rotor winding is at line frequency, and the inductive reactance of the rotor winding is at its peak. Consequently, the value of $\cos \varphi_2$ is low (of the order of 0.1 or 0.2) and, despite the high starting current, the starting torque is small.

At s_1 the largest torque will be produced. With any further decrease in the slip or, in other words, any further increase in the speed of the motor, its torque will rapidly decrease to zero at $s = 0$.

It should be noted that in practice an induction motor cannot have zero slip. This condition may occur only when the rotor is turned in the direction of the field by an extraneous mechanical force.

The starting torque can be increased by reducing the phase difference between the secondary current and emf at starting. From $\tan \psi_2 = X_{s2}/R_2$, it follows that for a constant inductive reactance of the rotor winding this can be accomplished by increasing its resistance. Then both ψ_2 and $\tan \psi_2$ will decrease, while $\cos \psi_2$ and the torque of the motor will increase. This is why, in starting up an induction motor, it is customary to connect the rotor circuit across a resistor (a starting rheostat), which is taken out of the circuit when the motor has come up to speed.

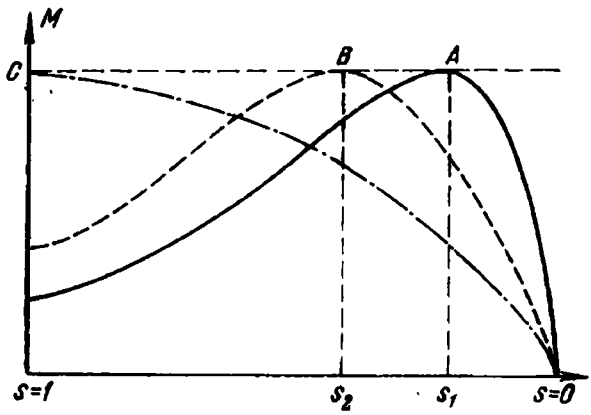


Fig. 243. Torque vs. slip

An increase in starting torque means that the maximum torque of the motor occurs at a greater slip (point s_2 on curve B in Fig. 243). Thus, the resistance of the rotor circuit can be increased to a point where the maximum torque will occur at starting ($s = 1$ on curve C).

The torque for any particular slip varies directly as the square of the voltage impressed on the stator. Therefore, even a negligible decrease in the applied voltage will sharply reduce the torque.

The power P_1 delivered to the stator winding of an induction motor is

$$P_1 = m_1 V_1 I_1 \cos \varphi_1,$$

where m_1 is the number of phases.

The power losses in the stator are:

(a) in the stator winding, $P_s = m_1 I_1^2 R_1$;

(b) the core losses (due to hysteresis and eddy currents), P_c.

The secondary input power is the power of the revolving magnetic field, also called the *electromagnetic power*, P_{em}.

The electromagnetic power is the difference between the primary input watts and the stator losses, or

$$P_{em} = P_1 - (P_c + P_s).$$

P_{em} may also be expressed thus:

$$P_{em} = \omega_1 M,$$

where ω_1 is the angular velocity of the revolving magnetic field, equal to $2\pi n_1/60$, and M is the torque developed by the magnetic field in joules.

The mechanical power developed by the rotor is

$$P_2' = \omega_2 M,$$

where ω_2 is the angular velocity of the rotor equal to $2\pi n_2/60$.

The difference between P_{em} and P'_2 is due to the electric losses in the rotor winding, P_r, if the losses in the rotor iron, which are very small, are neglected (the frequency at which the rotor is subjected to cyclic magnetization is usually very low):

$$P_r = P_{em} - P'_2 = m_2 R_2 I_2^2.$$

The slip of the motor is equal to

$$s = (\omega_1 - \omega_2)/\omega_1,$$

whence

$$s_1 \omega = \omega_1 - \omega_2.$$

P_r may also be expressed as follows:

$$P_r = P_{em} - P'_2 = M\omega_1 - M\omega_2 = M(\omega_1 - \omega_2) = M\omega_1 s = sP_{em}.$$

Thus, the losses in the rotor winding are proportional to the slip of the motor.

Subtracting the mechanical losses P_{mech} (which are due to friction in rotor bearings, windage, etc.) and also stray load losses P_{s1} (due to load, leakage reactance, field pulsation in the stator and rotor slots) from the mechanical power P'_2, will give the effective power output P_2 of the induction motor.

The efficiency of an induction motor can be found from the formula

$$\eta = P_2/P_1 = \frac{P_1 - (P_{mech} + P_c + P_s + P_{s1} + P_r)}{P_1}.$$

Both P_{em} and M may be expressed in still another way:

$$P_{em} = m_1 E'_2 I'_2 \cos \psi_2 = m_2 E_{2s} I_2 \cos \psi_2,$$

where E'_2 is the rotor emf referred to the stator emf and is equal to

$$E'_2 = E_2(T_1 k_1 / T_2 k_2),$$

where

T_1 = primary turns per phase;
T_2 = secondary turns per phase;
k_1 = winding factor of the stator;
k_2 = winding factor of the rotor;
I'_2 = secondary current in primary terms and equal to

$$I'_2 = I_2(m_2 T_2 k_2 / m_1 T_1 k_1);$$

ψ_2 = phase angle between the phase current and the emf in the rotor winding.

As has been noted

$$E_{2s} = 4.44 k_2 s f_1 T_2 \Phi_m.$$

Substituting the value of E_{2s} at $s = 1$ in the formula for P_{em} gives

$$P_{em} = 4.44 m_2 k_2 f_1 T_2 I_2 \Phi_m \cos \psi_2 = \omega_1 M,$$

whence

$$M = \frac{4.44 m_2 k_2 f_1 T_2}{\omega_1} \Phi_m I_2 \cos \psi_2.$$

If we let

$$c_2 = \frac{4.44 m_2 k_2 f_1 T_2}{\omega_1},$$

then we get

$$M = c_2 \Phi_m I_2 \cos \psi_2.$$

From the last expression it follows that the torque of an induction motor is proportional to the product of the revolving magnetic field, the secondary current and the cosine of the angle between the secondary emf and the current.

As follows from $P_r = M\omega_1 s$,

$$M = P_r/\omega_1 s = m_2 I_2^2 R_2/\omega_1 s = m_1 I_2'^2 R_2'/\omega_1 s.$$

Thus, the torque of an induction motor is proportional to the I^2R losses in the rotor winding.

The secondary current in primary terms can be obtained from an equivalent circuit by the formula (given without proof):

$$I_2' = \frac{V_1}{\sqrt{\left(R_1 + c_1 \frac{R_2'}{s}\right)^2 + (X_1 + c_1 X_2')^2}},$$

where

R_1 = resistance of the rotor winding;
X_1 = primary inductive reactance;
R_2' = secondary resistance in primary terms;
X_2' = secondary inductive reactance in primary terms;
c_1 ≈ coefficient (1.02-1.05).

Substituting the referred value of the secondary current in the latter formula, we get

$$M = \frac{m_1 V_1^2 R_2'}{s\omega_1 \left[\left(R_1 + c_1 \frac{R_2'}{s}\right)^2 + (X_1 + c_1 X_2')^2\right]}.$$

This equation shows that the torque of an induction motor is proportional to the square of the impressed voltage.

124. The Squirrel-cage Induction Motor

The squirrel-cage induction motor (Figs. 244 and 245) is one of the most widely used types.

It consists essentially of a stationary part or *stator*, *1*, (Fig. 245) carrying the primary winding, *2*, usually arranged for three-phase power supply. The starts of the three-phase coils are brought out to a terminal board, *3*, on the outside of the motor.

Since the stator winding carries an alternating current, an alternating magnetic flux will thread through the stator core. To reduce eddy currents in the stator it is assembled of silicon-steel punchings 0.35 and 0.5 mm thick. The punchings are insulated from one another by a coating of varnish and are held together by bolts in insulating sleeves. The stator winding is placed in slots on the inner periphery of the core. The core and winding are enclosed in a cast iron frame *3*.

The revolving part, or *rotor*, *4*, is also assembled of sheet steel stampings and carries the secondary winding in slots on its outer periphery. The secondary, or rotor, winding consists of copper bars solidly connected to copper end rings, *5*, on each end, thus forming a *squirrel cage* structure (at *a* in Fig. 246). In motors with ratings up to 100 kW, the squirrel cage structure is formed by aluminium cast (under pressure) into the slots of the rotor (at *b* in Fig. 246).

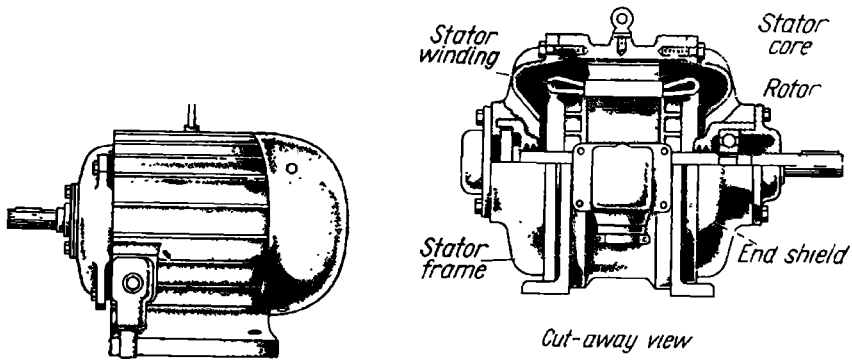


Fig. 244. General and cut-away views of a squirrel-cage induction motor

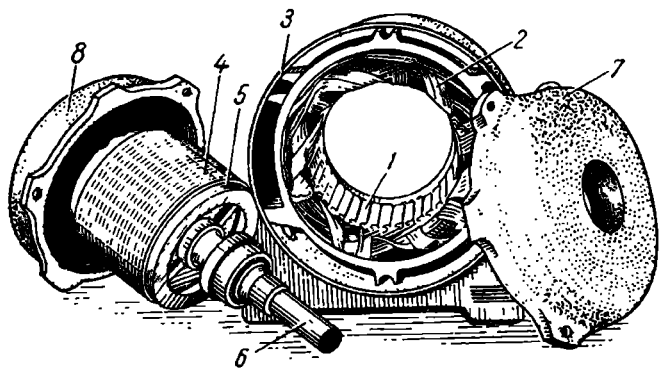


Fig. 245. Squirrel-cage induction motor taken apart

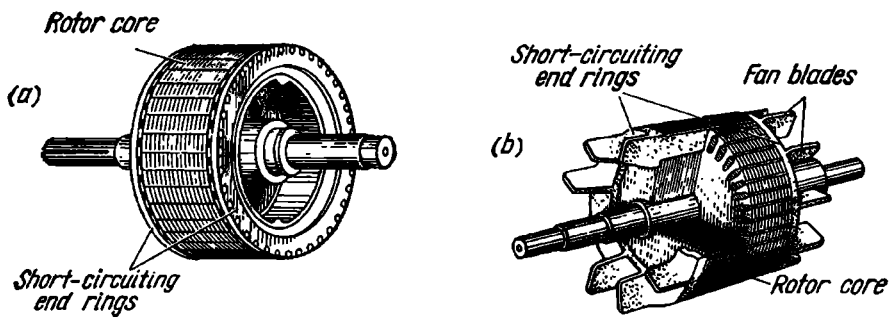


Fig. 246. General and cut-away views of a squirrel-cage rotor:

a—squirrel-cage structure of copper bars; *b*—aluminium-cast squirrel-cage structure

The shaft, *6*, of the rotor (Fig. 245) is carried by bearings fitted in the end shields *7* and *8*. The end shields are bolted to the motor frame. At one end of the shaft there is a pulley to transmit rotation via a V-belt. An induction motor may be started by connecting it across the line (full-voltage starting) by a knife switch or any other starting device. As has been explained, at starting, induction motors take 5 to 7 times their rated current. Because of this, there is a large voltage drop in the line which may

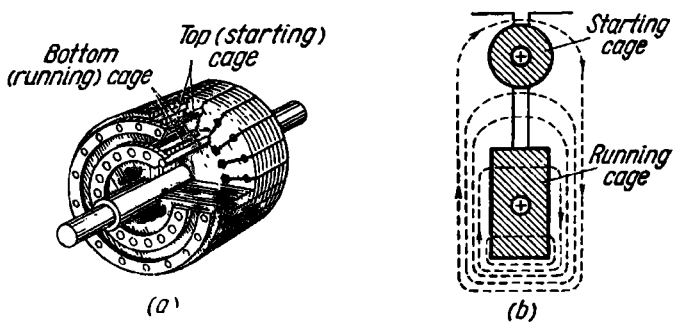


Fig. 247. Double squirrel-cage rotor:
***a*—cut-away view; *b*—section through a slot**

adversely affect the operation of other plant powered by the same line. On the other hand, the relatively low torque obtained when an ordinary squirrel-cage motor is started at reduced voltage seriously restricts its applications. Among the many devices developed for improving the starting performance, the simplest is the double squirrel-cage motor originally proposed by Dobrowolsky.

In a double squirrel-cage motor (Fig. 247), the rotor carries two squirrel-cage windings embedded in two rows of slots. The outer slots contain a high-resistance winding of low leakage reactance, and the inner slots contain a low-resistance winding of high leakage reactance under starting conditions.

The outer-winding bars are usually of manganese brass while the inner-winding bars are of red copper. The cross-sectional area of the inner winding is made greater than that of the outer structure so that the resistance of the outer slot winding is four or five times that of the inner-slot winding. The leakage reactance of the inner-slot winding is governed by the size of the neck between the two rows of slots (*b* in Fig. 247).

At starting, as in any induction motor, the rotor current has the same frequency as the supply current. Consequently, the leakage fluxes in the inner-slot winding are high, and so is its inductance. As a result, the phase difference between the emf and current in the inner winding will be great and the torque small. Owing to the high resistance and low leakage reactance of the outer winding, the phase difference between the current and

emf in it will be small and the torque due to this winding will be high. Thus, the starting torque is mainly supplied by the outer winding.

At normal speeds, the leakage reactance of the squirrel cages is of minor importance, and the current divides between them almost inversely as

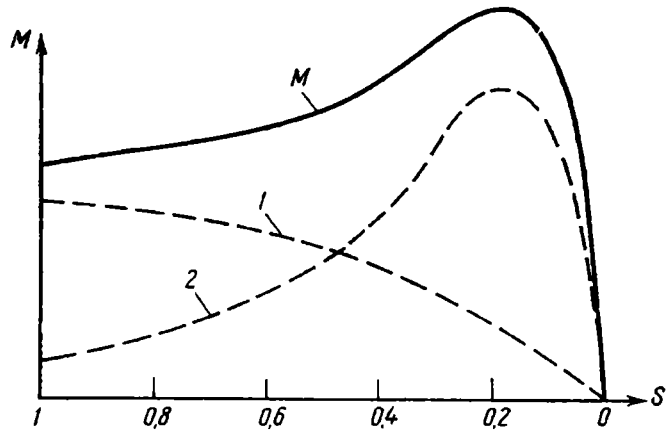


Fig. 248. Torque graph of a double squirrel-cage motor

their resistances. Since the resistance of the inner-slot winding is only a fraction of that of the outer winding, the bulk of the current flows through the former, and most of the torque comes from this winding.

Fig. 248 relates the torque of a double squirrel-cage motor to its slip. Curve *I* is for the outer winding and curve *II* for the inner winding. The sum of the instantaneous values of the two torques gives the total torque *M* of the motor.

Squirrel-cage rotors with pressure-cast aluminium conductors are simpler to make. One such rotor of double-squirrel-cage construction is shown in Fig. 249.

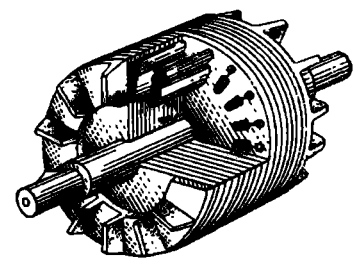


Fig. 249. Aluminium-cast double squirrel-cage rotor

The cost of a double squirrel-cage motor is 20 to 30 per cent higher than that of a single squirrel-cage unit. In the Soviet Union, double squirrel-cage motors are available in ratings from 5 to 2000 kW.

Another expedient serving to improve the starting characteristics of induction motors is the provision of deep-bar slots, such as shown in Fig. 250. In these the depth-to-width ratio is 10 or 12 to 1. At starting, the leakage reactance of the conductors at the bottom of the slots is greater because they are linked by greater leakage fluxes, and the current in the rotor winding is forced to flow in the top layers of the bars. The current density in the top layers increases, which amounts to a reduction in the

cross-section of the bars and an increase in their effective resistance. As a result, the torque increases and the starting current decreases. At normal speed, a deep-bar motor operates as an ordinary unit.

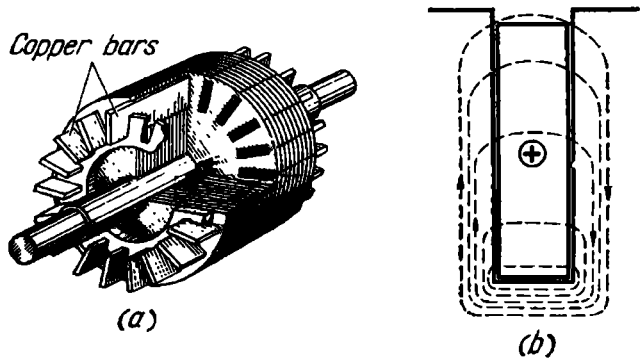


Fig. 250. Deep-bar rotor:
a—cut-away view; *b*—section through a slot

Table 17 summarizes the starting characteristics of induction motors with a single squirrel-cage, a double squirrel-cage and deep-bar slots, respectively. The starting characteristics are given as ratios of starting currents I_s to rated currents I_r and starting torques M_s to rated torques M_r.

Table 17

Starting Characteristics of Squirrel-cage Motors

Single squirrel-cage		Double squirrel-cage		Deep-bar	
I_s/I_r	M_s/M_r	I_s/I_r	M_s/M_r	I_s/I_r	M_s/M_r
4-7	0.8-1.2	3.3-5.5	1-2	4-4.8	1.2-1.5

125. The Slip-ring (Wound-rotor) Induction Motor

A general view of, and a section through, a slip-ring motor are shown in Fig. 251, while Fig. 252 shows a disassembled slip-ring motor.

The only difference in construction between the squirrel-cage and the slip-ring types lies in their rotors. The slip-ring rotor has a three-phase winding which may be either star or delta connected. The ends of the phases are brought out to three slip-rings on the rotor shaft and insulated both from each other and from the rotor core.

The starting current is reduced by a rheostat (Fig. 253) connected to the rotor winding by means of brushes bearing upon the slip-rings. At starting, the rheostat is brought fully into circuit (the position shown in Fig. 253).

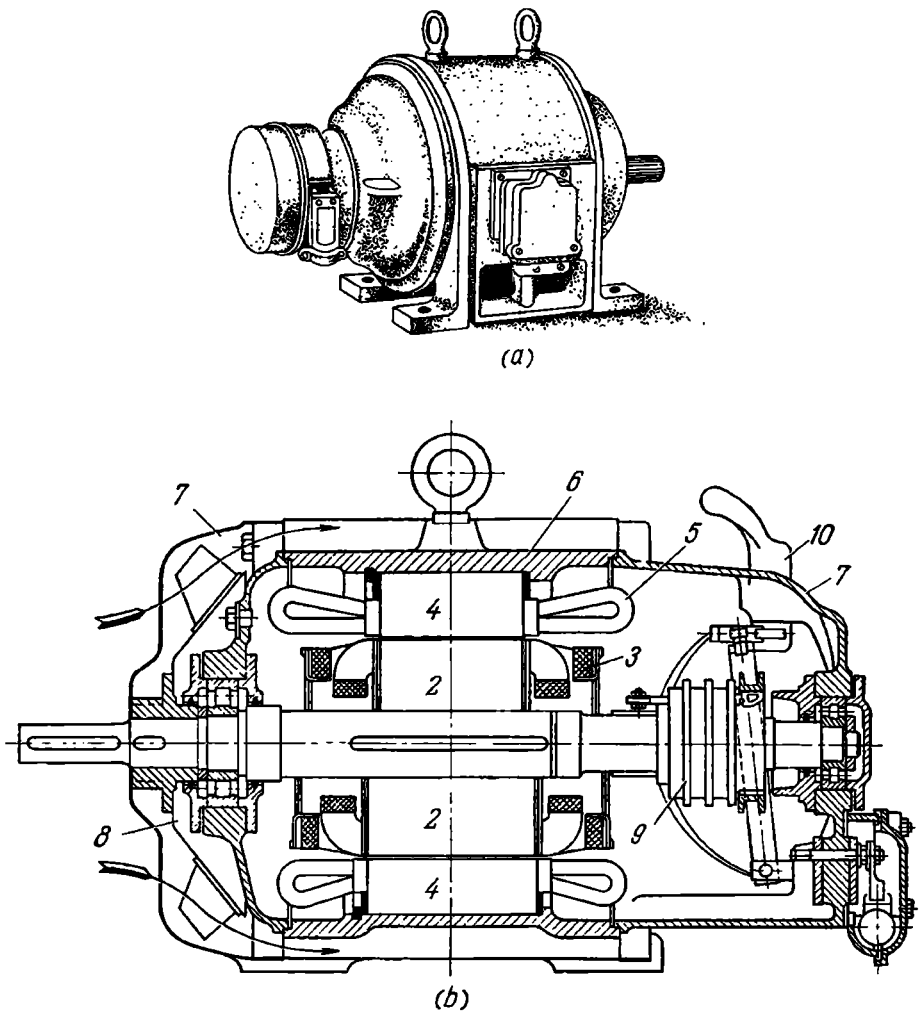


Fig. 251. Slip-ring (wound-rotor) induction motor:

a—general view; *b*—longitudinal section through a slip-ring motor; upper half for 1500 rpm; bottom half for 1000 rpm; *1*—shaft; *2*—rotor core; *3*—rotor winding; *4*—stator core; *5*—stator winding; *6*—frame; *7*—end shields; *8*—fan; *9*—slip-rings; *10*—brush-lifting handle

As the motor comes up to speed, the rheostat is gradually brought out of circuit until it is short-circuited at normal speed.

Some of the slip-ring motor designs have brush-lifting and ring-shorting devices.

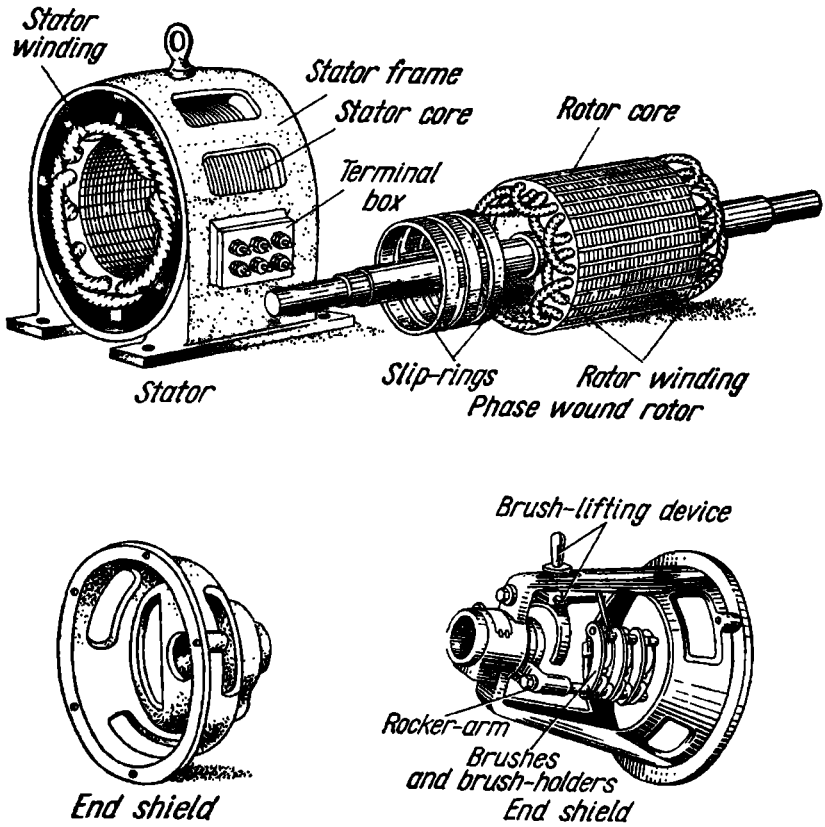


Fig. 252. Slip-ring (wound-rotor) motor taken apart

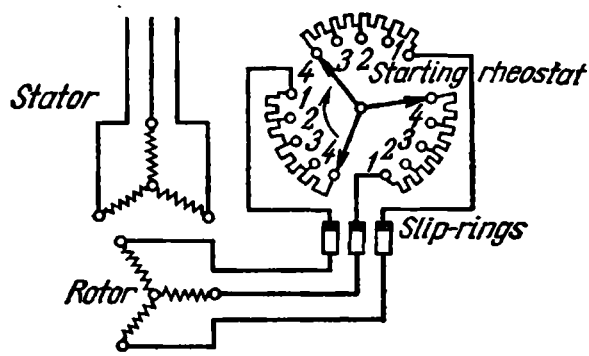


Fig. 253. Connection of a slip-ring motor and starting rheostat in a circuit

126. Advantages, Disadvantages and Applications of Induction Motors

The advantages of squirrel-cage motors are: (1) nearly constant speed at various loads; (2) high overload capacity; (3) simplicity and robustness of construction; (4) simplicity of starting, which may be made automatic; and (5) a higher power factor and efficiency in comparison with the slip-ring type.

Their disadvantages may be summed up thus: (1) their speed can hardly be regulated; (2) the starting current is very high; (3) the power factor is very low at underload; (4) they are very sensitive to fluctuations in supply voltage.

The advantages of slip-ring motors are: (1) high starting torque; (2) high overload capacity; (3) nearly constant speed at various loads; (4) low starting current in comparison with the squirrel-cage type; (5) adaptability to automatic starting.

The disadvantages are: (1) sensitivity to fluctuations in the supply voltage; (2) lower power factor and efficiency in comparison with the squirrel-cage type; and (3) low power factor at underload.

For the above reasons squirrel-cage motors should be employed where speed regulation is of minor importance and loads must be accelerated quickly. Where, on the other hand, loads must be started slowly or require a large starting torque, preference should be given to the slip-ring motor.

The overload capacity of a motor is the ratio of the maximum torque M_{max} to the rated torque M_r. Depending on the power rating and application of a motor, the ratio M_{max}/M_r ranges between 1 and 3.4.

127. Details of Stator and Rotor Windings in the Induction Motor

The three-phase stator winding which supplies a revolving magnetic field in the induction motor consists of a number of coils suitably connected. Each coil is wound with one or several turns of wire insulated from each other and from the core material. The winding insulation depends on the voltage and maximum safe temperature of the winding. Where a slot carries only one side of a coil, the winding is referred to as *single-layer*. Windings with two coil sides per slot are called *double-layer*. A coil may also consist of several sections, and the sections, of one or several turns. Fig. 254 shows a coil made of two sections, each section being wound with three turns. If z is the total number of stator slots and $2p$ is the number of poles, the number of slots per pole pitch will be

$$Q = z/2p.$$

The pole pitch, τ, is the circumferential distance between the centres of adjacent stator or rotor poles. Usually, there must be slots belonging to

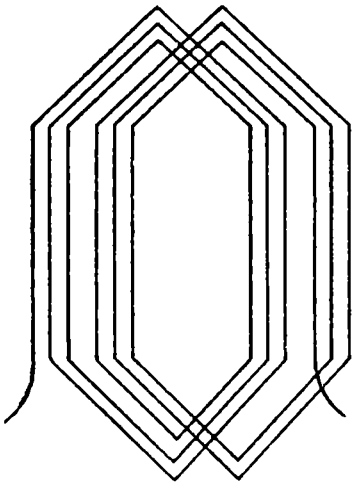


Fig. 254. Two-section coil

each of the three phases in one-pole pitch. Therefore, the number of slots per pole per phase is

$$q = Q/3 = z/3 \times 2p.$$

The number of slots spanned by a coil is called the *coil pitch, y*. Its numerical expression usually relates the number of slots spanned by a coil to that contained in one-pole pitch. For better utilization of the copper, two-layer windings are usually made short-pitched ($y < \tau$).

Fig. 255*a* shows the stator winding of an induction motor. In this winding each coil has two conductors. If many turns were thus wound, they would cover the stator from the inside. Instead, the conductors are arranged as shown in Fig. 255*b*.

The revolving magnetic field produced by a three-phase current completes one cycle in 360 electrical degrees. In two-pole machines, this amounts to one revolution of the rotor, which is equivalent to two pole pitches. At 50 c/s, the speed of a two-pole machine is 50 revolutions per second, or 3000 rpm. Figs. 255*c* and *d* show windings in which there are two conductors per coil side.

In a four-pole machine, the space of two-pole pitches occupies only half a revolution of the rotor. Consequently, the speed of a four-pole machine is half the speed of a two-pole unit, or 1500 rpm. A four-pole winding with

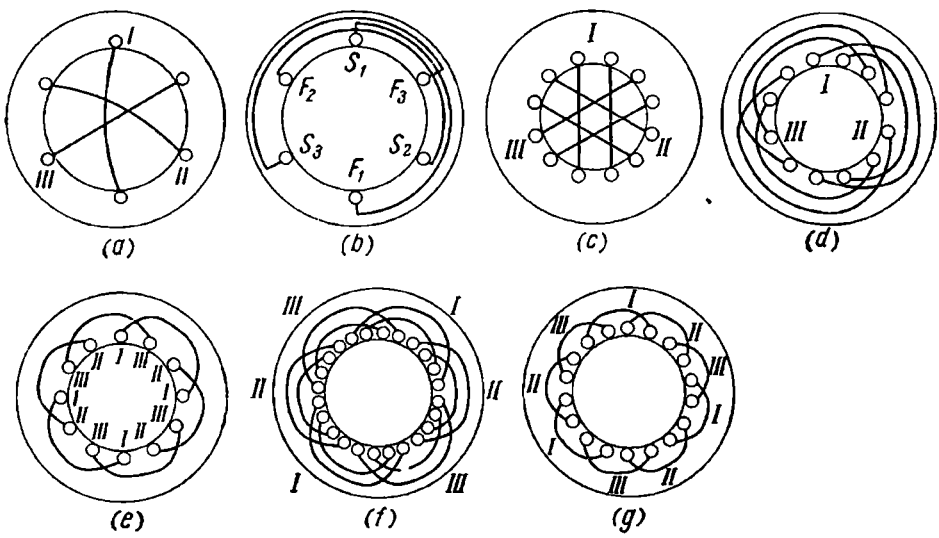


Fig. 255. Various types of stator winding of an induction motor

one conductor per pole per phase is shown in Fig. 255*e* and with two conductors per pole per phase in Fig. 255*f*. A six-pole machine has one-third of the speed of a two-pole unit, or 1000 rpm. A six-pole winding with one conductor per pole per phase is shown in Fig. 255*g*.

The number of slots in a stator is three times the product of the number of poles by the number of slots per pole per phase.

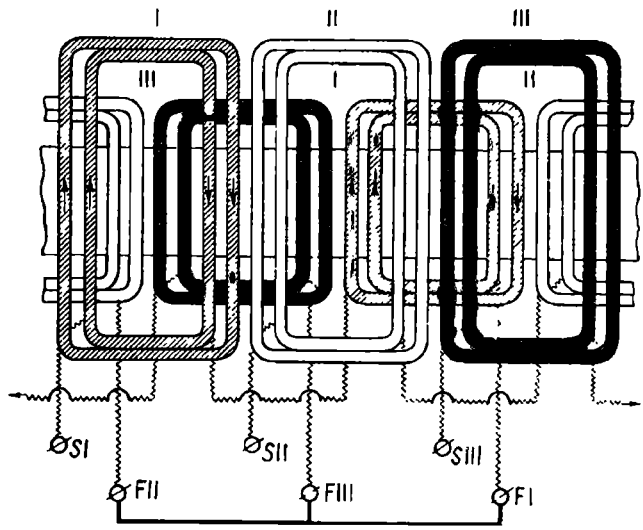


Fig. 256. Developed diagram of a three-phase single-layer winding

A developed diagram of a three-phase single-layer winding is shown in Fig. 256 and of a short-pitched two-layer lap winding in Fig. 257.

The six ends of a stator winding are brought out to a terminal panel.

128. Single-phase Induction Motors

As dictinct from three-phase induction units, the stator of a single-phase induction motor carries a single-phase winding (at *A* in Figs. 258 *a*, *b* and *c*). The rotor carries a three-phase winding either of the squirrel-cage or of the slip-ring type. As has been noted, a single-phase winding cannot produce a rotating field. Therefore, in single-phase induction motors the initial starting-up must be done by some other device. This usually takes the form of a starting winding on the stator, making an angle of 90° with the main winding (at *B* in Figs. 258 *a*, *b* and *c*). The two windings are connected to the same single-phase supply. The necessary phase angle between the currents of the two windings is obtained by connecting a resistance or a capacitance in series with the starting winding (at *a* and *b* in Fig. 258). The starting winding remains energized until the motor has come up to normal speed. Then it is disconnected from the supply line by a knife

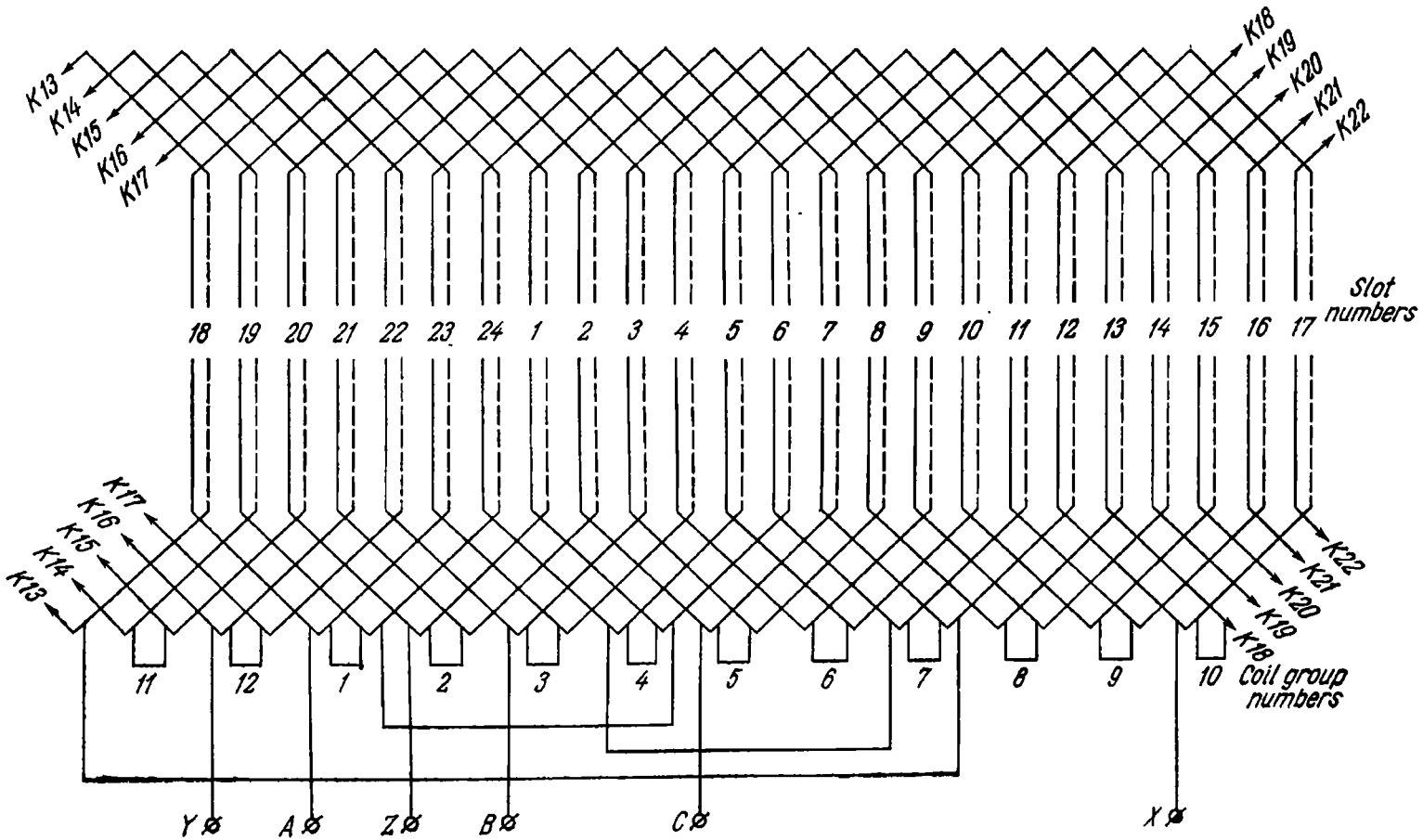


Fig. 257. Developed diagram of a short-pitch three-phase two-layer winding

switch (at *K* in Fig. 258), and the motor continues to operate on the main winding alone. This type of motor is called the *split-phase motor*, for the reason that the supply phase is split between the stator windings.

In low-power single-phase induction motors (0.5 to 30 watts) the starting coil consists of a short-circuited coil or turn. In such cases, the stator has salient poles, and the short-circuited turns (known as shading coils) embrace a portion of each pole face. The currents induced in the shading

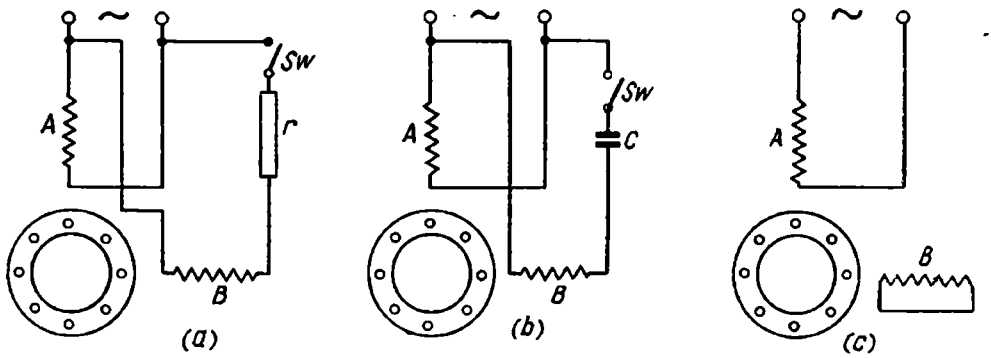


Fig. 258. Diagram showin ţthe starting of single-phase induction motors:

a—resistance in series with starting winding; *b*—capacitance in series with starting winding; *c*—starting-winding shorted turns in slot

coils by the main winding are out of phase with the main current, thereby producing the initial torque. This type of motor is called the *shaded-pole motor.*

As compared with three-phase units, single-phase induction motors suffer from the following shortcomings: (1) there is no starting torque; (2) overload capacity is low; (3) efficiency is low; (4) the power factor is reduced.

Review Questions

1. What is the operating principle of the induction motor?
2. What is the slip of a motor and how can it be determined?
3. What has the induction motor in common with the transformer?
4. What factors affect the torque of the induction motor?
5. Describe the construction of the squirrel-cage motor.
6. What are the main features of double squirrel-cage motors and deep-slot motors?
7. What are the constructional features of the slip-ring induction motor?
8. What is the purpose of the starting rheostat in slip-ring induction motors?
9. Describe the construction of the single-phase induction motor. How is it connected to the line?

Chapter Twelve

DIRECT-CURRENT MACHINES

129. The Direct-current Generator

A d-c generator is an electric machine which converts the mechanical energy of its prime mover into electric energy which is delivered to the consumer. A general view of a d-c generator is shown in Fig. 259; Fig. 260 gives a longitudinal and a cross-sectional view of a d-c generator.

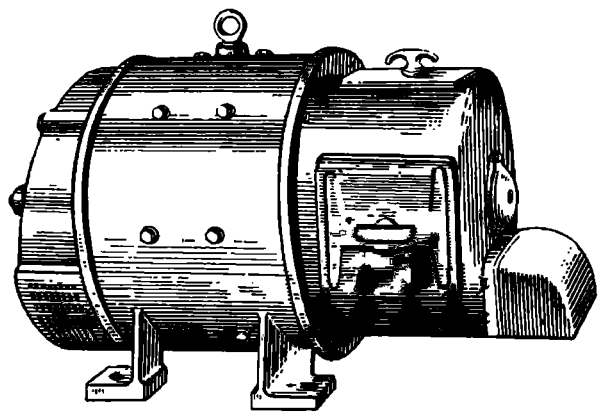


Fig. 259. D-c generator

For its operation, a d-c generator depends on electromagnetic induction. It consists essentially of a rotating armature carrying a winding, and a stationary field structure with electromagnets which produce the magnetic field.

The armature core is a cylinder assembled of separate 0.5-mm-thick silicon-steel punchings, which are insulated from one another by a coat-

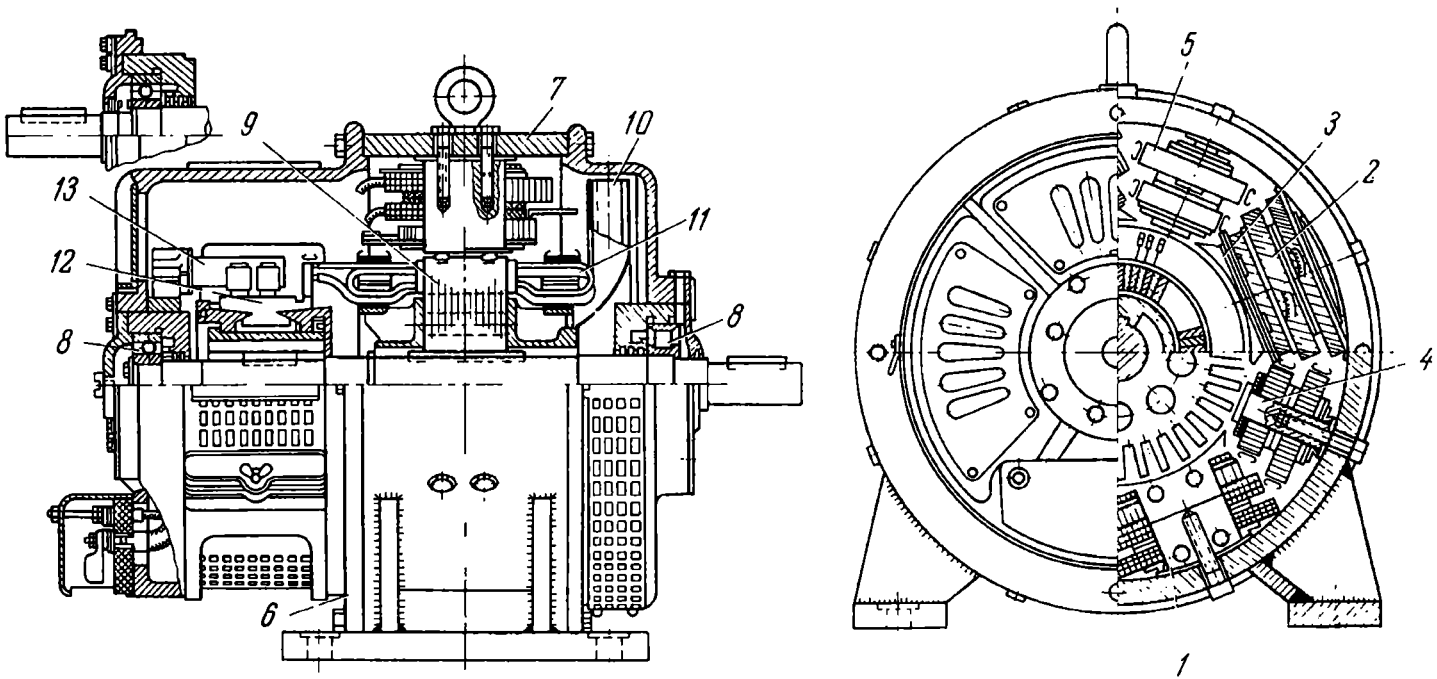


Fig. 260. Longitudinal section and cross-section through a d-c generator:

1—main-pole core; *2*—main-pole coil; *3*—pole shoe or face; *4*—intermediate (commutating) pole core; *5*—intermediate (commutating) pole winding; *6*—frame; *7*—yoke; *8*—end shield; *9*—armature core; *10*—fan; *11*—armature winding; *12*—commutator; *13*—brush stud

ing of varnish or thin paper. When the core is assembled, the notches on the punchings form slots which receive the armature winding.

The armature shaft carries a commutator consisting of separate copper segments each soldered to a definite portion of the armature winding. The commutator segments, or bars, are insulated from one another by mica or micanite. The purpose of the commutator will be discussed later.

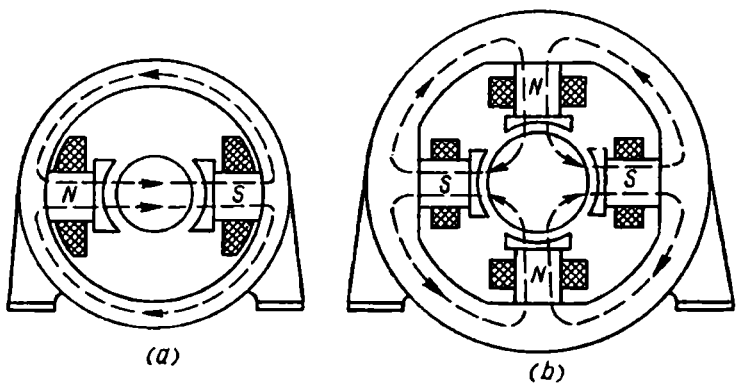


Fig. 261. Frame of a d-c machine:
***a*—two-pole; *b*—four-pole**

The poles of the electromagnets are of steel and are bolted to the yoke of the machine. The yoke is usually cast of steel. In small machines, the yoke and poles are cast integral. In the larger machines, the poles are assembled of silicon-steel punchings. The poles carry field coils wound with insulated copper wire. The current traversing the field coils sets up a magnetic field around the poles. For better distribution of the magnetic flux, the poles have pole shoes or faces made of separate steel sheets. In most cases, poles and pole shoes are pressed or stamped integral. Fig. 261 gives a diagrammatic representation of a yoke with two and four poles.

A d-c machine taken apart is shown in Fig. 262.

The armature circuit is connected to the load circuit by means of brushes. The brushes are carried in brush-holders which are mounted on brush-holder studs or brackets. In turn, the brush-holder studs are mounted on a brush yoke or rocker arm. The brush-holder studs are insulated from the brush yoke by means of insulating sleeves and discs. The brush yoke, brush-holders and brushes make up the brush-gear, which is shown in Fig. 263.

When the armature rotates, its winding cuts across the magnetic field. Due to electromagnetic induction, the emf induced in the armature winding will be

$$e = Blu \sin \alpha,$$

where B = magnetic induction of the poles $\left(\frac{\text{V} \cdot \text{sec}}{\text{m}^2}\right)$;
l = effective length of the armature conductors (m);
u = linear velocity of the armature conductors (m/sec);
α = angle between the direction of conductor velocity and the vector of the magnetic induction.

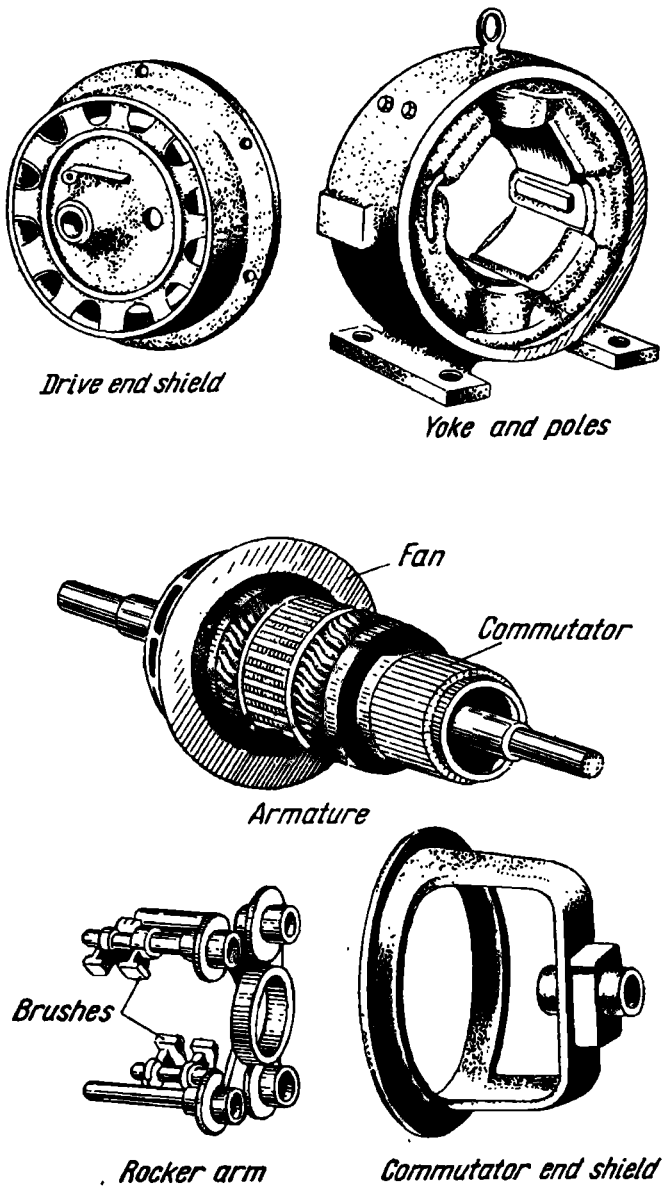


Fig. 262. D-c machine taken apart

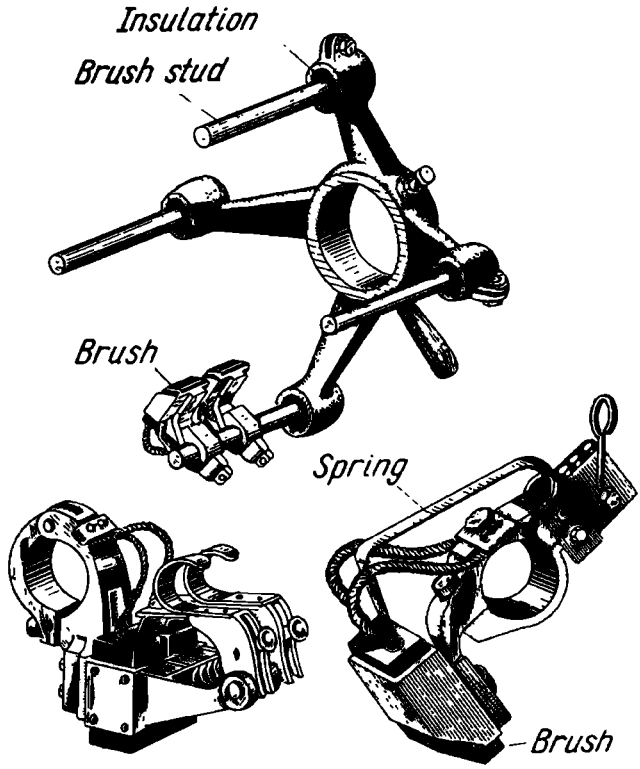


Fig. 263. Brush yoke or rocker arm and brush-holders

130. The Purpose and Construction of the Commutator in Direct-current Generators

The production of voltage in the armature winding of a d-c generator is exactly the same as in a-c generators. As the armature winding rotates, an alternating current is produced in it. So that the terminal voltage can always act upon the external load in the same direction, some device must be inserted between the armature winding and the terminals. This device must reverse the connection of the winding conductors to the external circuit at the instant when the voltage of the conductors is zero and changing in direction. Such a device is called a *commutator.*

An elementary form of commutator is shown in Fig. 265 where the coil leads are soldered to two copper half-rings called commutator segments. The way a d-c commutator operates is shown in Fig. 266. It shows a single-turn ring winding. Its start S is soldered to a commutator segment a while its finish F is soldered to another segment b. Bearing upon the commutator segments are two fixed brushes connected to the external circuit. By the right-hand rule the direction of current in the armature at a (Fig. 266) is from the start to the finish of the coil. It will enter the external circuit at the right-hand brush to flow around the circuit and return to the armature

at the left-hand brush. Therefore, the brushes may be respectively called positive and negative.

In the position *b* (Fig. 266) the coil is on the *neutral line*. The neutral line is the line drawn through the centre of the armature which makes a right angle with the pole axis. The side of the coil now moves along the magnetic lines without cutting them. Therefore, there is no emf induced in the coil and the load current will be zero. At this instant the brush is in contact with both segments, and the coil is short-circuited.

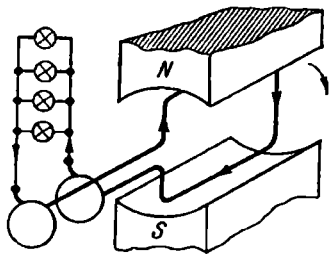


Fig. 264. Generation and collection of alternating current

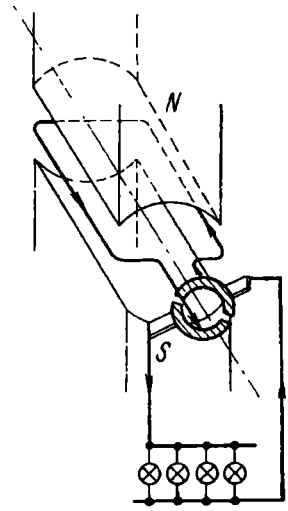


Fig. 265. Operation of a commutator

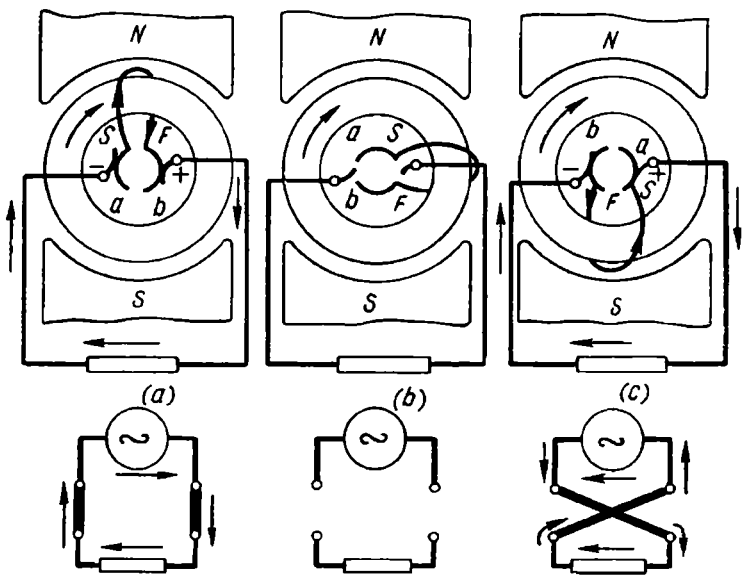


Fig. 266. Operation of a commutator in d-c generators

In the position *c* the coil is under the S pole, and the current in it flows from the finish to the start of the coil. In the external circuit, however, the current continues to flow as in the position *a* because the brushes have

changed contact with the commutator segments. Thus, in all positions the load current remains in the same direction.

The values of induced emf and, consequently, load current, vary as the armature rotates. Fig. 267 is the graph of a *pulsating load current* (i.e., a current of constant direction and varying magnitude).

If two coils are wound on the armature and connected in parallel, as shown in Fig. 268*a*, the terminal voltage of the generator will be equal to the voltage across one coil, but the current will be twice that of one coil. Let us take an armature carrying four coils spaced 90° apart and connected in series (at *b* in Fig. 268). As can be seen from the figure, there are as many commutator segments as there are coils.

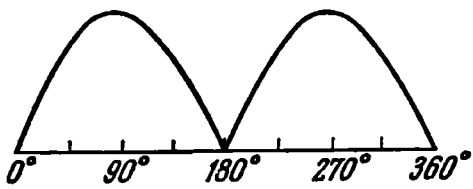


Fig. 267. Pulsating current

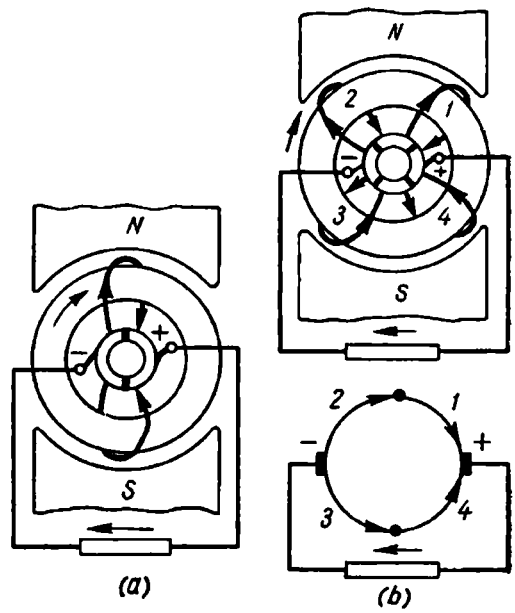


Fig. 268. Armature winding:
a—one pair of coils on the armature; *b*—two pairs of coils on the armature

Fig. 269 gives the graphs of the emfs induced in coils *1* and *2*. Since the coils are spaced 90° apart, the waveforms are likewise shifted through 90° in phase. Coils *3* and *4* have similar waveforms of induced emf, so our discussion may be limited to coils *1* and *2*.

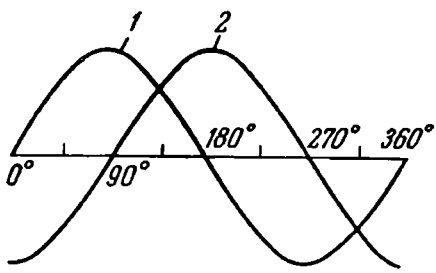


Fig. 269. Emfs of two adjacent coils arranged on the armature as shown in Fig. 268*b*

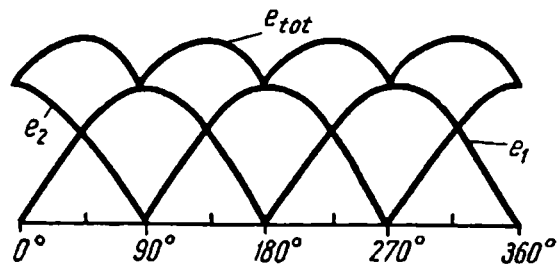


Fig. 270. The waveform of the total emf from two coils

Since the coils are connected in series, their total instantaneous emf, e_{tot}, is equal to the sum of the instantaneous emfs of each coil. As can be seen from Fig. 270, the waveform of the total emf has fewer ripples than the waveforms of each of the coils. The total emf of the remaining two coils has the same waveform but is in the opposite direction. Both emfs are connected in parallel to the brushes of the generator.

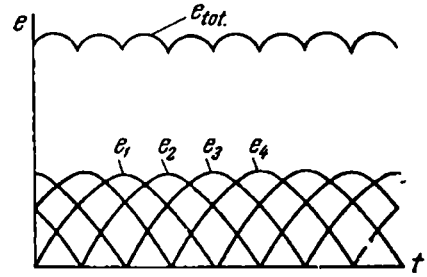


Fig. 271. The waveform of the total emf from four coils

In the case of eight sections (or four coils) the addition of their instantaneous emfs will give a total emf, e_{tot} (Fig. 271), which has still fewer ripples than in the previous case. Thus, by increasing the number of coils on the armature and the number of segments in the commutator, it is possible to obtain a current which will for all practical purposes be constant both in direction and magnitude (i.e., a direct current). For example, sixteen coils on the armature produce a ripple of less than one per cent. In modern d-c machines, the armature may carry over a hundred coils.

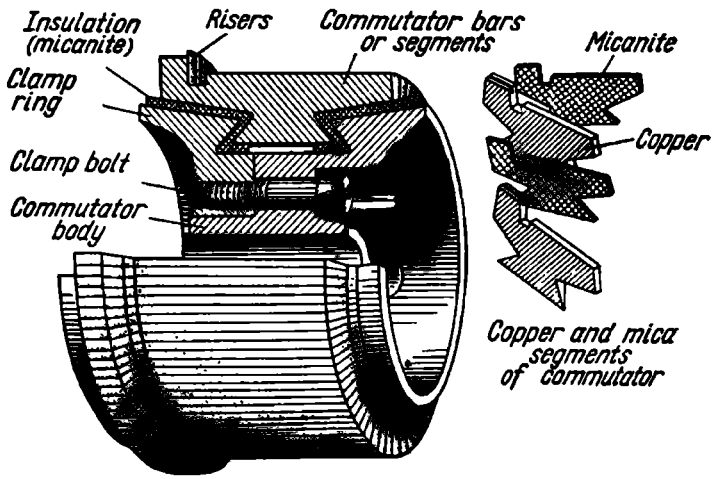


Fig. 272. Section through a commutator

If a d-c generator is fitted with both a commutator and slip-rings, it may be used as a source of both alternating and direct current.

The commutator shown in Fig. 272 is built up of hard-drawn copper bars insulated from one another by mica segments 0.5 to 1 mm thick. On the underside the commutator bars have dovetail projections which fit into a dovetail notch on the insulating clamp sleeve slipped onto the shaft on the armature side. On the opposite side the commutator bars are held in place by an insulating clamp ring which also fits into the dovetail projec-

tions of the bars. The clamp sleeve and ring are bolted so as to hold the bars together. The leads from the coils are soldered to commutator *risers*, also of copper.

131. The Armature Winding

Armature conductors must be connected together in such a way that a desired resultant emf may be maintained between two terminals connected to the external circuit. Thus connected, the conductors make up the armature winding which is placed in slots.

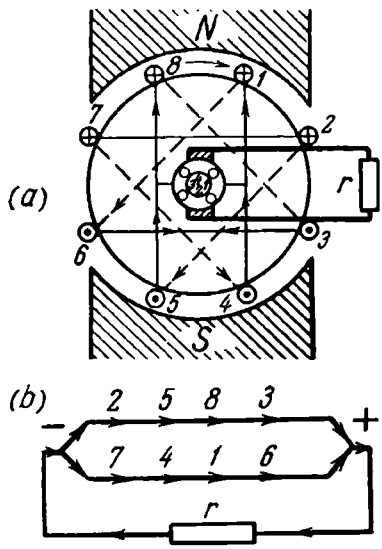
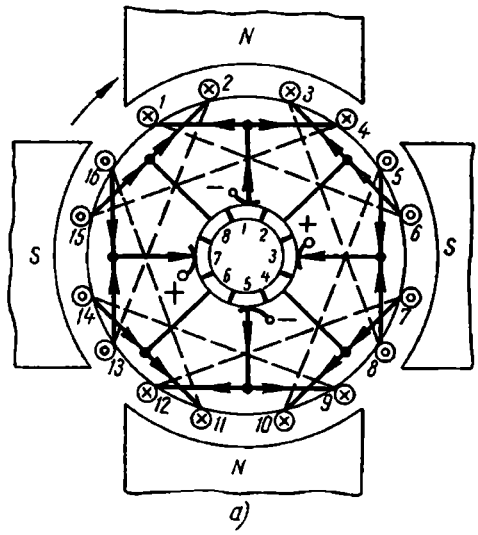


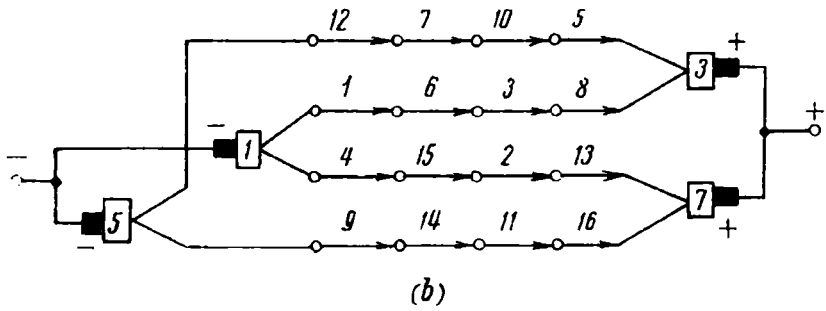
Fig. 273. Drum-type winding of a two-pole generator

There may be open, semi-closed and closed slots, the latter two forms being used almost exclusively in modern d-c machines. The reason for this is that the coils may be prewound on formers, taped with cotton tape or varnished cambric, bent to the required shape and then placed in the slots—a technique which cuts down both time and cost requirements of generator and motor manufacture. For higher dielectric strength the coil insulation is impregnated with varnish. Before the winding is put in place the slots are given U-shaped slot liners of leatheroid or presspahn to ensure mechanical protection of the coils. After the coils have been dropped in the slots, wedges of wood or hard fibre are driven in to hold them in place. Then rings of tinned steel banding wire are soldered to the coil ends.

There are two types of armature windings: the *lap winding* (Fig. 274*a*) and the *wave winding* (Fig. 275*a*). The difference between the two types is in the manner of connecting the armature coils to the commutator bars. A coil is the portion of the armature winding between successive connections to the commutator. In the lap type of winding the two ends of a coil are connected to adjacent commutator bars. In the wave type of winding, the two ends of a coil are connected to commutator bars spaced 360 electrical degrees apart (i.e., the distance between two consecutive connections where the field undergoes a complete cycle of changes).

The type of armature winding has a decisive bearing upon the voltage and current generated. This is because the number of parallel paths available to the current is different in different types. In the lap winding, there are as many parallel paths as there are pairs of poles on the machine. In the wave winding, there are always two paths in parallel between the armature terminals. The current paths in lap and wave windings are shown in Fig. 274*b* and Fig. 275*b*.





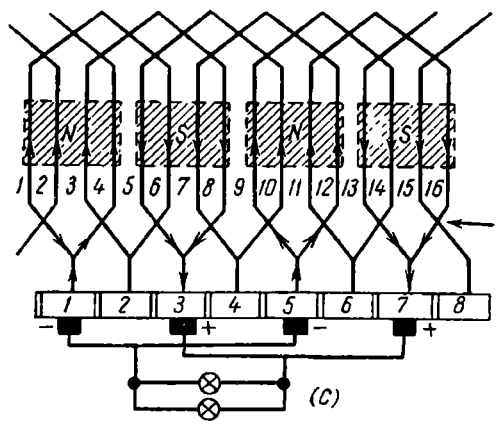


Fig. 274. Lap armature winding of a four-pole generator:

a—arrangement of the winding; *b*—distribution of current in the winding conductors; *c*—developed diagram

It is customary to give a developed winding diagram. From reference to Fig. 274*c* and Fig. 275*c* it is clear why the armature winding is called the lap winding in the former case (the conductors overlap one another) and the wave winding in the latter (the conductors progress like a wave).

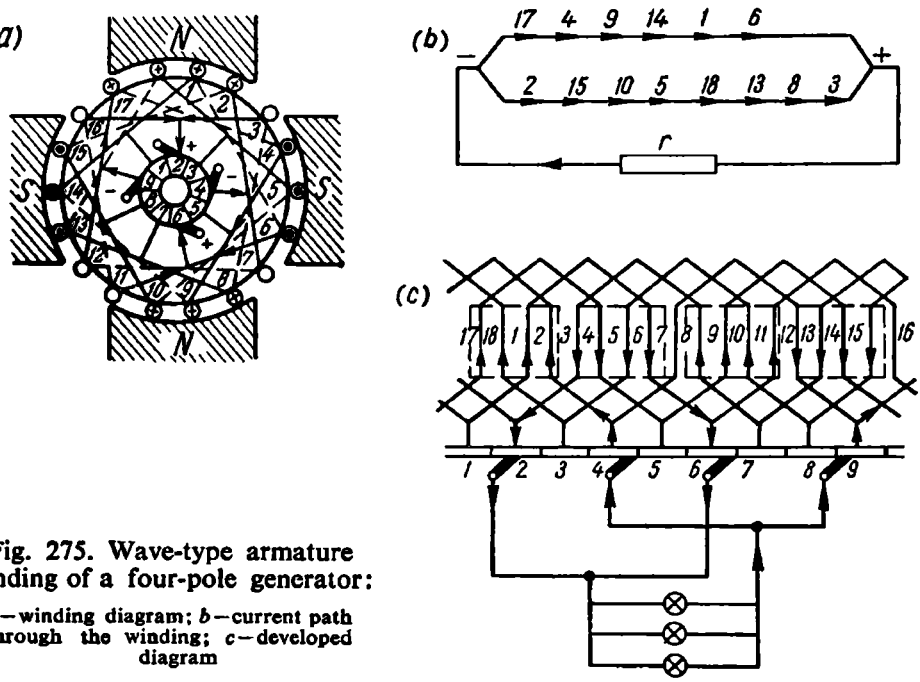


Fig. 275. Wave-type armature winding of a four-pole generator:

***a*—winding diagram; *b*—current path through the winding; *c*—developed diagram**

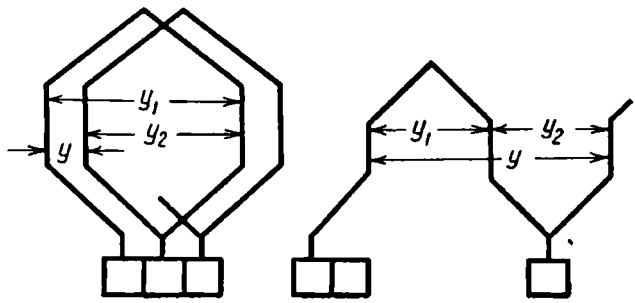


Fig. 276. Determining the pitch of lap-type and wave-type winding

In the case of generators designed for high voltages, preference is given to the wave type of winding. There are several reasons for this choice. The differences in the air gap, pole shape and armature centring combine to produce different emfs in the armature conductors. Since in the lap winding different coils come under the same pole, different emfs will be induced in the parallel circuits, and a *circulating current* will flow through the winding and brushes, causing sparking, additional loss and additional

heating. In the case of a wave winding, the circulating current due to non-uniform magnetic fluxes, poor centring of the armature, etc., is less felt, since different coils are placed under different poles and the difference in the emfs is equalized. For high-power, low-voltage machines, preference is given to the lap type of winding, since the load current is high and it is more advantageous to offer several parallel paths to its flow.

The length of one side in a coil is called the partial coil pitch (y_1 and y_2 in Fig. 276, since each coil has two sides). The algebraic sum of the two sides gives the coil pitch (y). Referring to Fig. 276, the coil pitch for the lap type of winding is

$$y = y_1 - y_2.$$

For the wave type of winding the coil pitch is

$$y = y_1 + y_2.$$

For the lap winding shown in Fig. 271*a* the partial pitch y_1 is five and the partial pitch y_2 is three (taking the distance between two adjacent conductors over the circumference of the armature equal to unity). Therefore, the coil throw (starting with the first conductor) will be

$$1+5 = 6;\ 6-3 = 3;\ 3+5 = 8;\ 8-3 = 5, \text{ etc.}$$

For the wave winding shown in Fig. 272*a*, $y_1 = 5$ and $y_2 = 5$. Therefore, the coil throw (starting with the first conductor) will be

$$1+5 = 6;\ 6+5 = 11;\ 11+5 = 16;\ 16+5 = 21;\ 21-18 = 3, \text{ etc.}$$

The sides of a coil are usually wound with several conductors which are insulated from each other and placed in respective slots.

132. The EMF Induced in the Armature Winding of a Direct-current Generator

As will be recalled, the induced emf is proportional to the time rate of change of the magnetic flux:

$$e = -(d\Phi/dt).$$

During one revolution of the armature in a two-pole generator each armature conductor cuts the magnetic field twice. If the speed of the machine is $n/60$ rpm, the number of magnetic lines cut by a conductor in unit time will be

$$2\Phi(n/60).$$

Consequently, the emf will be

$$e = 2\Phi(n/60) \text{ volts.}$$

If the armature winding has two parallel paths, the number of conductors in each path will be $N/2$. The emf induced in each of the circuits can be then expressed thus:

$$e = 2\Phi(n/60)\times(N/2) \text{ volts.}$$

In a machine having $2p$ poles (where p is the number of pole pairs), the number of magnetic lines cut by a conductor during one revolution of the armature will be $2p\Phi$, and the induced emf will be

$$e = 2p\Phi(n/60)\times(N/2) \text{ volts.}$$

In a machine having $2a$ parallel paths or circuits (where a is the number of pairs of paths through the armature), the number of conductors per circuit will be $N/2a$, and the emf of the generator will be equal to

$$e = 2p\Phi(n/60)\times(N/2a) \text{ volts.}$$

Finally, the emf of a d-c generator can be found thus:

$$E = (p/a)\times(n/60)\phi N.$$

Example. Calculate the emf of a generator if

$\Phi_{eff} = 0.0368$ V-sec; $N = 500$; $n = 750$ rpm; $2p = 6$; $2a = 6$.

$E = (p/a)\times(n/60)\Phi N = (3\div3)(750\div60)\times0.0368\times500 = 230$ V.

133. Armature Reaction. Commutation. Commutating Poles

When a d-c generator operates at no load, the armature current is very small or equal to zero, depending on the type of generator. Then the magnetic field produced by the field coils of a two-pole generator has the form shown in Fig. 277*a*. As has been noted, the line drawn through the centres of the pole shoes is called the pole axis. The axis of the magnetic field coincides with the pole axis. The line at right angles to the axis of the magnetic field, or the physical neutral line, coincides in this case with the *geometrical neutral line* (line *ab*).

When the secondary circuit of a d-c generator is completed through a load, a current flows through the armature winding, setting up a magnetic field of its own, as shown in Fig. 277*b*. The superposition of the armature field upon the main field produces a resultant magnetic field. The diagram in Fig. 277*c* shows the resultant magnetic field when both the armature and main exciting fields exist together. This effect is termed *armature reaction*. The consequences of armature reaction may be summed up as follows.

1. The flux density is decreased at the leading pole tip where the two mmfs subtract, while it is increased at the trailing pole tip where the two mmfs add. Where the magnetic induction of the pole core is small, the

increase in the flux density at the leading pole tip is equal to the decrease at the trailing pole tip. Where, on the other hand, the magnetic induction is great, the pole cores are saturated, and the increase in flux density at the leading pole tip is less than the decrease at the trailing pole tip, so that the total flux is diminished and the terminal voltage of the generator drops off.

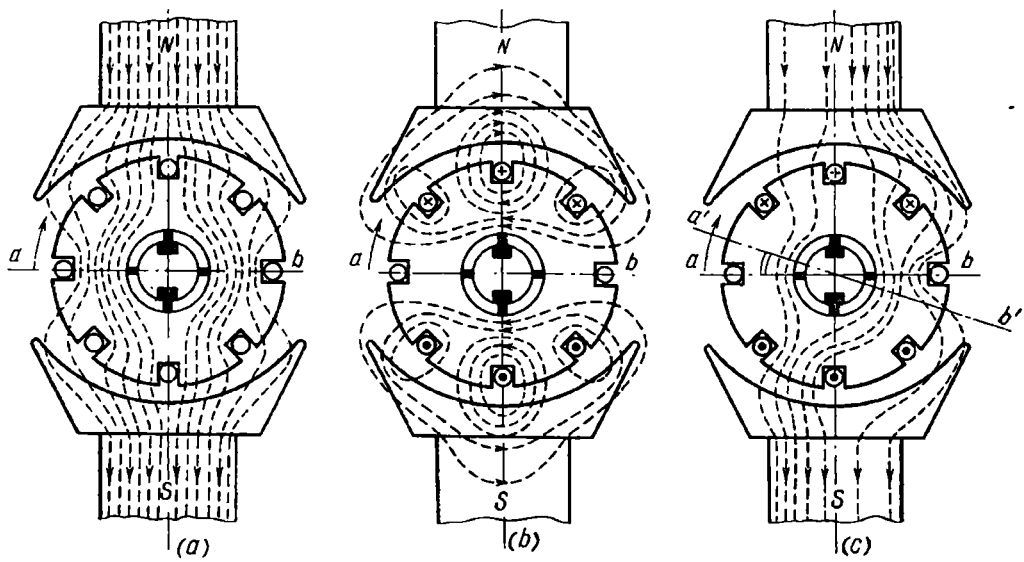


Fig. 277. Armature reaction in d-c generators

2. The magnetic field of the machine is distorted and the physical neutral line is shifted in the direction of rotation (positions *a'* and *b'* in Fig. 277*c*). The displacement of the neutral line depends on the magnitude of the armature flux, which is in turn dependent on the armature, or load, current.

3. As a result, the brushes have to be shifted in the direction of rotation to avoid sparking. To avoid continually shifting the brushes with changing load, additional poles (called commutating poles) have to be provided.

The emfs induced in all the armature conductors under the north pole of a d-c generator, i.e., the conductors in one of the parallel circuits, are all in the same direction while those generated in the conductors under the south pole, or in the other parallel circuit, are in the opposite direction and so are the respective currents. As the armature rotates, the conductors that have been under the north pole come under the south pole and vice versa and the direction of the emfs and currents in the respective circuits is automatically reversed. This is known as *commutation* and occurs when the conductors of a parallel circuit cross the neutral line and come under a pole of opposite sign.

Since the armature coils are embedded in magnetic material, they are inductive, and the current changing during commutation develops emfs

of self-induction. By Lenz's Law, these emfs of self-induction tend to slow down the desired reversal of current, or commutation, before the coil leaves the brush.

Stated in simple terms, commutation takes place as follows: Fig. 278*a* shows a coil *abc* in a commutating zone. The current from two adjacent parallel circuits flows to commutator bar *1* and is taken by a positive brush to the external circuit. The width of the brush is taken equal to the width of the commutator bar. In the position shown in Fig. 278*b* the coil

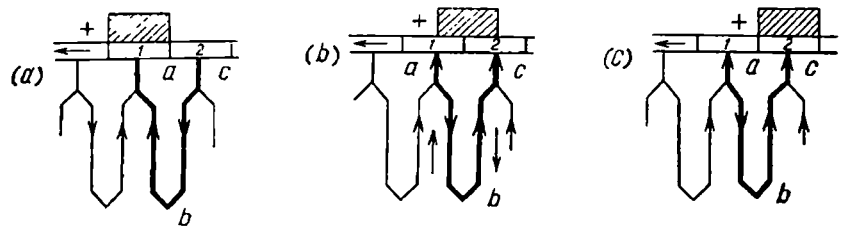


Fig. 278. Commutation in a winding coil

abc has moved some distance, and the brush is now in contact with commutator bar *2*. It will be noted that the direction of the current has reversed: instead of flowing from *c* to *a*, it now flows from *a* to *c*. The current reverses just as the brush changes contact from one commutator bar to another.

Fig. 278*c* shows the coil in an intermediate position between commutator bars *1* and *2*.

When the brush is in equal contact with both commutator bars *1* and *2*, the coil *abc* is on the physical neutral line. If commutation were not accompanied by other phenomena, the current in the coil would be zero, and the currents in connecting leads *a* and *c* would be inversely proportional to the contact resistance between the brush and the commutator bars or, which is the same, directly proportional to the contact areas between the brush and the commutator bars. Then equal currents adding up to the current flowing from the positive brush into the external circuit would flow in the connecting conductors *a* and *c*. As the brush leaves bar *1* and comes onto bar *2*, the contact area decreases in the former case and increases in the latter. In an ideal case, the currents in the respective connecting conductors would follow the same trend, and the current in the coil would increase. Actually, commutation is complicated by the emf of self-induction, which establishes a current opposing the currents in the coil conductors by Lenz's Law. This condition is shown by dotted lines in Fig. 278*c*. Consequently, the currents in the conductors *a* and *c* flow in opposite directions. So there is an increase in current density at the trailing pole tip and a decrease in current density at the leading pole tip. This increase may result in heating and sparking detrimental to the brushes and commutator.

From the above it follows that for good commutation the current in the coil within the commutating zone (i.e., when it is short-circuited by

the brush) should be zero despite the generation of emf of self-induction. This requirement is satisfied by shifting the brushes through some angle from the physical neutral line in the direction of armature rotation. Then in the coil undergoing commutation an emf will be induced opposite in direction to the emf of self-induction, and the two will cancel each other.

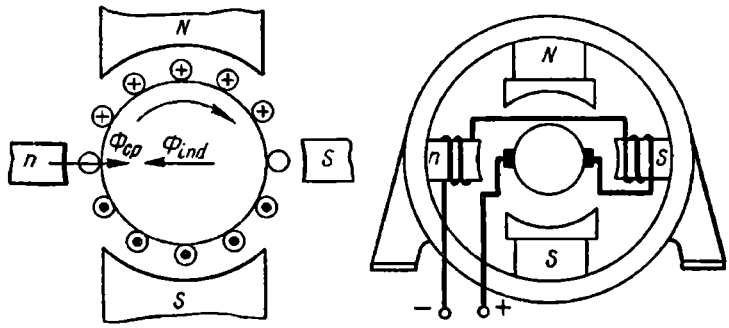


Fig. 279. Arrangement of commutating (intermediate) poles in a generator

Since the emf of self-induction varies with the load current, the brushes would have to be shifted continually, a requirement which cannot be possibly met. Instead, commutating poles are inserted between the main poles of d-c machines, and the brushes are left on the neutral line. The commutating poles induce in the coil undergoing commutation an emf which cancels the emf of self-induction, thereby preventing sparking and other adverse effects of commutation. The exciting windings on the commutating poles are in series with the armature winding, or the load, so that their magnetic field is always proportional to the load, or the armature current. The connection and arrangement of commutating poles and their exciting windings are shown in Fig. 279.

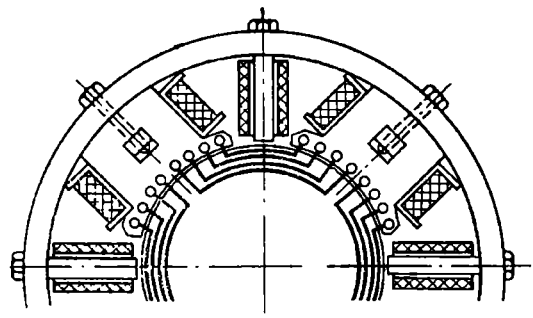


Fig. 280. Diagram of compensating winding

Generators that have to carry a sharply varying load (in hoists, cranes, rolling mills, etc.) sometimes incorporate compensating windings. A compensating winding is placed in slots in the main pole faces. The current flowing in a compensating winding should oppose the current in the armature winding. Then it will set up an mmf equal and opposite to that of the armature. This will reduce, or completely prevent, distortion of the field due to the armature reaction.

Like the exciting windings of commutating poles, compensating windings are connected in series with the load. A diagrammatic representation of a compensating winding is shown in Fig. 280.

134. Types of Direct-current Generators

The mmf necessary to establish the flux in the magnetic circuit of a d-c generator can be obtained by means of (1) a permanent magnet; (2) field coils excited by the generator itself; and (3) field coils excited by some external source.

A permanent-magnet generator consists of an armature and one or several permanent magnets encircling the armature. In view of the low power thus generated, permanent-magnet generators have not found industrial application.

In the case of separately excited generators, the field coils may be excited by any d-c constant-potential source such as a storage battery, a d-c generator (called an exciter), a rectifier, etc.

Where the field coils are excited by the generator itself, it is said to be self-excited. Self-excitation consists in the following. In the absence of current in the field coils, the armature rotates in the weak magnetic field produced by the residual magnetism of the pole cores. The weak emf induced in the armature winding sends a slight current through the field coils. The magnetic field of the field coils builds up, and so does the emf in the armature winding. This raises the exciting current. This continues until the current in the field coils attains a value corresponding to the resistance of the field circuit. Therefore, self-excitation will take place only if and when (a) the current in the field coils produces a field increasing the field produced by residual magnetism and (b) the resistance of the field circuit does not exceed a definite (critical) value.

Self-excited generators are further subdivided into: (1) shunt-wound units with the field coils connected in parallel with the armature winding; (2) series-wound units, in which the field coils are connected in series with the armature winding; and (3) compound-wound machines, in which there are two windings on each pole, one connected in parallel and the other in series with the armature winding.

The type of connection of the two windings determines the construction and performance of the generator.

135. Separately Excited Generators

A schematic diagram of a separately excited generator is shown in Fig. 281. The current energizing the field coils is supplied by an external source and is independent of the operation of the generator. A rheostat included in the field circuit makes it possible to vary the exciting current at will. This *field rheostat*, as it is called, has a third contact enabling the rheostat

to be short-circuited when the field coils are de-energized. In this way, the contacts of the rheostat are protected from burning, because when the field circuit containing considerable inductance is opened, the rapidly vanishing current induces a considerable emf of self-induction that maintains the arc between the contact arm and the last contact of the rheostat.

When the armature rotates at constant speed and at no-load, the emf of the generator will depend solely on the exciting current. This relation is known as the *no-load saturation characteristic of a generator* (Fig. 282). This characteristic can be plotted by varying the excit-

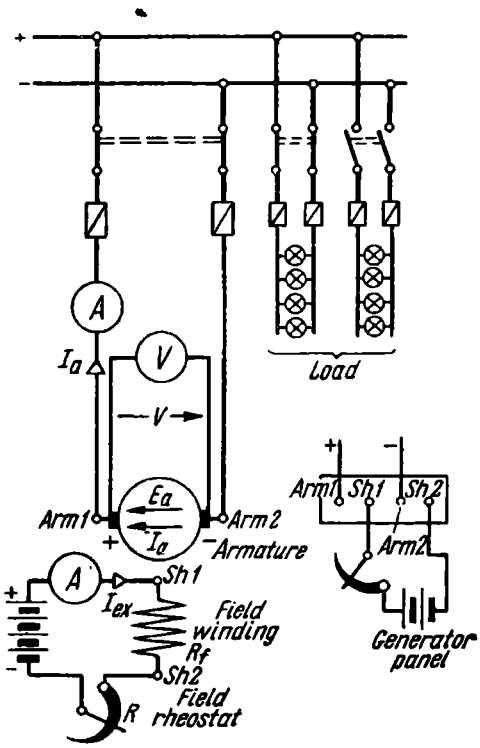


Fig. 281. Diagram of a separately excited generator

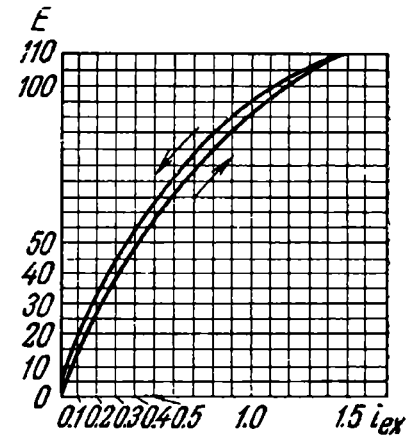


Fig. 282. No-load saturation characteristic of a separately excited generator

ing current by means of the field rheostat and taking simultaneous readings of an ammeter connected in the field circuit and of a voltmeter connected across the generator brushes.

When the generator is magnetized for the first time and in the absence of exciting current ($I_{exc} = 0$), the voltmeter will read zero at any rpm. The rising exciting current will at first be accompanied by an emf rising in proportion, and the corresponding portion of the curve will be a straight line. The emf will continue to increase with the exciting current until the magnetic circuit becomes saturated. This condition is marked by a bend in the characteristic curve. If the exciting current is now decreased, the emf of the generator will be higher for the same exciting current than during magnetization, and the demagnetization curve will be located above the magnetization curve. This phenomenon is due to hysteresis. When the exciting current is reduced to zero, the generator will continue to generate some emf due to the residual magnetism of the poles. The farther the point,

corresponding to the emf of the generator in normal operation, lies past the bend on the no-load excitation curve, the less is the dependence of the emf of the generator on the exciting current. There is a limited possibility to regulate the voltage in operation past the bend on the curve. Conversely, during operation on the straight portion of the curve, even the slightest change in exciting current will appreciably change the emf of the generator. Thus, the no-load saturation characteristic of a generator gives an idea of its magnetic properties.

The principal requirement that any generator must meet is constant voltage at any load. Since, however, the voltage of a generator does vary with the load, the best machine will be one in which the voltage will change less for the same change in the load.

The brush voltage of a separately excited generator changes with the load for two reasons:

1. Due to the voltage drop across the armature winding and the contact resistance of the brushes.

The emf, E, of a generator differs from the brush voltage, V, by the voltage drop across the armature winding, I_aR_a:

$$E = V + I_aR_a, \quad \text{or} \quad V = E - I_aR_a.$$

If, for example, the emf of a generator is 120 V and the resistance of the armature winding is 0.01 ohm, the brush voltage of the generator at 50 A will be

$$V = E - I_aR_a = 120 - 50 \times 0.01 = 119.5 \text{ V};$$

at 100 A

$$V = 120 - 100 \times 0.01 = 119 \text{ V},$$

at 200 A

$$V = 120 - 200 \times 0.01 = 118 \text{ V}.$$

In order to reduce the voltage drop across the armature winding, it is wound with copper wire or bars of large diameter, so the resistance of the armature winding can be kept to a value of the order of several tenths, hundredths or even thousandths of one ohm.

At no-load, $I_a = 0$. Therefore,

$$V = E.$$

2. Due to the armature reaction, which reduces the magnetic flux and the emf of the machine.

This relationship is known as the external characteristic of a generator (Fig. 283). It is plotted by increasing the load on the generator gradually and taking simultaneous readings of an ammeter connected in the armature circuit and a voltmeter across the brushes of the generator, while keeping its speed and exciting current constant. The decrease in brush voltage with increasing load may be as great as 5 to 8 per cent of the rated voltage.

The brush voltage of a generator can be maintained constant by adjusting the exciting current by means of the field rheostat.

When the direction in which a separately excited generator rotates is reversed, the polarity of its brushes is also reversed.

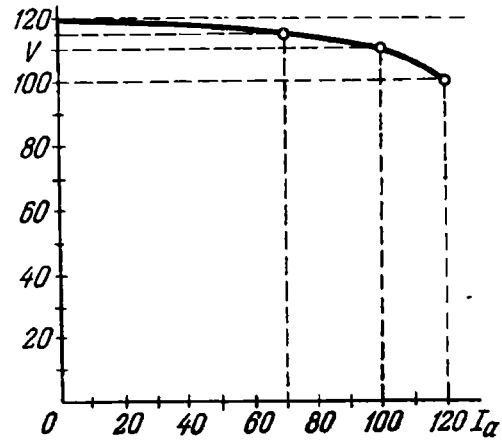


Fig. 283. External characteristic of a separately excited generator

136. The Shunt-wound Generator

Fig. 284 shows diagrammatically a shunt-wound generator. The field coils of the generator are connected in parallel with the armature winding. Therefore, in operation, the total armature current, I_a, flowing from the positive brush divides between two parallel paths: the external circuit and the field circuit. The line current I and the exciting current I_{ex} close the circuit at the negative brush, and

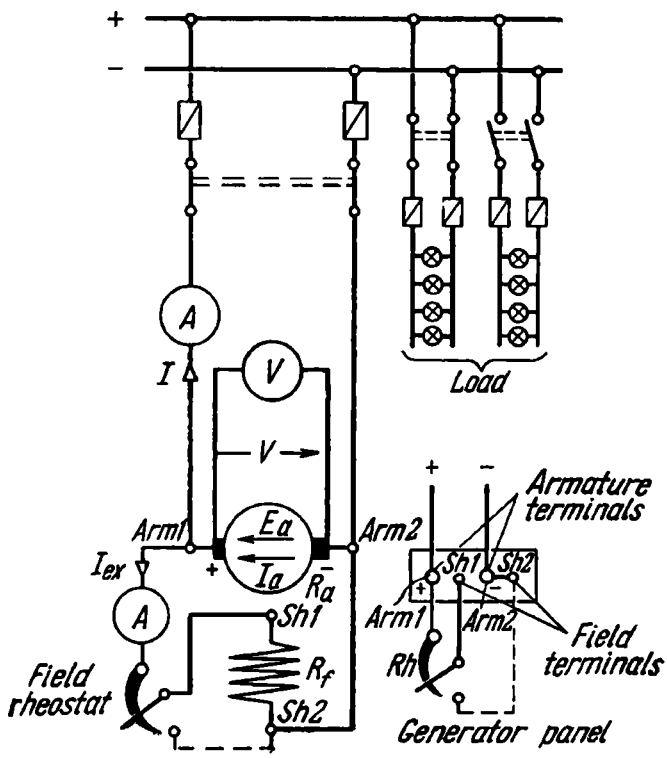


Fig. 284. Diagram of a shunt-wound generator

their sum is equal to the armature current. Therefore, we may write

$$I_a = I + I_{ex}.$$

The effective power of a generator is proportional to the current delivered by the machine into the line. Therefore, the exciting current must be kept as low as possible. For shunt-wound generators it is maintained at about 2.5 per cent of the rated armature current. To obtain the desired mmf, the field coils are wound with a large number of turns of fine insulated copper wire.

The no-load saturation characteristic of a shunt-wound generator is similar to that of the separately excited generator and is obtained by varying the exciting current by means of a field rheostat at constant speed and taking simultaneous readings of a voltmeter connected across the generator brushes.

The voltage of a shunt-wound generator varies with the load. This is due to three causes:

(1) the voltage drop across the armature winding and the contact resi tance of the brushes;

(2) the decrease in magnetic flux as a result of the armature reaction;

(3) the decrease in the exciting current as a result of the first two factors. At a constant resistance of the field circuit, the exciting current is proportional to the brush voltage of the generator. Therefore, an increase in the brush voltage will bring about a decrease in the exciting current. This will in turn cause the emf and brush voltage of the generator to drop too. The shunt-wound generator is thus again different from the separately excited generator, in which the field coils are supplied by an independent external source.

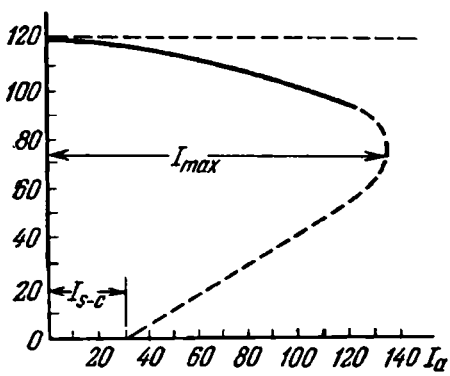


Fig. 285. External characteristic of a shunt-wound generator

Fig. 285 shows the external characteristic of a shunt-wound generator. It is plotted by varying the load (i.e., the resistance of the external circuit) at constant speed and constant resistance of the field circuit and by taking simultaneous readings of an ammeter connected in the armature winding and a voltmeter connected across the generator brushes. As can be seen from the characteristic, the voltage of a shunt-wound generator decreases with increasing load. Within the normal operating range, this decrease is insignificant (shown by the solid line). For generators having commutating poles, it is 8 to 15 per cent of the rated voltage at rated load. Outside the operating range, an increase in the load current brings about a considerable decrease in brush voltage (as shown by the dotted line).

When the load current has attained some critical value, I_{max}, it will drop even though the resistance of the external circuit is reduced. This occurs because the line current is determined not only by the resistance of the external circuit but also by the generator voltage.

The voltage of a shunt-wound machine can be kept fairly constant by providing additional resistance in the field circuit (the field rheostat). This rheostat may be cut out as the brush potential falls. Then more current will flow through the field coils, a stronger magnetic field will be set up, and a greater emf will be induced in the armature, restoring the brush potential.

In the case of a short-circuit, the voltage of a shunt-wound machine drops to zero, and the emf induced in the armature winding by residual magnetization will establish a short-circuit current.

When a shunt-wound generator is started up for the first time, it may be that no voltage will be produced: either because the poles have no residual magnetization or the poles have retained some residual magnetism but the connection of the field coils is reversed, so that the magnetism developed by the field coils on starting has destroyed the residual magnetism and the machine cannot "build up". In both cases, the field coils must be connected to a d-c source (a storage battery) for a short while to magnetize the poles. Then the generator will generate some emf with the field coils disconnected. If there is any doubt about the connection, the polarity of a shunt-wound machine can be determined as follows. The generator is started up, the leads of the field circuit are brought in contact with the armature terminals on the generator terminal panel, and readings are taken on the voltmeter connected across the generator terminals. If the voltmeter deflects to the right, the field coils are connected correctly; if it deflects to the left, the leads of the field circuit should be interchanged. After the polarity of the field circuit has been verified, the generator is stopped, and the leads of the field circuit are clamped securely on the terminal board.

The poles of a shunt-wound generator may also be inadvertently demagnetized by a reversal in the rotation of the machine, since this brings about a reversal in the emf induced in the armature winding, in the polarity of the brushes and in the current through the field coils. This is why it is important to indicate the direction of rotation for shunt-wound generators.

137. The Series-wound Generator

A diagram of a series-wound generator is shown in Fig. 286. As can be seen, the generator has its armature winding, field coils and external circuit all connected in series with each other so that the same current flows through all parts of the circuit. For this reason the field coils of medium- and high-power series-wound machines are wound with a small number of turns of large-diameter wire.

If a series-wound generator is operated at no-load (the external circuit is open), there will be no current through the field coils. If the external circuit is closed, both the armature winding and the field coils will be traversed by the load current. As the load current is increased, the voltage will also increase (the solid line in Fig. 287) until the magnetic circuit becomes saturated. With further increase of load, the voltage will decrease. This is due to an increased armature reaction and a greater voltage drop across the windings (the dotted line in the diagram). Thus, a series-wound gener-

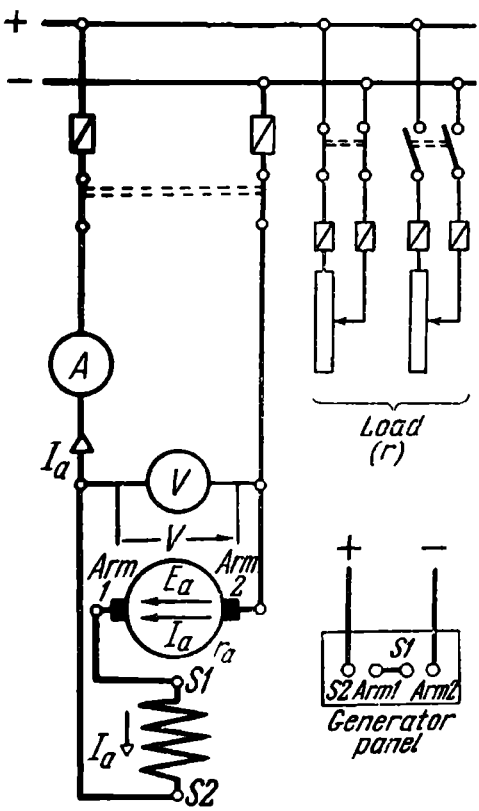


Fig. 286. Diagram of a series-wound generator

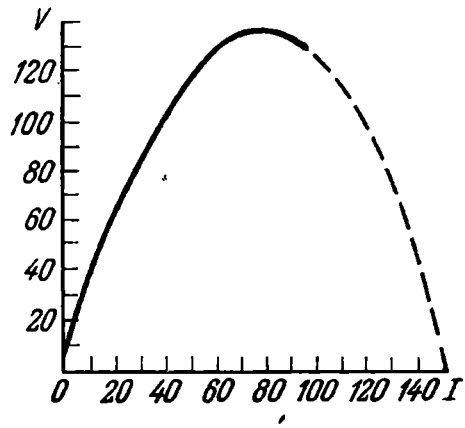


Fig. 287. External characteristic of a series-wound generator

ator, in which the voltage increases with the load, differs markedly from a shunt-wound generator, in which the voltage decreases with increasing load.

In the case of a short-circuit in the armature circuit, a fairly heavy current will be established while the generator voltage will be zero.

Because of the sharp increase in generator voltage with increasing load, series-wound generators are rarely employed.

138. The Compound-wound Generator

The compound-wound generator is shown diagrammatically in Fig. 288. Since its main poles carry both shunt and series field coils (Fig. 289), this type of generator possesses the qualities of both types discussed ear-

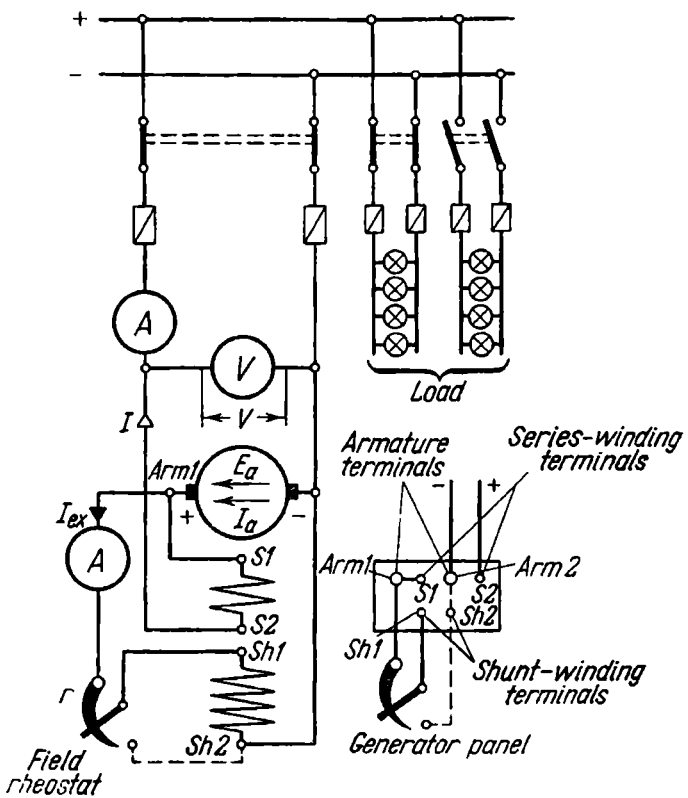


Fig. 288. Diagram of a compound-wound generator

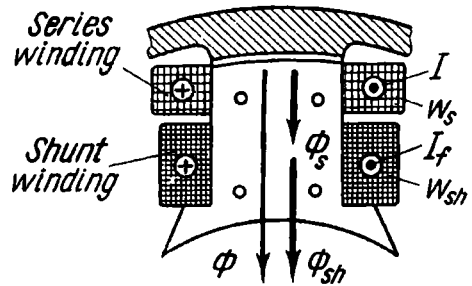


Fig. 289. Main pole of a compound-wound machine

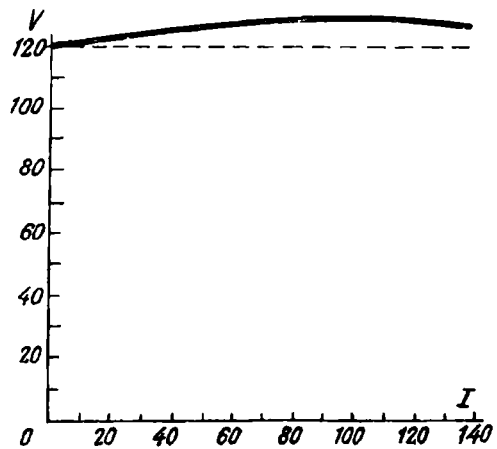


Fig. 290. External characteristic of a compound-wound generator

lier. Its no-load saturation characteristic resembles that of the shunt-wound generator, since the series coils are de-energized at no-load.

There are three cases of compounding: flat compounding, overcompounding, and undercompounding.

A flat-compounded generator is one having its series winding so proportioned with respect to the shunt winding that the terminal voltage of the generator remains practically constant at all loads.

An overcompounded generator has its series winding so proportioned that its full-load voltage is greater than its no-load voltage. This is necessary in order to be able to compensate for the voltage drop across the line wires and to maintain a practically constant voltage across the consumer's terminals.

An undercompounded generator is one with a relatively small series winding; its voltage decreases with increasing load.

The external characteristic of an overcompounded generator is shown in Fig. 290.

139. Parallel Operation of Shunt-wound Generators

If shunt-wound generators are to operate successfully in parallel the following requirements must be met:

1. They must operate at exactly the same voltage.
2. They must be of the same polarity (the "+" terminal of one generator must be connected to the "+" terminal of the other, and the same for their "−" terminals).

The voltages of the generators to be connected in parallel are adjusted as follows. With one of the generators already running, the prime mover of the incoming generator is started and brought up to rated speed. Next, the voltage of the incoming generator is adjusted by means of the field rheostat until it is equal to the voltage of the running generator or the line. The voltages are checked for equality with voltmeters connected across the line and the incoming generator or with one voltmeter fitted with a selector switch.

The polarity of connections is determined only when a generator is paralleled for the first time. The terminals are then so located as to make a repeated polarity test unnecessary. The polarity test is carried out either with a moving-coil voltmeter or incandescent lamps. With the knife-switch of the incoming machine cut out, any pair of line and generator terminals are short-circuited by a strap. Then a voltmeter designed for twice the line voltage is connected to the other pair of terminals. If the connection is correct, the voltmeter will read zero, otherwise it will read twice the line voltage. Fig. 291 shows the connection of two shunt-wound generators for parallel operation.

The incoming generator, when just put on the line, will not generate any current, since its emf is equal and opposite to the emf of the running gener-

ator. For the incoming generator to produce current, its emf must be increased by means of the field rheostat. The current delivered to the line traverses the armature winding and reacts with the magnetic field of the field coils, producing a moment of forces opposite to the torque of its prime mover. Therefore, an increase in load current must be accompanied by an increase in the mechanical power output of the prime mover.

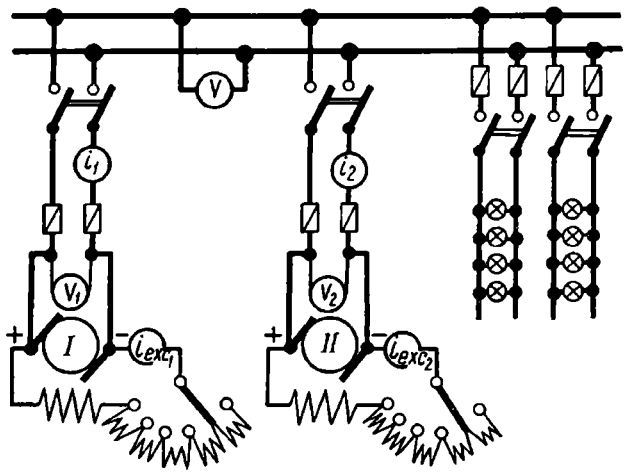


Fig. 291. Two shunt-wound generators in parallel operation

Where the load has to be shifted from one generator to the other, both field rheostats should be operated so that the exciting current of the generator which is to take the greater share of the load is increased and that of the generator to be unloaded is decreased.

In the parallel operation of two generators, it may happen that the emf of one of them drops (due to, say, a decrease in the speed of its prime mover or a breakdown of the V-belt from the prime mover to the generator). Then this generator will draw current from the line and will tend to operate as a motor. To avoid this, automatic circuit breakers are included in the circuit to switch it off as soon as the current through the generator has reversed.

140. Operation of a Direct-current Machine as a Motor

If the armature winding of a d-c machine is connected to a d-c source, it will operate as a motor, converting electrical energy into mechanical energy. This *reversibility* of operation is due to the identity of the generator and motor structure.

For its operation a d-c motor depends on the interaction of the current traversing the armature winding and the magnetic field produced by the

field coils. Therefore, the torque of a d-c motor may be expressed thus:

$$M_r = cI_a\Phi,$$

where c = coefficient of proportionality depending on the constants of the motor (number of pairs of poles, number of conductors in in the armature winding, number of parallel circuits in the armature);
I_a = armature current;
Φ = magnetic flux.

At constant speed, the torque of a d-c motor is equal to the opposing torque due to the mechanical load applied to the shaft of the motor, or

$$M_r = M_{op}.$$

The input power taken from the supply line differs from the output power by the power losses due to friction in bearings, between the brushes and commutator, windage, hysteresis and eddy-current losses, I^2R losses in the motor and rheostat windings, etc. The efficiency of a motor varies with the load. At rated power, the efficiency ranges between 70 and 93 per cent, depending on the rating, speed and construction of the motor.

The rotation of a motor can be reversed by reversing the current through either the armature winding or the field coils. If the current through both is reversed, the motor will continue to rotate as before.

All d-c motors must receive their excitation from an outside source. Therefore, they are separately excited. Their field and armature windings are connected, however, in one of the three different ways employed for self-excited d-c generators, and so a d-c motor may be shunt-wound, series-wound or compound-wound.

141. The Commutator of a Direct-current Motor

The direction of the force acting upon a current-carrying coil in a magnetic field depends on the direction of the current through the coil. In the case of a d-c motor, it is necessary, therefore, that the current through the coils of the armature winding be reversed as a particular coil leaves, say, the north pole, crosses the neutral line and comes under the south pole. This purpose is served by the commutator. The operation of the commutator is illustrated in Fig. 292.

Let there be a single-turn coil placed in a magnetic field. Its leads are soldered to commutator segments *a* and *b*, each carrying a brush. The plus side of the supply line is connected to the left-hand brush and the minus side to the right-hand brush. In position *I* the line current arrives at the commutator segment *a*, flows through the upper side *1* of the coil away from the reader (as shown by the cross in the circle) and then through the bottom side *2* of the coil towards the reader (as shown by the dot in

the circle), reaches the commutator segment *b* and flows again into the line through the other brush. By the left-hand rule, the coil will tend to rotate counterclockwise.

In position *II* the coil is on the neutral line; there is no contact between the commutator segments and the brushes, and there is no current flowing through the coil. The coil crosses the neutral line by inertia. If there are many turns in the coil, those not on the neutral line will supply the necessary torque.

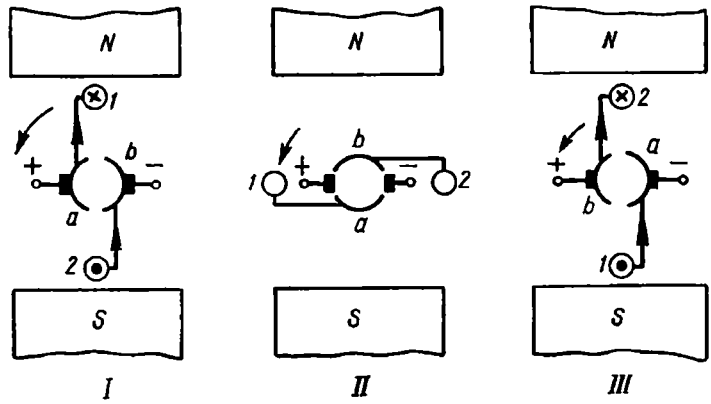


Fig. 292. Commutator of a d-c motor

In position *III*, the two sides of the coil, *1* and *2*, have changed poles, and the current through them has reversed. The commutator segments, however, have also changed contact with the brushes. So, by the left-hand rule, the coil will continue to rotate in the same direction as before, i.e., counterclockwise.

142. The Counter-EMF of the Armature

Since the motor armature revolves in a magnetic field, an emf is generated in the armature conductors. The direction of this emf, as determined by the right-hand rule, is opposed to the direction of the line voltage. Hence, it is called the *counter-emf of the armature* (Fig. 293).

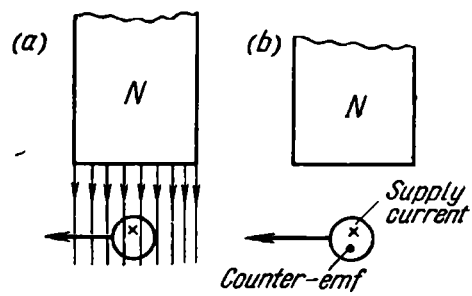


Fig. 293. Generation of counter-emf

The applied emf, or line voltage, must be large enough to overcome this counter-emf and to send the current through the resistance of the armature:

$$V = E + I_a R_a,$$

where V is the applied voltage, E is the counter-emf, and R_a is the resistance of the motor armature.

Then the current through the armature will be

$$I_a = (V-E)/R_a.$$

Example 1. Calculate the current taken by a d-c motor from a line if the line voltage is 120 V, the counter-emf is 119.5 V, and the resistance of the armature winding is 0.01 ohm.

$$I_a = (V-E)/R_a = \frac{120-119.5}{0.01} = 50 \text{ A}.$$

The counter-emf depends on the speed of the motor and the magnetic flux Φ:

$$E = cn\Phi,$$

where c is the coefficient of proportionality depending on the constants of the motor (the number of pole pairs, the number of conductors on the armature, and the number of parallel circuits in the armature winding).

At starting, the speed of the motor is zero; therefore, the counter-emf is also equal to zero. The starting current at this instant is equal to the line voltage divided by the armature resistance—a value dangerous to both the armature winding and the commutator. This is why some additional resistance must be inserted in the armature circuit at starting to limit the current to a safe value. The resistance usually takes the form of a starting rheostat wound with nickeline or constantan wire. The wire, which may be in round or strip form, is wound on an insulating frame or bobbin.

Starting rheostats for crane motors use resistances of cast iron put on insulated steel bars and clamped by nuts. When traversed by a current, starting rheostats grow hot. They may be cooled by air or oil. When air-cooled, rheostats are enclosed in a perforated metal casing, the exchange of hot and cold air being through the perforations. When oil-cooled, rheostats are immersed in an oil-filled tank. Since the thermal conductivity of oil is greater than that of air, oil-cooled rheostats are smaller in size.

The starting rheostat is gradually cut out as the motor gains speed and the counter-emf becomes available for limiting the armature current. If the rheostat were not cut out, it would reduce the torque, and the motor would not gain speed properly. At the end of the starting period the rheostat must be fully cut out. It should be remembered that starting rheostats are designed for short-time duty only. If left in the circuit, the remaining portion of the rheostat may burn.

If the counter-emf is to build up quickly at starting, the motor must be given full excitation (i.e., there must be no resistance or an open-circuit in the field circuit). A starting rheostat should be so chosen that it reduces the starting current to 2 to 2.5 times full-load current. This effects an economy in terms of rheostat cost and materials and also leaves enough current for the motor to come up to speed rapidly.

Example 2. Calculate the resistance of the starting rheostat for an electric motor with a rated current of 20 A and an armature winding resistance of 0.02 ohm. The supply voltage is 220 V.

The starting current may be taken twice the rated current, or $20 \times 2 = 40$ A. Then the resistance of the armature circuit will be

$$R = V/I_{st} = 220 \div 40 = 5.2 \text{ ohms.}$$

This also includes the resistance of the armature winding. Therefore, the resistance of the starting rheostat will be

$$5.2 - 0.02 = 5.18 \text{ ohms.}$$

When a motor runs at constant speed, its torque and mechanical load are balanced. If the load increases, the balance is upset, and motor slows down. Since it is speed-dependent, the counter-emf will also decrease, while the armature current will increase. This will continue until the increased armature current develops an increased torque equal to the increased load. Now the motor will run at a lower but constant speed. In the case of a decrease in the load, the balance is again upset, and the motor will pick up speed. The counter-emf will increase, decreasing the armature current; the torque of the motor will be decreased until it balances the load.

143. Speed Regulation of Direct-current Motors

As has been established, the counter-emf is determined by the speed and magnetic flux of the motor:

$$E = cn\Phi,$$

whence

$$n = E/c\Phi = V - I_a R_a/c\Phi.$$

Since $I_a R_a$ is very small (due to the low R_a), we may write (approximately)

$$n = V/c\Phi.$$

As follows from the above expression, the speed of a d-c motor is directly proportional to the applied voltage and inversely proportional to the magnetic flux. Putting it another way, the speed of a d-c motor can be controlled by varying the applied voltage and by varying the excitation of the motor by means of a rheostat connected in the field circuit.

144. Armature Reaction in Direct-current Motors

The armature reaction in d-c motors is due to the same causes as in d-c generators (Fig. 294). Since, however, a d-c motor rotates in a direction opposite to that of a d-c generator for the same direction of current in the armature winding and for the same polarity of the brushes, the effects of the armature reaction in d-c motors will be somewhat different.

1. The flux density will be decreased at the trailing pole tip and increased at the leading pole tip. At saturation of the magnetic circuit, the resultant magnetic flux will decrease and the motor will speed up.

2. The axis of the resultant magnetic field is displaced relative to the pole axis. The physical neutral line is displaced in a direction opposite to the rotation of the motor through an angle dependent on the motor load.

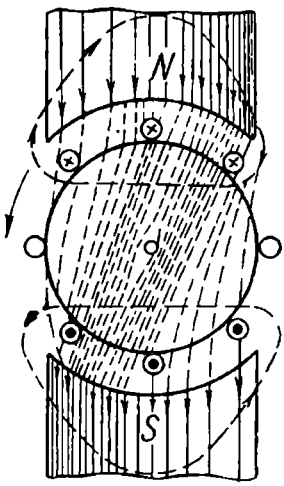


Fig. 294. Armature reaction in a d-c motor

3. The brushes must also be shifted on the commutator in a direction opposite to the rotation of the motor to make up for the displacement of the neutral line.

When an armature coil crosses the neutral line, the current in it is reversed by the commutator. Also, the coil is short-circuited during commutation by the brush. The change of current in the coil gives rise to an emf of self-induction which tends to slow down the reversal of the current. As has been stated earlier (Sec. 133), the emf of self-induction can be counteracted (1) by shifting the brushes or (2) by fitting commutating poles without the brushes being shifted. While in a d-c generator the brushes are shifted forward in the direction of rotation, in a motor they must be shifted backward. Furthermore, in d-c generators, a commutating pole following a main pole is of different polarity (alternate polarity), while in d-c motors a commutating pole following a main pole in the direction of rotation is of the same polarity (consequent polarity).

Heavy-duty motors (in hoists, cranes, mine and traction motors, rolling mills, etc.) are also fitted with compensating windings which reduce sparking in commutation and prevent distortion of the flux due to the armature reaction.

145. The Shunt-wound Motor

The shunt-wound motor with a starting rheostat connected in the armature circuit is shown diagrammatically in Fig. 295. Since the field coils are connected in parallel with the supply line, the magnetic flux of the motor at constant resistance of the field circuit and constant applied voltage must be constant. If we ignore the voltage drop across the armature winding, the speed of a d-c motor depends solely on the applied voltage and magnetic flux. If they are likewise constant, the speed of a shunt-wound motor will remain unchanged with varying load. However, as follows from the formula

$$n = E/c\Phi = (V - I_a R_a)/c\Phi,$$

the counter-emf decreases with increasing armature current, and the motor slows down too. On the other hand, as the load increases, the armature

reaction reduces the magnetic flux, and there is some increase in the speed of the motor. In practice, the voltage drop across the armature winding is chosen so that its effect on the speed of the motor is fully compensated by the armature reaction.

Thus, the speed of a shunt-wound motor remains nearly constant with varying load. Usually, there is a 3 to 5 per cent drop in the speed of a shunt-wound motor at full load.

The torque of a motor is

$$M = cI_a\Phi.$$

Since changes in the magnetic flux due to the armature reaction are negligibly small, it may be said

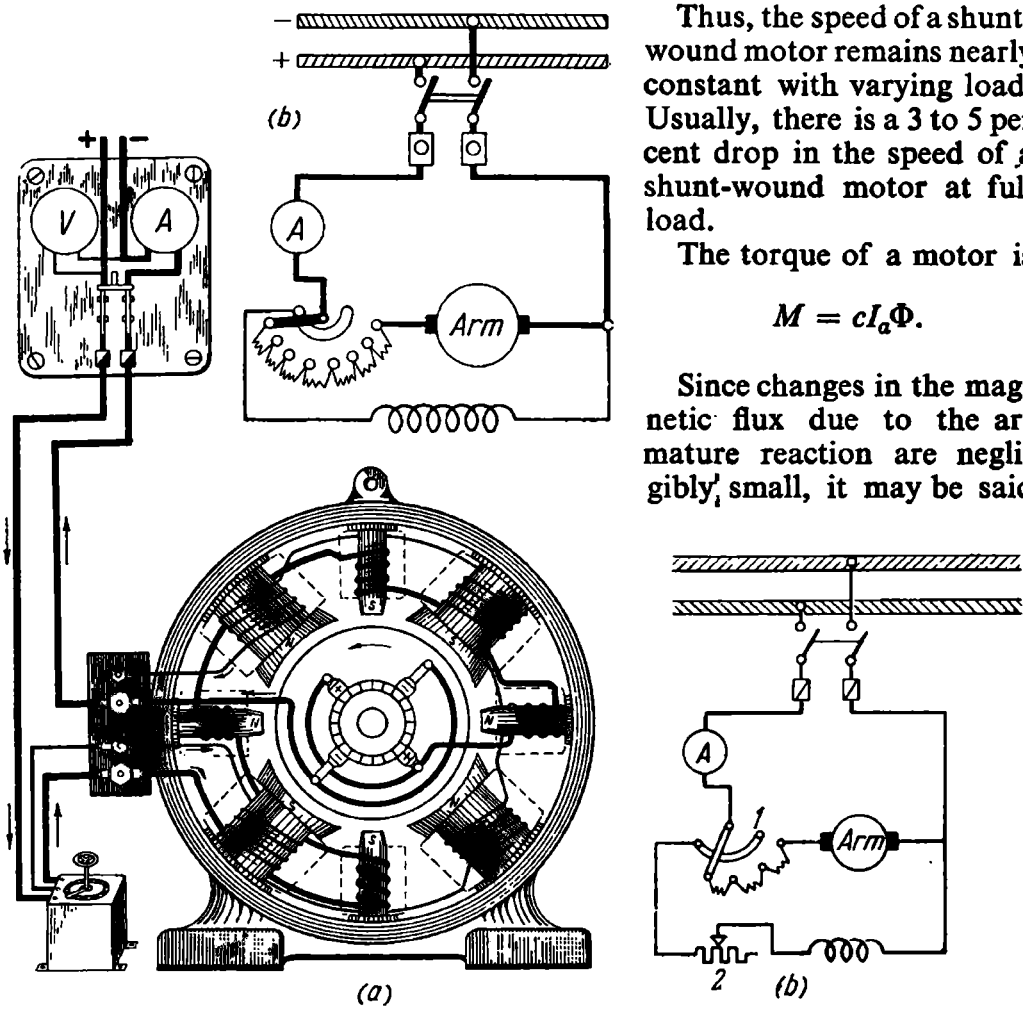


Fig. 295. Shunt-wound motor:

a—construction; *b*—circuit diagram; *1*—starting rheostat; *2*—field rheostat

that the torque of a shunt-wound motor is proportional to the armature current:

$$M = c_1 I_a.$$

Therefore, an ammeter connected in the armature circuit will indicate the load on the motor.

The speed of a shunt-wound motor is usually regulated by varying the magnetic flux by means of a field rheostat. For some motors, the speed can

be regulated in the ratio 1 : 3. The highest speed will be attained at no-load, with the field rheostat fully brought out.

The direction of rotation of a shunt-wound motor can be reversed by reversing the flow of current through the field coils or through the armature winding. As a rule, the second method is preferable, since any break in the field circuit will reduce the magnetic flux, and the resulting emf of self-induction may rupture the winding insulation and burn the contacts, causing the motor to run at a speed dangerous to its mechanical strength.

Shunt-wound motors find application as drives for machine-tools, cableways, pumps, fans, looms, rolling mills, and mine hoists, where constant speed and broad speed regulation are essential.

146. The Series-wound Motor

The series-wound motor is shown diagrammatically in Fig. 296. In this type of motor, the armature winding, field coils and supply line are all connected in series. Therefore, the same current flows through both windings.

At low saturation of the magnetic circuit, the magnetic flux is proportional to the armature current:

$$\Phi = c_1 I_a.$$

Therefore, the torque of the motor will be

$$M = cI_a\Phi = c_2 I_a^2,$$

i.e., the torque of a series-wound motor varies approximately as the square of the armature current. Doubling the armature current will make the torque four times as great. This enables a series-wound motor to build up its torque very rapidly with the load, which is of special importance at starting.

Other things being equal, the series-wound motor will develop a greater torque at the same starting current than the shunt-wound motor, since in the latter the torque is in linear proportion to the armature current.

The speed of a series-wound motor varies sharply with varying load, because any change in the armature current causes the magnetic flux to vary. As follows from

$$n = (V - I_a R_a)/c\Phi,$$

when the applied voltage is kept constant, the speed of the motor is inversely proportional to the magnetic flux. Therefore, a loaded motor will take considerable current from the line, develop a large magnetic flux, and run at a low speed. As the load decreases, the armature current decreases, the magnetic flux is reduced, and the speed of the motor rises. If the load is greatly reduced or thrown off completely, the armature current and the magnetic flux will considerably decrease, and the speed of the motor will increase to a value dangerous to the mechanical strength of the motor: the

motor is said to run away, and so series-wound motors are unsuitable for applications where there is a danger of the motor running light. For the same reason, series-wound motors should not be used with a belt drive, because the motor would run away if the belt broke or slipped off the pulley.

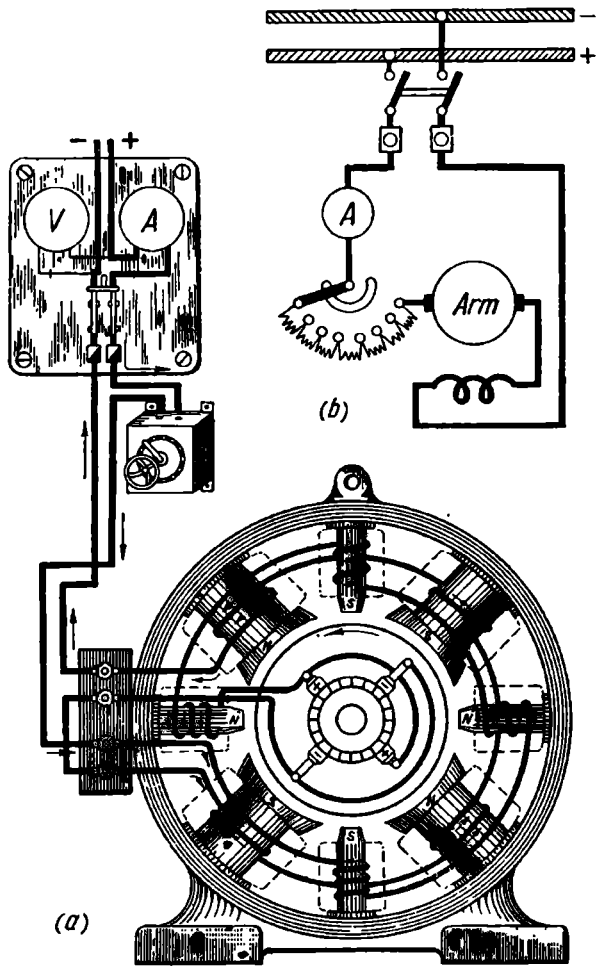


Fig. 296. Series-wound motor:
a—construction; *b*—circuit diagram

The speed of a series-wound motor can be regulated (1) by varying the applied voltage or (2) by changing the magnetic flux of the field poles. In voltage control, a separate rheostat is connected in the motor circuit (in addition to the starting rheostat), or a single rheostat is used which can serve both as starting and adjusting rheostat. This method is not economical, however, since considerable heat is dissipated in the rheostat winding. Speed regulation by field control is carried out by means of a rheostat,

called the diverter, which is connected in parallel with the field coils; varying the resistance of the diverter changes the flux and, consequently, the speed of the motor. An alternative method of speed regulation through field control is to arrange the field coils into several groups and to connect them in series or parallel, as the case may be. Another method of field control requires a field winding with several tappings so that a portion of the winding can be cut out of circuit, thus reducing the flux.

If two series-wound motors are connected mechanically in parallel, they may be connected electrically in series with half voltage across each motor or in parallel with full voltage across each motor, thus giving two speed characteristics. This method is called series-parallel control.

Series-wound motors are suitable for driving hoists and cranes, for traction on railways and underground lines, etc.

147. The Compound-wound Motor

The compound-wound motor is shown diagrammatically in Fig. 297. It has two field windings, one connected shunt and the other series, and so combines the advantages of both the shunt-wound and the series-wound motor: constant speed and high torque at starting. The shunt field winding does not allow the motor to run away at low load or no-load.

If the series winding is arranged to assist the shunt winding (so that their fluxes add), the motor becomes a shunt or a series motor depending on which of the two windings develops a greater mmf. Such units are called cumulatively-compound motors. If the series field is connected so that it tends to produce flux in the opposite direction to that produced by the shunt field, the machine is said to be differentially connected. As the flux of the opposing series field increases, that of the shunt field decreases, and the speed of the motor goes up.

The speed of compound-wound motors is regulated by means of a field rheostat connected in the shunt circuit.

148. Conversion Equipment

The conversion of alternating current to direct current can be effected with: (a) motor-generator sets; (b) synchronous converters; and (c) motor converters. Also, this can be done by rectifiers which are discussed in Chapter 13.

A motor-generator set (Fig. 298) consists of a motor mechanically coupled (by a V-belt or a clutch) to a generator. The motor and the generator are mounted on the same bedplate and on the same shaft usually. Sets converting alternating current to direct current (so-called direct converters) use induction or synchronous motors and externally or self-excited d-c generators.

The efficiency of a motor-generator set can be determined as follows.

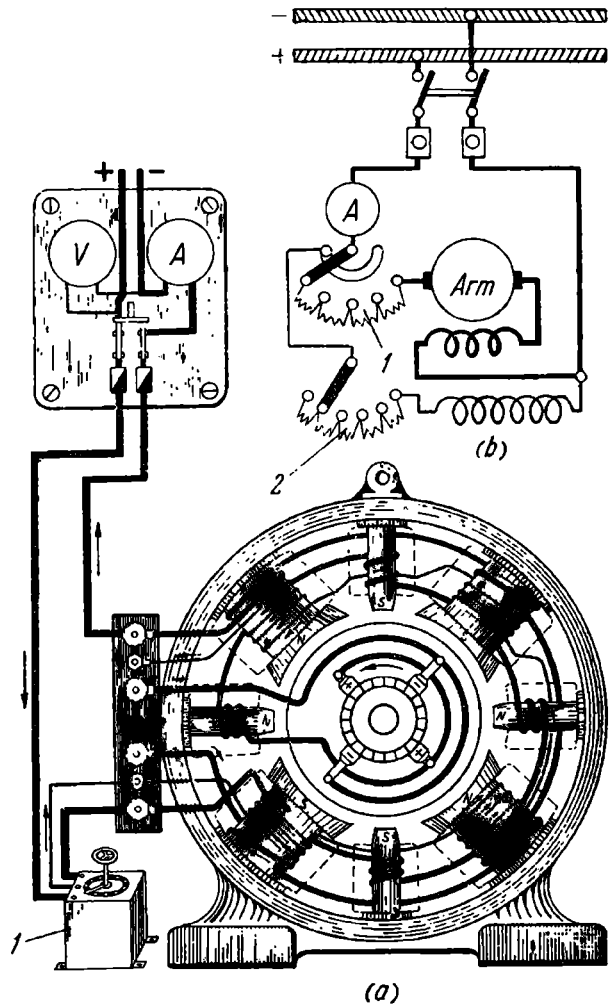


Fig. 297. Compound-wound motor:
a—construction; *b*—circuit diagram; *1*—starting rheostat; *2*—field rheostat

Let P_1 be the input power of the motor and P_2 the output power. Then the efficiency of the motor is

$$\eta_m = P_2/P_1,$$

whence

$$P_1 = P_2/\eta_m.$$

Denoting the input power of the generator by P_3 and the output power by P_4, we have

$$\eta_g = P_4/P_3,$$

whence $P_4 = \eta_g P_3$.

The overall efficiency of a motor-generator set is

$$\eta = P_4/P_1 = \frac{\eta_g P_3 \eta_m}{P_2}.$$

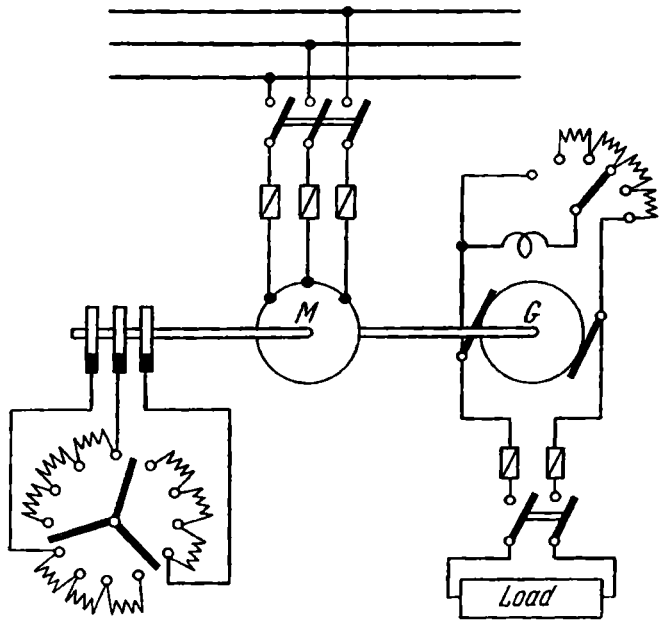


Fig. 298. Diagram of a motor-generator set

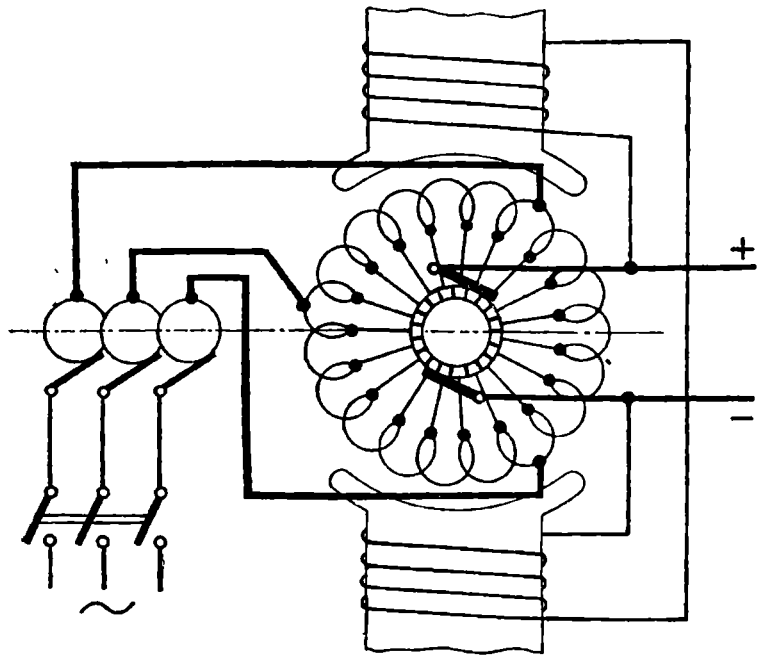


Fig. 299. Synchronous (rotary) converter

If the two units are mounted on a common shaft, it may be assumed that $P_2 = P_3$. Then, finally,

$$\eta = \eta_g \eta_m.$$

If the two units are coupled by a V-belt or any other type of transmission, the efficiency of the transmission will be $\eta_t = P_3/P_2$, and the overall efficiency of the set will be equal to the product of the efficiencies of the motor, transmission and generator:

$$\eta = \eta_m \eta_g \eta_t.$$

In motor-generator sets, the d-c voltage is independent of the a-c voltage.

A synchronous (or rotary) converter (Fig. 299) is generally the same as an ordinary d-c machine and has a stationary field-magnet system and a rotating armature. The armature winding is connected to a commutator, the brushes on which are connected to the d-c terminals. At the opposite end of the armature are slip-rings connected to equidistant tappings on the armature windings, and the brushes on these rings are connected to the a-c terminals. If a-c power is supplied, the converter will run as a synchronous motor, and direct current can then be taken from the d-c terminals (in which case the machine is converting from an a-c supply to d-c). Similarly it can be supplied with d-c power and an alternating current can be taken from the slip-rings, in which case the machine is an inverted converter (or simply an inverter).

The supply given to the a-c side may be one-, three-, six- and more phase. Respectively, there will be two, three, six, or more slip-rings, and the tappings on the armature winding will be spaced 180, 120, 60, etc., degrees apart. Fig. 299 shows a synchronous converter arranged for a three-phase supply. The field is usually self-excited from the d-c side. The magnetic flux produced by the field winding is common to the emfs of the direct and alternating current induced in the synchronous converter. The two currents bear definite relations to each other. Without going into mathematical details, these relationships are as follows:

$V_{ac} = 0.72\ V_{dc}$ for a single-phase supply;

$V_{ac} = 0.62\ V_{dc}$ for a three-phase supply;

$V_{ac} = 0.354\ V_{dc}$ for a six-phase supply.

As can be seen, the d-c voltage is always greater than the a-c voltage.

Six-phase synchronous converters are the most economical, since for the same weight they develop a greater power than single- and three-phase units.

The methods for starting a synchronous converter are (1) from its d-c side; (2) from its a-c side (if polyphase); and (3) by means of an auxiliary motor mounted on its shaft.

Starting from the d-c side is possible if there is an external d-c supply. The converter is started as a d-c motor and is synchronized onto the a-c supply; the main-line switch is closed. Then the starting source is disconnected and the load is put on the converter.

Starting from the a-c side is possible only if the converter is polyphase. The converter is then started as an induction motor by closing the polyphase line switch. The armature winding acts as the primary and the damping windings embedded in the pole faces, as the squirrel-cage winding. The interaction of the revolving field produced by the armature winding and the current induced in the squirrel-cage structure causes the converter to rotate. The field is self-excited. Started as an induction motor, the converter then pulls into synchronism due to the presence of a constant magnetic field, and continues to run as a synchronous motor.

Starting by means of an auxiliary motor does not require any explanation.

A motor converter consists of a wound-rotor induction motor mounted on the same shaft as a rotary converter. The secondary output of the induction motor is fed directly, or cascaded, into the armature of the rotary converter. Hence another name for the machine is the *cascade converter.*

The set runs at half (or some other fraction) of the synchronous speed of the induction motor and the slip energy from the motor rotor is fed to the armature of the rotary converter, where it is converted to direct current. The other part of the energy input to the induction motor is transmitted mechanically to the converter which then acts as a d-c generator.

Review Questions

1. What are the main parts of a d-c machine and what purpose do they serve?
2. Describe the construction of the commutator in d-c generators. What purpose does it serve?
3. What types of armature winding are there?
4. What is armature reaction? What are the effects of the armature reaction upon the operation of a generator (motor)?
5. What is commutation? What are the conditions for ideal commutation?
6. How can the armature reaction be controlled?
7. How can commutation be improved in d-c machines?
8. Describe the separately excited generator, indicating its basic structural features, advantages and disadvantages.
9. Describe the shunt-wound generator in the same way.
10. Same about the series-wound generator.
11. Same about the compound-wound generator.
12. What is the procedure for connecting shunt-wound generators for parallel operation?
13. Why is it that a d-c motor cannot be connected across the line without a starting rheostat?
14. How can the speed of a d-c motor be regulated?
15. Describe the shunt-wound motor, indicating structural features, advantages and disadvantages.
16. Same about the series-wound motor.
17. Same about the compound-wound motor.

Chapter Thirteen

RECTIFIERS

149. General

Direct-current power is frequently necessary for such applications as electrolytic work (galvanoplasty, electroforming, refining of metals), charging of storage batteries, electric traction, and magnetic-field production, to mention but a few.

Direct-current power can be obtained from d-c generators, storage batteries and primary cells. Frequently, however, only a-c power is available. Then it has to be converted into d-c power. This is where rectifiers come into the picture. In effect, in these applications rectifiers have superseded motor-generator sets and motor converters.

The two types of rectifiers that have found wide industrial use are (1) *metal rectifiers* and (2) *mercury-arc rectifiers.* There are also mechanical-contact rectifiers, electronic rectifiers (such as rectifier diodes or kenotrons and gas-filled valves), and electrolytic rectifiers.

The term "*metal rectifiers*" is widely accepted as covering a whole range of dry-contact rectifiers in which rectification takes place at the junction between a conductor and a semiconductor or between two semiconductors of different properties. The area of the rectifying junction is called the barrier layer; hence another name for metal rectifiers is *barrier-layer rectifiers.* Two common types of these rectifiers are the *copper-oxide rectifier* and the *selenium rectifier.*

150. The Copper-oxide Rectifier

The copper-oxide, or rectox, rectifier is made up of a series or parallel combination of unit cells. A diagram of a single cell of the copper-oxide rectifier is shown in Fig. 300. A thin film *2* of cuprous oxide with an outer

layer of cupric oxide is produced by heating a copper disc or plate *1* to a high temperature and quenching it in water. The cupric oxide is then removed, leaving a layer of cuprous (red) oxide on the base copper.

Contact with the oxide surface is made by holding a lead disc *3* against the oxide surface at a definite pressure. The junction between the base copper and the red oxide is the barrier, or blocking, layer a few hundred-thousandths of one millimetre thick.

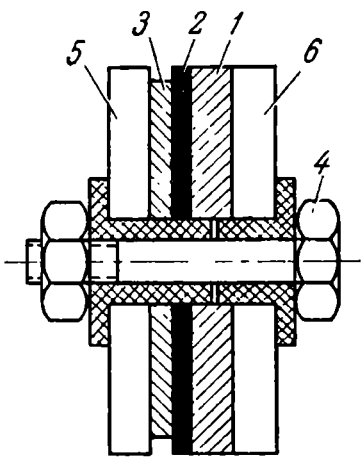


Fig. 300. Diagram of a copper-oxide rectifier cell

Thus, a rectifier cell consists of three layers: the base metal, high in free electrons; the barrier layer having no free electrons; and the semiconductor layer, low in free electrons. When a potential difference is established between the outer layers, a strong electric field is produced in the barrier layer, stimulating the liberation of free electrons from the adjacent layers. When the copper is negative and the oxide positive, a large number of free electrons will be liberated from the copper and an electric current will flow from the oxide to the copper. If the polarity is reversed, very few free electrons will be liberated by the copper, and practically no current will flow. Another way of stating the same thing is that the junction between the copper and oxide offers a high resistance to the passage of current from copper to oxide and a relatively low resistance to the passage of current from oxide to copper. It is this property that makes current rectification possible.

Separate rectifier cells are assembled in contact on an insulated metal spindle *4* and clamped between brass washers 5, 6.

The voltage that may be applied to a single rectifier element ranges between 4 and 8 V. If a higher voltage has to be handled, rectifier elements are connected in series. The maximum current density in the easy, or "forward", direction is 0.1 to 0.15 A/cm^2 at an allowable voltage of 8 V per element. Where greater currents are involved, larger plates are made or individual rectifier elements are connected in parallel.

For heat abstraction, metal washers of large diameter, called radiators, are placed between the rectifier discs.

The service life of copper-oxide rectifiers is 12 to 15 years and their efficiency is 70 per cent.

Copper-oxide rectifiers find use in railway telecommunication systems, radio engineering, battery-charging, and measuring instruments.

151. Selenium Rectifiers

The element selenium (Se) is found in sulphur deposits. Commercially, selenium is obtained from the wastes of sulphuric-acid production and also from the slime of electrolytic copper refining. Under normal conditions, selenium is a solid grey material with specific gravity 4.8 and melting point 220°C. The resistance of selenium varies with illumination—the reason why it is employed in photoelectric devices.

A selenium rectifier element is shown diagrammatically in Fig. 301. It consists of a supporting plate *1* made of nickel-plated iron, upon which is deposited a thin coating of selenium *2*. The second terminal of the cell, known as the counter electrode *3* is then applied. The counter electrode normally takes the form of a sprayed metallic film of a low-melting alloy of bismuth, tin and cadmium. The counter electrode is in contact with a brass washer *4*. The cell is connected into circuit by means of tags *5* and *6*, which are in contact with both electrodes. In contrast to the copper-oxide rectifier, the barrier or blocking layer is formed not between the base and the semiconducting layer, but between the selenium and the counter electrode.

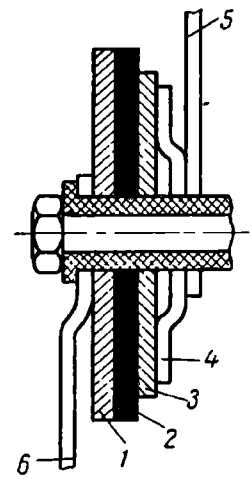


Fig. 301. Diagram of a selenium rectifier cell

The maximum current density for selenium rectifiers is 0.2 to 0.3 A/cm². The voltage per element should not exceed 16-18 V (depending on the diameter of the discs). The efficiency is about 80 per cent.

Selenium rectifiers have the same applications as copper-oxide rectifiers, except in instruments where they cannot be used because of unstable performance.

152. Semiconductor Rectifiers

Semiconductor rectifiers made of germanium and silicon came into wide use after the second world war.

Germanium and silicon rectifiers do not dissipate heat as valve rectifiers do, they are insensitive to shock and vibration; they are simple to manufacture and are compact in size (a few millimetres across); they are reliable in operation and have a long service life. While the efficiency of copper-oxide rectifiers does not exceed 70 to 75 per cent, and that of selenium rectifiers is about 80 per cent, the efficiency of germanium rectifiers is over 95 per cent and the current density is many times as great. Apart from their use as current rectifiers, germanium and silicon diodes serve as amplifiers and generators in radio circuits.

As has been noted elsewhere, the type of conduction of a semiconductor depends on the kind of impurity present. The atoms of donor impurities,

such as phosphorus, arsenic or antimony (with five valence electrons) can replace germanium or silicon atoms in the crystal lattice. Only four of the electrons can then enter the inner atomic bonds; the remaining electron is only lightly bonded to the nucleus and can be broken free. The resulting current flow is known as electron (negative, or *N*-type) conduction.

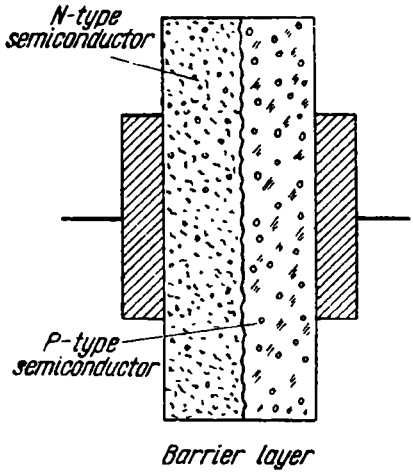


Fig. 302. Operating principle of the germanium (silicon) rectifier

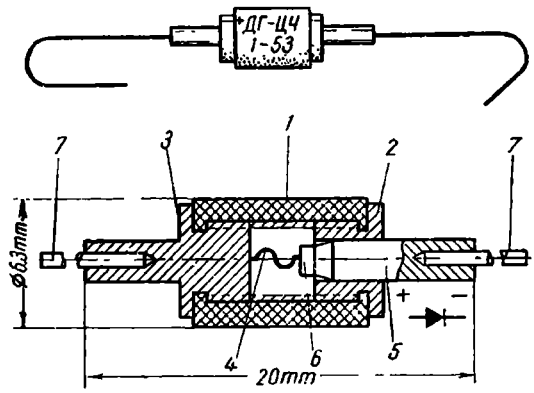


Fig. 303. General view and construction of a germanium rectifier:

1—ceramic container; *2*, *3*—top and bottom electrodes; *4*—cat's whisker; *5*—crystal-holder; *6*—germanium wafer; *7*—lead wires

Acceptor impurities (such as aluminium and indium) have only three valence electrons. When any acceptor atoms are present in the germanium crystal, there is an electron deficiency (hole) in the bond. Electrons can jump from bond to bond into an adjacent hole. This kind of conduction is known as hole (positive, or *P*-type) conduction.

The barrier layer where rectification takes place is formed at the junction of an *N*-type and a *P*-type semiconductor, called a *P-N* junction (Fig. 302). If the *P*-region is made positive with respect to the *N*-region, both holes and electrons are driven together and recombine. This movement constitutes forward current. If, on the other hand, the *P*-region is kept negative with respect to the *N*-region, holes and electrons disperse and leave the junction area devoid of charge carriers. The *P-N* junction is then an insulator which blocks the reverse flow of current.

Large-power semiconductor rectifiers are usually made of silicon. Small-power rectifiers, widely used in radio engineering as diodes, are made of germanium. Fig. 303 shows a germanium diode.

153. Rectifier Circuits

A single-phase alternating current can be rectified (changed into direct current) by using the circuit shown in Fig. 305. Since, owing to unilateral conductivity (or valve action), a rectifier presents a different resistance to the flow of an electric current when the direction of the current is reversed, only the positive half-wave of the current is allowed to pass through the circuit, the negative half-wave being eliminated. The current delivered into the circuit is a series of impulses recurring at half-cycle intervals. This is *half-wave rectification.*

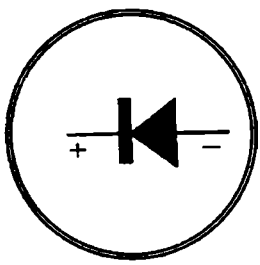


Fig. 304. Symbol for rectifiers

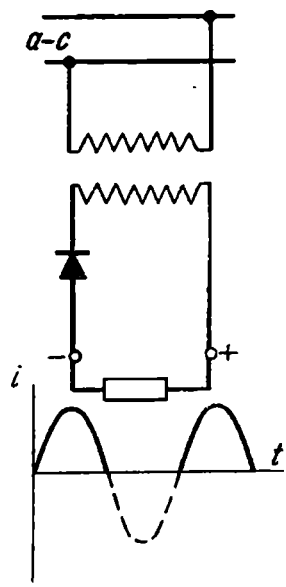


Fig. 305. Diagram and output waveform of a half-wave rectifier for single-phase current

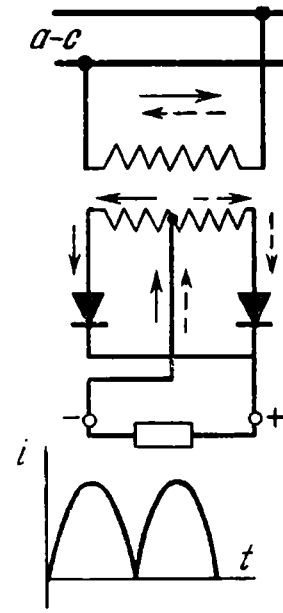


Fig. 306. Diagram and output waveform of a full-wave rectifier for single-phase current

Fig. 306 shows a circuit for the rectification of a single-phase alternating current using a centre tap of the transformer secondary (hence the name of the circuit—a centre-tap rectifier). The solid lines show the flow of current during a positive half-cycle and the dotted lines, during a negative half-cycle. The current in the circuit is a series of impulses, recurring continuously since both the positive and the negative half-waves are utilized. This constitutes *full-wave rectification.*

Fig. 307 shows a bridge circuit consisting of four rectifiers. This is also a full-wave rectification circuit, since it utilizes both halves of the sine wave. The flow of current during a positive half-cycle is shown by the solid lines and during a negative half-cycle, by the dotted lines. Fig. 307*a* shows a rectifier circuit diagram and Fig. 307*b* illustrates the actual connection of the elements.

Fig. 308 gives a circuit for the rectification of a three-phase current.

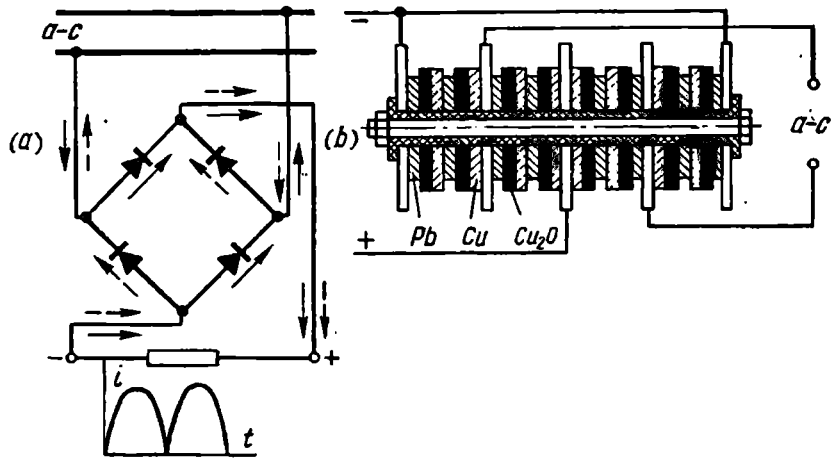


Fig. 307. Four rectifier cells connected in a bridge rectifier circuit:
a—diagram of a full-wave rectifier; *b*—stack of rectifier cells connected in a full-wave bridge circuit

154. The High-vacuum Rectifying Diode (Kenotron)

Fig. 309 shows a glass envelope *1* evacuated to a residual pressure as low as 0.0000001 mm Hg; thus, the space within the envelope may be taken as completely devoid of any gas. Within the envelope are two electrodes, one the cathode *2* located at the bottom, and the other, the anode *3*, usually of nickel or molybdenum, located at the top. The cathode is a filament of a high-melting-point metal (such as tantalum, molybdenum or tungsten) connected across a storage battery *4* or any other d-c source of 4 to 6 V, which heats the filament. The filament and battery make up the cathode circuit.

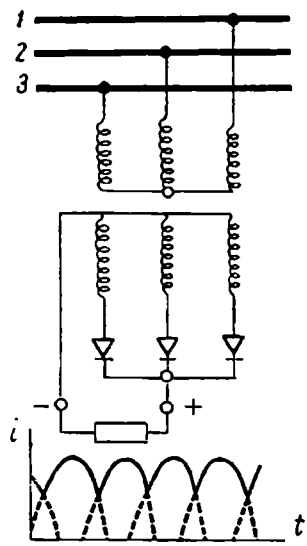


Fig. 308. Diagram and output waveform of a three-phase rectifier

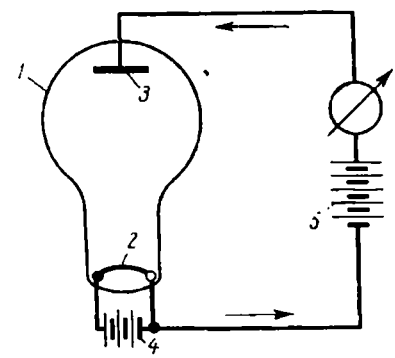


Fig. 309. Diagram showing the operating principle of the kenotron

The heated filament releases electrons due to thermionic emission. Gradually a "cloud" of electrons (a space charge) forms around the cathode, preventing further emission from the cathode surface. If another battery *5* with a voltage of 80 to 100 V is connected so that its plus side makes contact with the anode and the minus side with the cathode, an electric field will be set up between the anode and cathode within the glass envelope. This field will cause the electrons emitted by the cathode to flow towards the anode. In the external circuit, the electrons will flow from anode to cathode, and so a current flow will be established in what is known as the anode circuit. This anode current will increase in value with

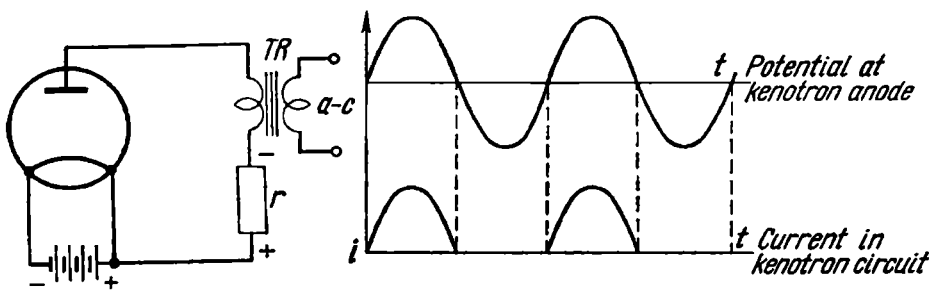


Fig. 310. Connection diagram and output waveform of kenotron

increasing anode potential until a *saturation point* is attained, when the current will remain constant in spite of any increase in the anode potential. This is called *saturation current.*

The number of electrons emitted by a hot cathode depends on the surface area and temperature of the cathode. Cathodes of tungsten should preferably be heated to 2300-2500°C. Higher temperatures take too much power and sharply reduce the service life of the cathode.

The current practice is to make tungsten cathodes with a coating of thorium. These produce a very high saturation current at relatively low temperatures (1600° to 1850°C). There are also oxide cathodes made of nickel as the base metal and an oxide of barium, calcium or strontium as the coating. They operate at 900° to 1200°C.

If the anode is made negative with respect to the cathode, the emitted electrons are repelled, and there is no anode current flowing through the space within the glass envelope. Thus, a current can only flow in one direction through the anode circuit; namely, when the anode is positive with respect to the cathode. This property of unilateral conduction, or valve action, is utilized for rectification of alternating current. The device used for this purpose is called a *kenotron* in the United States or a high-vacuum high-voltage hot-cathode rectifier diode in the British literature.

The kenotron may be connected as shown in Fig. 310. Since only one half of the wave is utilized in this arrangement, it is called a half-wave rectifier. Full-wave rectification is obtained with circuits such as those

shown in Fig. 311, where use is made of two kenotrons, and in Fig. 312, where the circuit contains one kenotron with two anodes (a double diode). In the latter, the cathode takes an alternating current from a special transformer winding. The pulsating d-c waves from the rectifiers shown in the above figures are smoothed out by filters consisting of capacitors and iron-cored coils (chokes).

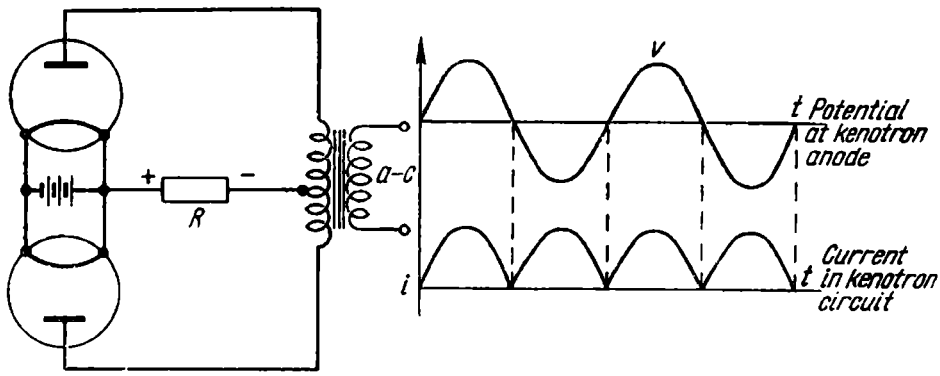


Fig. 311. Connection diagram and output waveform of a double-kenotron rectifier

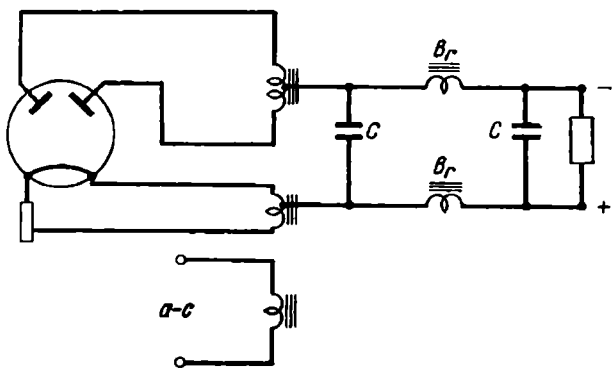


Fig. 312. Diagram of a two-anode kenotron

The pulsations in the output of a rectifier can be considered the resultant of an alternating current superimposed upon a steady direct current. From this viewpoint, the filter may be considered to consist of a shunting capacitor, which shortcircuits the a-c component while not interfering with the flow of the d-c component, and a series choke, which readily passes direct current but impedes the flow of the a-c component. This kind of arrangement is shown in Fig. 312. Thus, the current flowing in the load circuit is in effect a direct current.

In circuits using two kenotrons or a double-anode kenotron, the current at any instant will be conducted through the anode which is positive with respect to the cathode. The centre tap on the transformer winding supply-

ing the anodes is the minus terminal of the d-c circuit. The centre tap on the transformer winding supplying the cathode is the plus terminal of the d-c circuit.

Kenotrons are employed in a-c receivers, cable testers, X-ray equipment, etc.

155. Glass-bulb Mercury-arc Rectifiers

The mercury-arc rectifier (Fig. 313) is a glass vessel *1* with a pool of mercury *2* at the bottom and a graphite block *3* at the top. Wires sealed into the glass serve as terminals for connecting the device into a circuit. The vessel is evacuated and sealed.

Suppose a storage battery *4* is so connected to the device that its plus side makes contact with the graphite block (which is then the anode) and its minus side, with the mercury pool (which thus becomes the cathode). An electric field will then be set up between the anode and cathode.

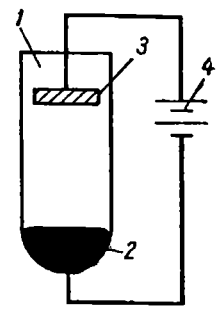


Fig. 313. Diagram of a glass-bulb mercury-arc rectifier

Some of the mercury within the vessel will vaporize even at room temperature, and its vapours will fill the vessel. If by some unspecified means an arc is drawn on the surface of the mercury pool, a small spot on the mercury surface, called the cathode spot, will be raised to incandescence and will emit electrons which will be caused by the electric field to move towards the anode. On their way the emitted electrons will shock-ionize the molecules of mercury vapour to produce positive and negative ions. Ionization produces the characteristic blue luminosity. The electrons and the negative ions of the mercury vapour proceed to the anode, while the positive ions are attracted by the cathode, thereby constituting an electric current. The positive ions driven downwards to the cathode spot bombard it, maintaining it at emission temperature. On coming in contact with the cool walls of the glass vessel, the mercury vapour condenses into drops of mercury which flow down into the mercury pool. This action is not reversible, for if the cathode be made positive and the anode negative, no electron movement can take place since there are none produced at the anode, although there may be an incandescent spot on the cathode surface. To sum up, a mercury-arc device possesses unilateral conductivity, or valve action, for which reason it converts alternating current into direct current.

Fig. 314 shows the graph of a single-phase alternating current. If we assume that the positive half-waves correspond to current flow from anode to cathode, there will be no current flowing through the rectifier during negative half-waves, since the rectifier is then nonconducting. Therefore, the output of a mercury-arc rectifier will be a series of current impulses, or a pulsating current.

To maintain the d-c output reasonably steady, mercury-arc rectifiers are usually fitted with two anodes. Fig. 315 shows a double-anode mercury-arc rectifier connected into an a-c circuit. Such rectifiers are usually manufactured for currents up to 50 A and voltages up to 500 V. The pressure within the glass vessel is reduced to 0.001 to 0.05 mm Hg. A molybdenum rod *2* sealed into the glass is dipped in the mercury pool at the bottom of the vessel. Two graphite anodes *3* are sealed into two arms of the glass vessel. There is also an ignition anode *4* with a small pool of mercury for striking arcs.

In starting up a mercury-arc rectifier, the push-button is pressed in the ignition circuit and the glass vessel is tilted so that the mercury pool of the ignition anode merges with the mercury pool of the cathode. When the vessel is brought back to the normal position, a break is produced between the two bodies of mercury and an arc is drawn on the surface of the cathode spot. The electrons breaking loose from the cathode spot move towards the main anodes. The cathode of the rectifier serves as the plus terminal, and the central tap of the transformer *5* as the minus terminal of the load circuit. The solid lines show the path of current during positive half-cycles and the dotted lines, during negative half-cycles.

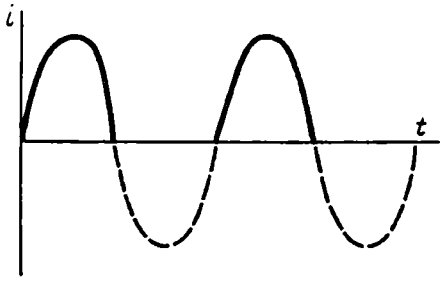


Fig. 314. Output waveform of a mercury-arc rectifier

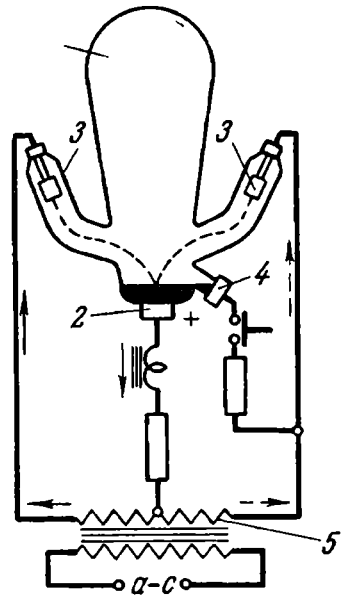


Fig. 315. Connection diagram of a single-phase glass-bulb two-anode mercury-arc rectifier

As can be seen from Fig. 316*a*, there are times when the rectified current drops to zero. In order that the arc may not die out, mercury-arc rectifiers are usually designed with an induction coil to smooth the ripples of the d-c output. The d-c output from a mercury-arc rectifier fitted with an iron-core coil is shown in Fig. 316*b*.

Sometimes, especially at high temperatures, a cathode spot may be formed on the anode, and a current may flow from the cathode to the anode (in the case of a single-anode rectifier) or from the other anodes to

the incandescent anode. In this condition, termed an *arc-back*, (also known as backfire), a mercury-arc rectifier will conduct on both (positive and negative) half-cycles. There are several different causes of arc-backs: poor

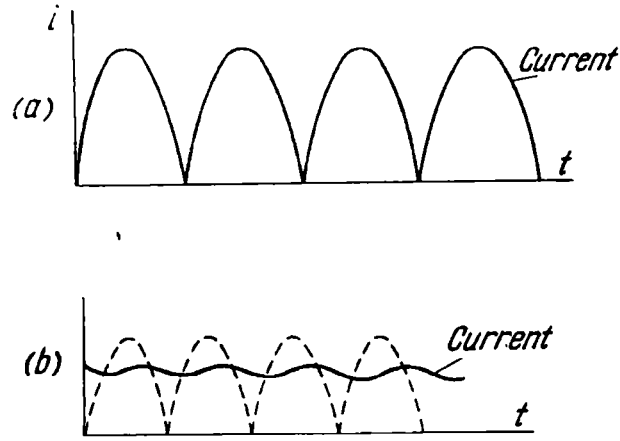


Fig. 316. Output waveforms of a single-phase two-anode mercury-arc rectifier:

a—without a choke; *b*—with a choke

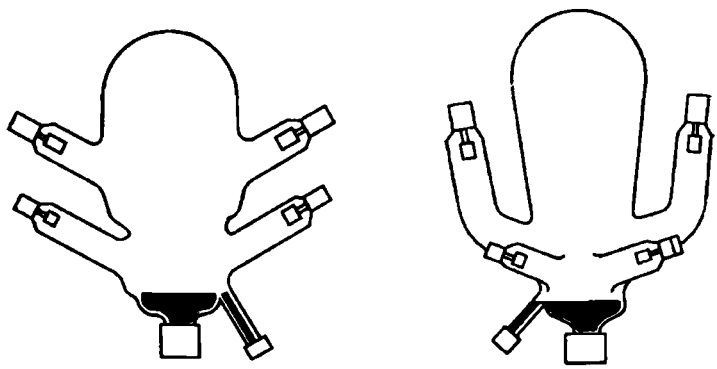


Fig. 317. Four-arm mercury-arc rectifier and separately-excited mercury-arc rectifier

cooling of the rectifier, overload, presence of foreign gases in the glass vessel, etc. An arc-back is actually a short-circuit in the secondary of the supply transformer. Arc-backs are extinguished by suitable circuit-breakers which promptly disconnect the rectifier.

Sometimes glass-bulb mercury-arc rectifiers have four arms (on the left of Fig. 317). They usually supply current at different voltages to two separate circuits.

High-voltage mercury-arc rectifiers have their anodes sealed in long arms. Such rectifiers are manufactured for voltages of 3000 and 10,000 V.

The cathode spot may be initiated every cycle or, once started, maintained even though the load current ceases. In the latter case this is accomplished by keeping a short arc alive between the cathode and, usually, two auxiliary anodes, called "excitation" or "keep-alive" anodes powered from a separate transformer (Fig. 317).

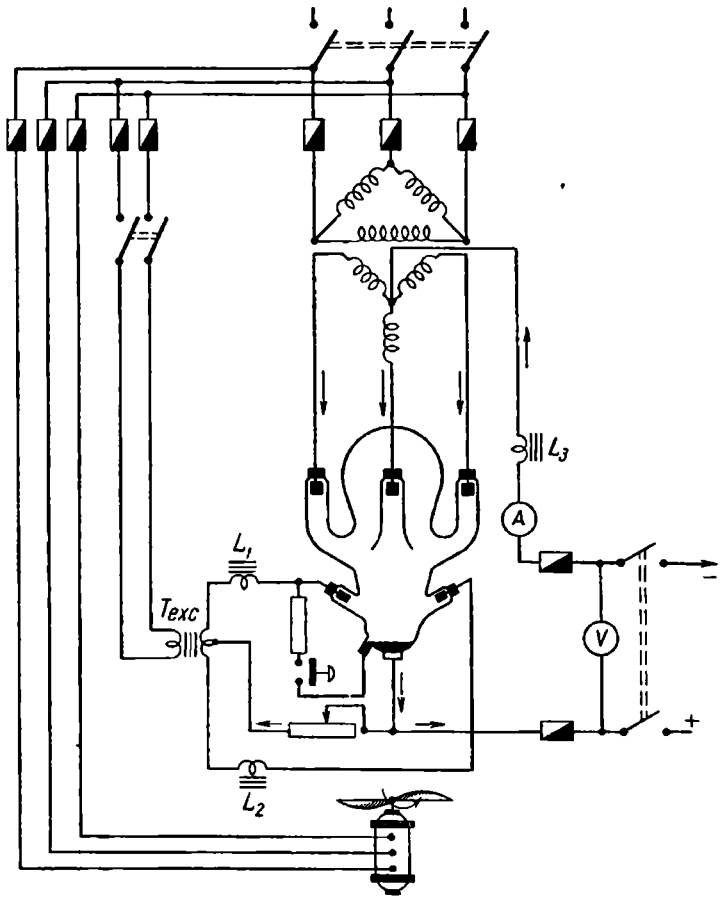


Fig. 318. Connection diagram of a three-phase glass-bulb mercury-arc rectifier

Low-voltage rectifiers for currents from 20 A upwards and high-voltage rectifiers are usually cooled by a propeller type of fan driven by an electric motor.

Three-phase current is converted into direct current by three-phase mercury-arc rectifiers (Fig. 318). The main anodes are connected to a three-phase transformer, the centre tap of which serves as the negative terminal for the d-c circuit, while the cathode is its positive terminal. The excitation anodes are energized by a separate transformer, T_{exc}, via chokes L_1 and

L_2. The resistance limits the current in the ignition circuit. The motor-driven fan is started simultaneously with the transformer.

In the case of a three-phase supply current, the waveforms of the d-c output overlap each other, and the ripples are less noticeable than in single-phase circuits (Fig. 319). The sole purpose of the induction coil connected in the rectifier circuit is to smooth the ripples.

The type designations for Soviet-made rectifiers consist of letters and numerals. The letter B (Russian alphabet) stands for the vessel of the rectifier; the numeral preceding the letter B indicates the number of main anodes. The letter H indicates provision of excitation anodes. The numeral following the letters gives the maximum d-c output in amperes. For high-voltage rectifiers the voltage of the d-c output is given by the numeral following the current rating.

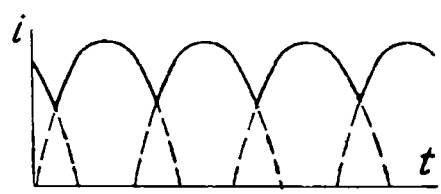


Fig. 319. Output waveform of a three-phase mercury-arc rectifier

Example. "Rectifier Type 2B-12" stands for a glass-bulb two-anode mercury-arc rectifier for a d-c output of 12 A. "Rectifier Type 4B-6" is a glass-bulb four-anode mercury-arc rectifier for a d-c output of 6 A. "Rectifier Type 3BH-6-10000" is a glass-bulb three-anode mercury-arc rectifier with excitation anodes, for a current of 6 A at up to 10 kV.

The efficiency of mercury-arc rectifiers is very high (up to 86 per cent) due to a very low voltage drop across the arc (which ranges between 16 and 25 V depending upon rectifier design).

Glass-bulb mercury-arc rectifiers find use in battery charging equipment, in power supplies for sine projector arc-lamps, d-c power and lighting installations, electrolytic work, control circuits of substation and station equipment, etc.

156. Metal-tank Mercury-arc Rectifiers

For currents over 500 A, use is made of metal-tank mercury-arc rectifiers. Diagrammatically, a metal-tank mercury-arc rectifier is shown in Fig. 320. The metal tank *1* of the rectifier is water-cooled. The mercury pool forming the cathode is contained within a protective cylinder *2*. The main anodes *3* are passed through anode shields *4* which protect the anodes from mercury condensing from the vapour. There is an ignition anode *5* and an excitation anode *6*, also placed inside the tank. The top end of the ignition anode is connected to the steel plunger of a solenoid. When the solenoid is energized, the plunger is pulled in, dipping the ignition anode into the mercury pool for a short while. An opposing spring brings the ignition anode back into the initial position. The arc that strikes between the ignition anode and the mercury pool at this instant jumps over to the excitation anode, which keeps the arc alive.

All the parts of a metal-tank rectifier have vacuum joints. Since operation of a rectifier depends on the degree of vacuum, metal-tank mercury-arc rectifiers are furnished with vacuum pumps.

The output voltage is controlled by selecting the desired tap on a sectionalized transformer or an autotransformer with several taps. This varies the supply voltage and, consequently, the output voltage. There are also mercury-arc rectifiers in which the d-c output is varied by *control grids 7* placed around, and insulated from, the anodes (Fig. 320).

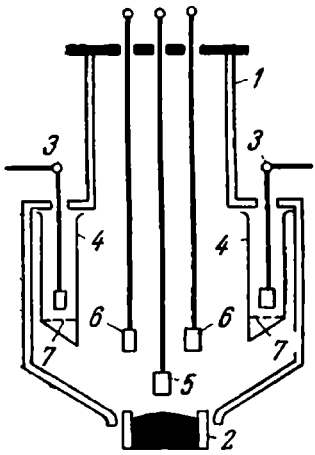


Fig. 320. Diagram of a metal-tank mercury-arc rectifier

Metal-tank mercury-arc rectifiers used in three-phase circuits may have three, six or even twelve main anodes. Polyphase rectifiers draw their a-c voltage from a transformer whose secondary is connected to yield a six- or a twelve-phase current.

Figs. 321*a* and *b* show schematic diagrams of mercury-arc rectifiers operating three- and six-phase.

Where all the anodes are sealed in a common tank, the rectifier is called multi-anode. If, on the other hand, each anode is placed in a separate tank, each unit is termed a single-anode rectifier.

The type designation of Soviet-made metal-tank mercury-arc rectifiers consists of the letter P for "mercury", the letter M for "metal-tank", the letter H for "pump", and the letter B for "water-cooled". The numeral following the letter designation gives the maximum current at 600 V, and the next numeral indicates the number of single-anode units in the rectifier. For example, "PMHB-1000×6" is a metal-tank water-cooled mercury-arc rectifier with a vacuum pump, intended for a current of 1000 A at 600 V and having six units.

Single-anode rectifiers offer the following advantages over multi-anode units: (1) the possibility of using ordinary three-phase transformers in conjunction with bridge rectification instead of special-purpose polyphase transformers; (2) a small voltage drop across the arc; (3) separation of the arcs in isolated vacuum tanks; (4) reduced dimensions; (5) higher efficiency; and (6) better cooling.

In the Soviet Union, tramway, trolleybus and underground traction substations as well as the traction substations of suburban and main line electrified railways use Type PMHB-500×6 rectifiers (Figs. 322 and 323). This rectifier consists of six single-anode units mounted on a common frame. Each unit communicates with a vacuum manifold by means of a vacuum cock. The vacuum within the tanks is maintained at 0.001 to 0.005 mm Hg. Air that gets into the tanks and the gases released in operation are withdrawn by two pumps: a mercury diffusion pump and a fore-vacuum

pump. The mercury pump is connected to the vacuum manifold through which the residual air and gases are fed into a fore-vacuum reservoir, whence they are discharged into the atmosphere by the fore-vacuum pump. The pressure in the rectifier tanks is indicated by mercury-in-glass manometers.

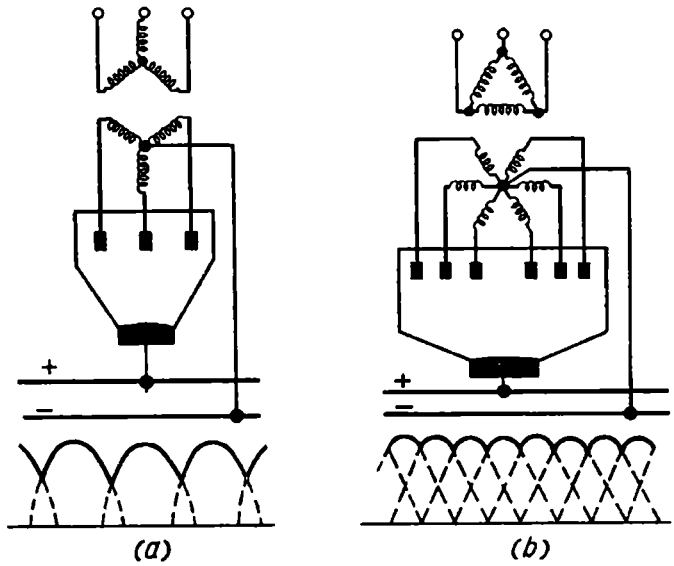


Fig. 321. Circuit diagrams and output waveforms of metal-tank mercury-arc rectifiers:

a—for three-phase circuits; *b*—for six-phase circuits

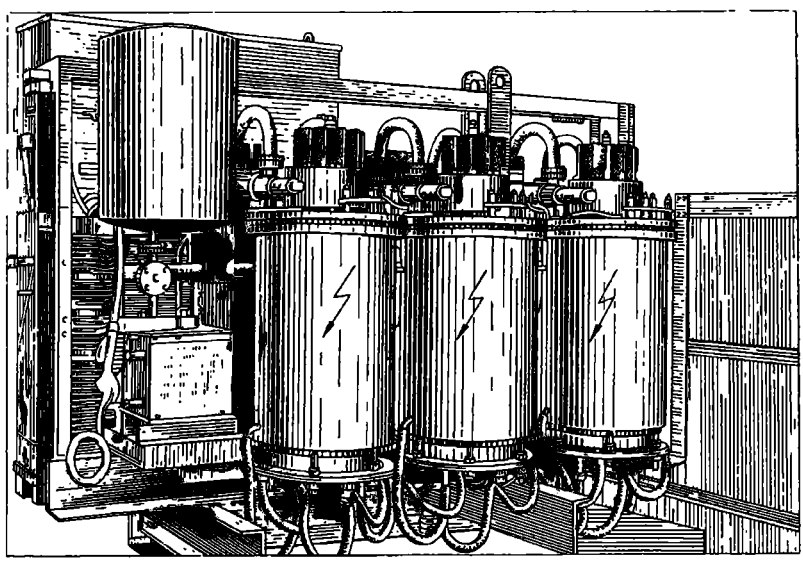


Fig. 322. A bank of single-anode metal-tank mercury-arc rectifiers

The ignition and excitation apparatus as well as supplies for control grids are arranged in a rectifier control cabinet.

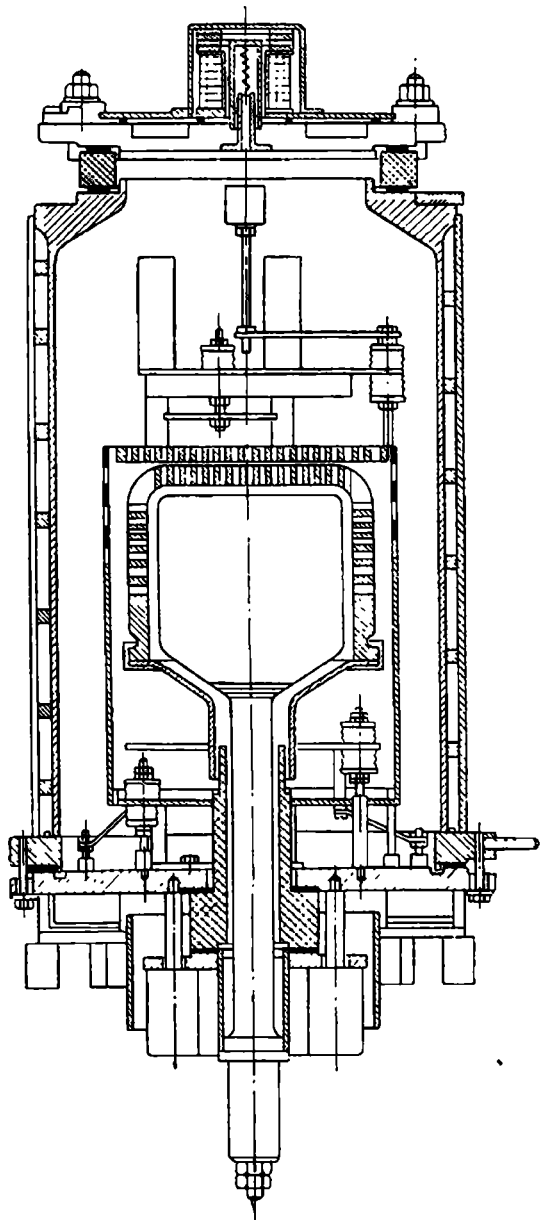


Fig. 323. Section through a Type PMHB-500×6 single-anode metal-tank mercury-arc rectifier

Multi-anode metal-tank rectifiers use an open-circuit water-cooling system. Rubber tubes deliver cold water to the tank; the hot water coming from the tank is discharged into the sewage system.

Single-anode rectifiers use a closed-circuit water-cooling system operating on distilled water. A centrifugal pump forces the water through the system; the hot water enters a coiled-tube heat exchanger where the heat is abstracted by running water. The cooled distilled water is then returned to the system.

Metal-tank mercury-arc rectifiers can supply up to 600 V direct current for tramways and trolleybuses, 750 V for underground railways, 1500 and 3000 V for suburban and mainline trains.

Review Questions

1. What are rectifiers used for? Name the different types.
2. Describe the construction, operating principle and applications of copper-oxide rectifiers?
3. Same about selenium rectifiers.
4. What is the construction of germanium and silicon rectifiers and what advantages do they offer in comparison with copper-oxide and selenium rectifiers?
5. What is the operating principle of mercury-arc rectifiers?
6. How are the pulsations of rectified direct current smoothed out in mercury-arc rectifiers?
7. What is the construction of metal-tank mercury-arc rectifiers and where are they employed?
8. What is the difference between multi- and single-anode metal-arc rectifiers?

Chapter Fourteen

ELECTRICAL MEASUREMENTS AND MEASURING INSTRUMENTS

157. General

Electrical quantities are measured by means of special electrical instruments.

An electrical measuring instrument consists of a moving system and a stationary system. The force necessary to rotate the moving system can be produced by any of the various effects of electric current (thermal, magnetic, or mechanical).

The actuating force is proportional to the electrical quantity under measurement, and so the deflection of the moving system, and of the pointer connected to it, will also be proportional to this quantity. To keep the pointer from swinging off the scale, there must also be a restoring force. This is usually produced by a spring of phosphorus bronze. When the two forces are equal, the moving system (or the movement) is in equilibrium.

Still another force present in any instrument is that of friction. It acts against the actuating force and would distort instrument readings if not reduced. Friction is usually reduced by mounting the movement on pivots which are carried by jewels of high hardness (ruby, sapphire or agate). A diagrammatic representation of a shaft, pivot and jewel is given in Fig. 324.

Some of the instruments are fitted with locks to clamp the movement in transit so as to protect the jewels from mechanical injury.

For one reason or another, the restoring force may change. Thus, the restoring spring may develop different torques as ambient temperature varies, and the pointer will deflect from zero all by itself. As a countermeasure, instruments are fitted with *zero adjusters*. Fig. 325 shows the movement of an instrument equipped with a zero adjuster.

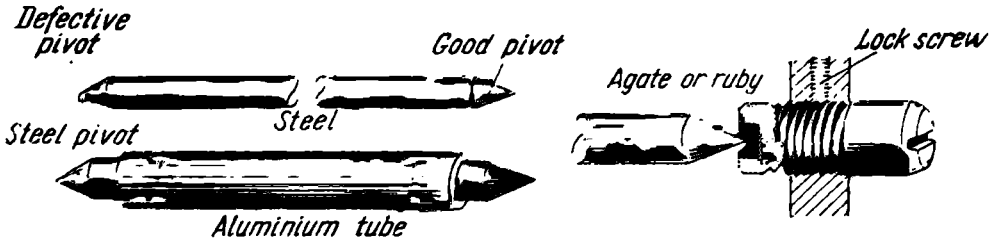


Fig. 324. Shaft and pivots of an electrical instrument

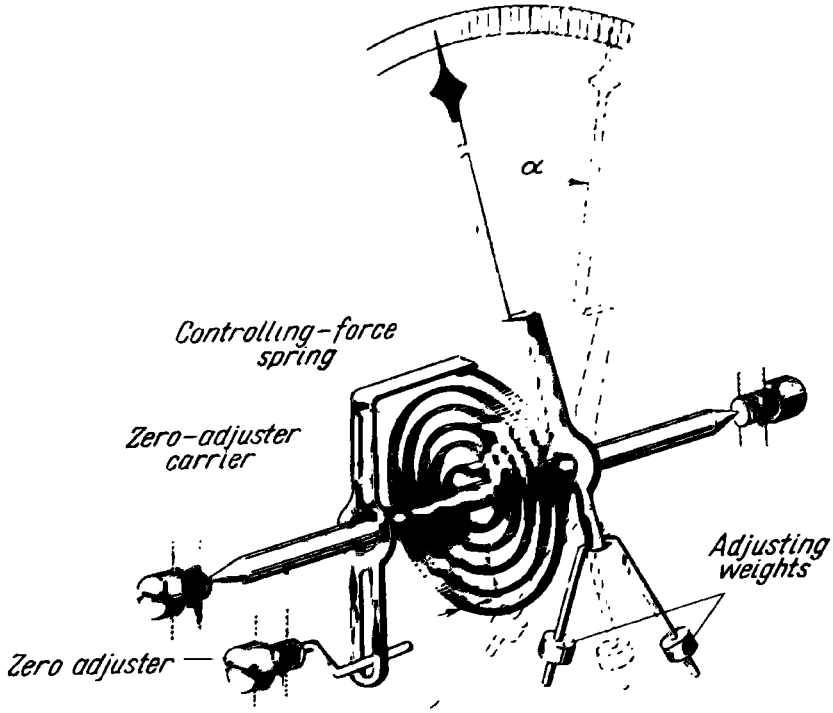


Fig. 325. Diagram of the moving system of an electrical instrument

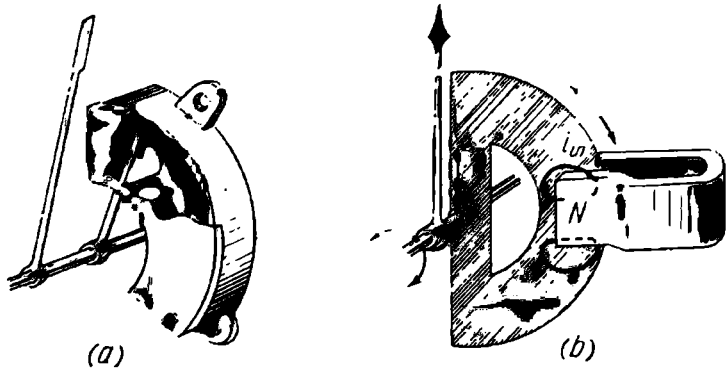


Fig. 326. Air dashpot (a) and eddy-current damper (b)

A good instrument should give a rapid indication. This is obtained by providing a fourth force, called a *damping force*. Damping may be by an air or oil dashpot or by eddy currents induced in a moving conductor. Fig. 326 shows an air dashpot and an eddy-current damper.

The movement of an instrument is enclosed in a case which gives it protection against mechanical injury, ingress of dust, humidity and fumes.

The maximum value indicated on the scale of an instrument is called its *range*. The closeness with which a measurement approaches the true value of the variable being measured is termed *accuracy* of measurement. It is usually expressed in terms of error. When the units are those of the quantity measured, this is an *absolute error* or *absolute deviation*. Thus, the accuracy of an ammeter reading 5 A for a current of 4.9 A (true value) will be $5-4.9 = 0.1$ A.

The ratio of the absolute deviation or error to the true value of the measured quantity gives the *fractional deviation* or *error*:

$$0.1 \div 4.9 = 0.02.$$

When expressed in per cent, it becomes the percentage deviation or error (0.02=2 per cent).

The accuracy of an instrument (as distinct from the accuracy of measurement) is expressed by the ratio of the absolute deviation or error to the range (full-scale value) of the instrument and is termed the limit of error (or accuracy limit). A relevant U.S.S.R. State Standard also specifies a *standard accuracy limit*: the limit of error determined under standard or normal conditions (with the instrument in the normal position, at 20°C in the absence of external or stray magnetic fields and large ferromagnetic masses). Classification of instruments by accuracy limits has become universal practice.

Instruments differ in sensitivity, which is the ratio of the response of an instrument to a change in the measured variable.

A good instrument should draw little power from the circuit under measurement, or else it would affect the operation of the circuit and yield erroneous results.

The principal considerations that govern the choice of an instrument are: (1) reliability; (2) simplicity of operation; (3) accuracy; (4) power consumption; (5) shape; (6) weight; (7) cost.

158. Classification of Instruments

Electrical instruments can be classed in various ways as follows:

(1) According to the quantity being measured.
(2) According to the kind of current.
(3) According to accuracy limits.
(4) According to the principle of operation.

(5) According to the type of indication.
(6) According to application.

They can be further subdivided:

(a) according to type of mounting;
(b) according to protection against stray magnetic or electric fields;
(c) according to overload capacity;
(d) according to the working temperature range;
(e) according to size, etc.

(1) *According to the quantity being measured,* instruments are classed into *ammeters* for measuring the magnitude of the current; *voltmeters* for measuring voltages; *ohmmeters* and *resistance bridges* for measuring resistances; *wattmeters* for power measurements; *watthour-meters* for energy measurements; *frequency meters* for frequency measurements; and *power-factor meters* for power-factor measurements.

(2) *According to the kind of current,* instruments are classed into d-c, a-c and a-c/d-c instruments.

(3) *According to accuracy limits,* instruments are divided in the U.S.S.R. into seven classes: 0.1; 0.2; 0.5; 1; 1.5; 2.5; and 4, the numerals giving limits of accuracy in per cent of the full-scale value.

(4) *According to the principle of operation,* instruments are grouped into moving-coil, moving-iron, electro-dynamic, induction, hot-wire, thermoelectric, and rectifier types.

(5) *According to the type of indication,* instruments may be indicating and recording.

(6) *According to application,* instruments are classed into switchboard and portable.

159. Moving-coil Instruments

For its operation, a moving-coil instrument depends on the interaction of a current-carrying coil and a permanent-magnet field. Diagrammatically, this type of instrument is shown in Fig. 327. The magnetic field is produced by a strong horseshoe-shaped permanent magnet *1* made of cobalt, tungsten or nickel-aluminium steel. The magnet is furnished with pole pieces *2* of soft iron, shaped so as to give a uniform field. A soft iron core *3* is mounted in the central space enclosed by the pole pieces. The core reduces the reluctance of the magnetic circuit and helps to produce a uniform magnetic field. The core is embraced by a coil of fine insulated copper wire *5* wound on a light-weight aluminium frame *4*. The coil is carried by a shaft *6* pivoted in jewelled bearings *7*. The pointer *8* is also attached to the shaft. The current is introduced into the coil by two spiral springs *9*, which also provide the restoring (or controlling) force. The springs are connected by conductors to terminals brought out of the case.

When the coil is energized, a mechanical force acts upon it in a direction which can be determined by the left-hand rule. This force causes the coil to rotate. The actuating force is determined by the magnetic induction B, the current I through the coil, the number of turns, T in the coil, and the effective length, l, of the coil:

$$F = BIlT.$$

When the coil is deflected, an emf is induced in the aluminium frame opposing the deflection of the coil by Lenz's Law and acting as a damper on the coil. Since the magnetic field is uniform throughout the space occupied by the coil, the deflection is proportional to the current through the coil, and a linear, i.e., evenly divided scale, may be used.

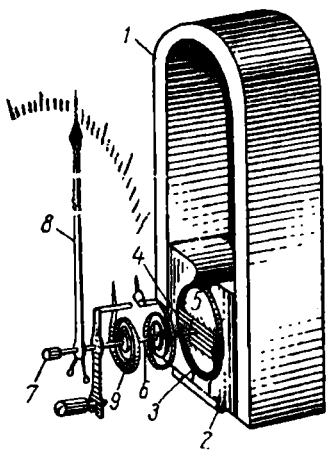


Fig. 327. Diagram of a moving-coil instrument

Moving-coil instruments are the most sensitive and accurate of all existing direct-reading instruments. They are available as laboratory multi-range units of accuracy classes 0.1 and 0.5 and also as commercial units of accuracy classes 1 and 1.5.

The advantages of moving-coil instruments are high accuracy, high sensitivity, a linear scale, low power-consumption (0.5 to 3 watts), insensitivity to external or stray magnetic fields, and good damping. This is why they have found wide use as ammeters, voltmeters, milliammeters, millivoltmeters, etc., for use in d-c circuits.

The disadvantages of moving-coil instruments are high cost, low overload capacity, and applicability to direct current only. If energized by an alternating current, the coil would tend to oscillate at the line frequency. But owing to the high inertia of the moving system, the pointer would remain stationary, since the frequency of the current is very high.

Small charges, low values of current and voltage are usually measured with moving-coil galvanometers, especially where measurements are taken by the null method in order to detect the presence, or otherwise, of a current in some portion of a circuit. In order to reduce the weight of the moving system and friction in bearings, the coil is wound with fine copper wire and impregnated with varnish for strength since it has no supporting frame. Low-sensitivity galvanometers have their coils pivoted in jewelled bearings. High-sensitivity galvanometers have their coils suspended by flat phosphor-bronze strips or helixes, which also conduct current to and out of the coil. The coils of high-sensitivity (reflecting-type) galvanometers carry mirrors. A suitable light source throws a beam of light upon the mirror, which reflects it to the scale set up a certain distance from the galvanometer.

Galvanometers can measure currents from 10^{-6} to 10^{-12} A directly.

160. Moving-iron Instruments

Moving-iron instruments utilize the reaction between the current of a fixed coil and the magnetic field of a moving, temporarily magnetized piece of soft iron or any other ferromagnetic material. The fixed coil may be either of the flat type or of the round type. A flat-type coil instrument, also known as the attraction type, is shown in Fig. 328.

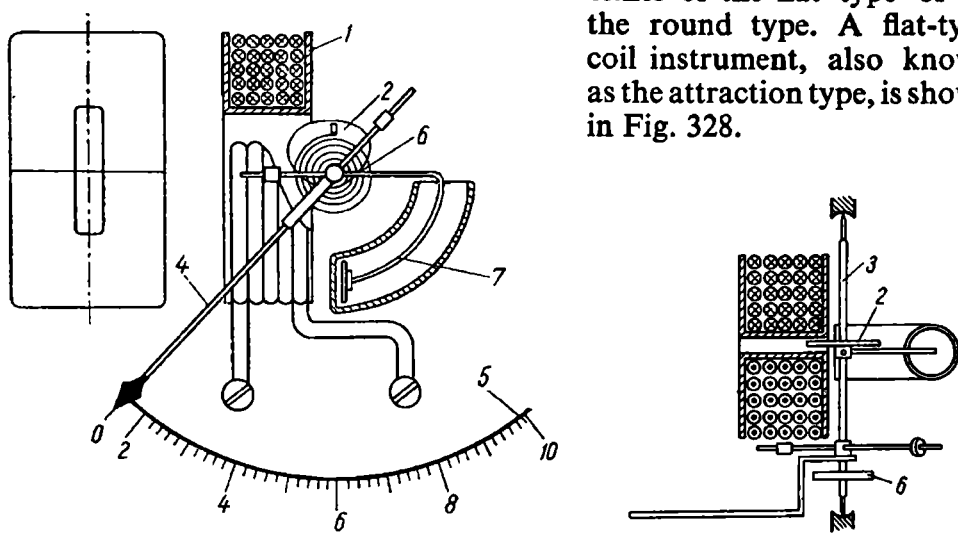


Fig. 328. Diagram of an attraction-type moving-iron instrument

The flat coil *1* of a voltmeter has a slot and is wound with a large number (2000 to 10,000) of turns of fine copper wire (0.05 to 0.1 mm in diameter). In ammeters for currents up to 30 A the coil is wound with a small number of turns of heavy wire. In ammeters for currents up to 200 A the coil is wound with copper strip, while for currents in the range 300-500 A the coil is simply a single turn of flat copper conductor.

The temporarily magnetized piece of ferromagnetic material (usually permalloy), or the vane, *2*, is pivoted eccentrically on the shaft *3* of the instrument. When the coil is energized the vane is attracted into the slot of the coil, the attraction increasing with increasing current through the coil. The pointer *4* mounted on the shaft sweeps over the scale *5*. The restoring, or controlling, force is provided by a spiral spring *6* connected to the shaft at one end and to the stationary part of the instrument at the other. Damping is by air dashpots *7*. The piston of a dashpot, made fast to the shaft, travels in a curved cylinder against the resistance of air, and thus the pointer is brought to a standstill after a few oscillations. The actuating force is applied as a torque proportional to the square of the current flowing through the coil. For this reason, moving-iron instruments usually have nonlinear (square-law) scales.

A round-coil instrument, usually called the repulsion type, is diagrammatically shown in Fig. 329. It has two iron vanes, one, *2*, fixed inside the

coil *1* and the other, *3*, pivoted on the coil shaft. The near tips of the vanes are magnetized similarly and repel each other, rotating the coil shaft.

The effect of stray magnetic fields upon the operation of moving-iron instruments is minimized by an iron housing. Moving-iron instruments are used in both d-c and a-c circuits, since reversal of the current through the coil does not affect the reaction between the two vanes. In the latter case, root-mean-square values of current and voltage are indicated. Power consumption in moving-iron ammeters is 2 to 8 watts, while in voltmeters it is 5 or 6 watts. The advantages of moving-iron instruments are simple design, low cost, high overload capacity, and applicability to both direct and alternating current. Their disadvantages are low accuracy, a nonlinear scale, and susceptibility to stray magnetic fields and frequency variations.

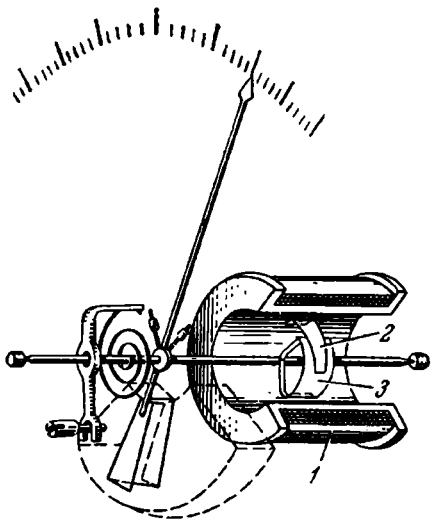


Fig. 329. Diagram of a repulsion-type moving-iron instrument

Moving-iron instruments are usually made as commercial switchboard units of accuracy classes 1, 1.5 and 2.5. The scale of moving-iron instruments can be made to approach a linear scale by suitably shaping and positioning the vanes with respect to the coil.

161. Electrodynamic Instruments

The operating principle of electrodynamic instruments is the interaction between the current in a movable coil mounted on a shaft and a fixed coil.

Diagrammatically, the construction of an electrodynamic instrument is shown in Fig. 330. At *1* is the fixed coil consisting of two halves; at *2* is the movable coil mounted on a shaft *3*. Current to the movable coil is conducted by spiral springs *4*, which at the same time provide the controlling force. When the two coils are energized, their magnetic field will interact, and the resulting torque will tend to rotate the movable coil so that its field coincides in direction with that of the fixed coil. The magnetic field in each coil is determined by the current, and so the torque will be proportional to the square of the current. This is why electrodynamic instruments use nonlinear, or square-law, scales. Damping is by an air dashpot, since magnetic damping would affect the accuracy of the instrument.

Electrodynamic instruments are equally applicable to direct and alternating currents, since reversal of current through the coils will not affect

their interaction and the resulting torque. In a-c circuits, electrodynamic instruments read rms values of current and voltage. Owing to this, high accuracy (class 0.2 to 0.5) electrodynamic instruments are widelyused as reference instruments in a-c measurements. They suffer, however, from high power consumption (5 to 10 watts in ammeters and 7 to 15 watts in voltmeters), low overload capacity, and susceptibility to stray magnetic fields for the reason that their own magnetic fields are fairly weak. In the case of an overload, the spiral springs may lose elasticity or burn.

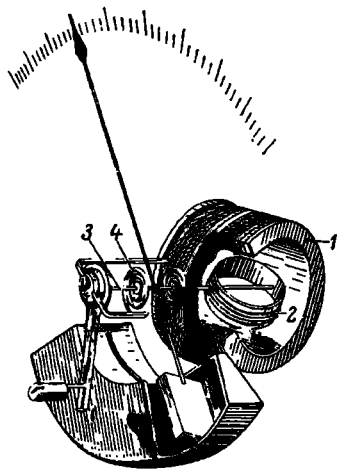


Fig. 330. Diagram of an electrodynamic instrument

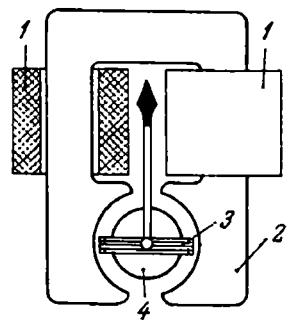


Fig. 331. Diagram of an iron-cored electrodynamic instrument

Of late, iron-cored electrodynamic instruments have been developed (Fig. 331). The fixed coil *1* is wound on a yoke *2* built up of silicon-steel punchings; inside the movable coil *3* is a steel core *4*. The core builds up the magnetic field of the instrument, thereby increasing the torque and minimizing the effect of stray magnetic fields. On the other hand, the presence of an iron core involves additional losses due to hysteresis and eddy currents in a-c measurements. Therefore, the accuracy of iron-cored electrodynamic instruments is lower than that of air-cored units. Possessing a high torque, electrodynamic instruments are often employed in recorders, in which case the pointer is fitted with a pen which traces out curves of current, voltage, etc., on a paper chart moved by clockwork.

162. Hot-wire Instruments

For their operation, hot-wire instruments depend on the elongation of a current-carrying conductor; this motion is then transformed into the rotary motion of the moving system. A hot-wire instrument is shown diagrammatically in Fig. 332. In this instrument, the circuit current passes through a wire *1* of a platinum-iridium or platinum-silver alloy 100 to 160 cm long and 0.03 to 0.05 mm in diameter, which is stretched between points

A_1 and A_2. At the point *B*, this metal wire is kept taut by another wire *2* anchored at *C*. At *D* the cross-wire is pulled by a silk thread *3*, which is passed around a small pulley *4* attached to a shaft carrying a pointer. The other end of the silk thread is made fast to a steel leaf spring *5*. The circuit current heats the platinum-iridium wire (to 300°C in some instruments), and the wire elongates, pulling the cross-wire, silk thread and leaf spring. The dotted lines in Fig. 332 show the position of the instrument parts when the platinum-iridium wire is hot. The quantity of heat produced by a current is proportional to the square of the current and the resistance of the wire (I^2R). Neglecting changes in the resistance of the wire due to heating, the expansion of the wire may be taken proportional to the square of the current, for which reason hot-wire instruments use a square-law scale. This type of instrument can be equally used in d-c and a-c circuits. In a-c circuits it will read true rms values for any waveforms.

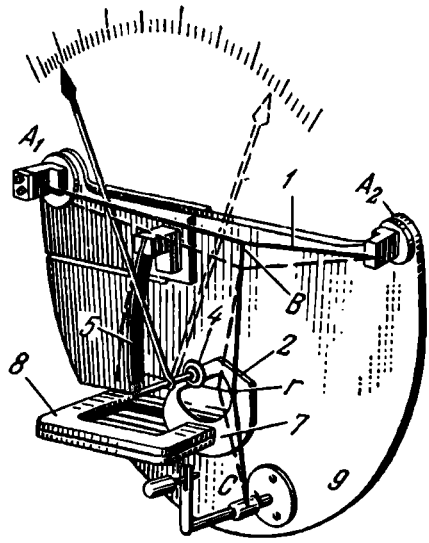


Fig. 332. Diagram of a hot-wire instrument

Since hot-wire instruments do not utilize magnetic fields, stray magnetic fields have no effect on their operation. The absence of any iron and the low inductance of the wire make their readings independent of variations in current frequency within a broad range. Damping in most cases is magnetic. To this end an aluminium vane *7* is mounted on the shaft of the instrument between the tines of a strong U-shaped permanent magnet *8*. When the shaft rotates, the aluminium vane cuts across the magnetic lines and eddy currents are induced in it. The eddy currents react with the magnetic field of the magnet, and the moving system is quickly damped.

Of the disadvantages of hot-wire instruments, the most important is the dependence of their readings on variations in ambient temperature. Temperature errors are corrected by mounting the hot wire on a base-plate *9* having a coefficient of expansion very close to that of the wire, so that both expand or contract (under variations in ambient temperature) by the same amount. There are also other methods of reducing temperature errors. Hot-wire instruments also have a low overload capacity. In the case of an overload the hot wire may either burn or acquire a permanent set, and the scale will have to be recalibrated.

The power consumption of hot-wire instruments is very high, reaching 1 to 3 watts in ammeters for 5 A and 20 to 30 watts in voltmeters for 150 V.

The accuracy is low, the errors being 1.5 to 2 per cent. As a result, this type has now largerly been replaced by thermoelectric instruments fitted with air or vacuum transducers.

163. Induction Instruments

Induction instruments operate on the principle that an alternating field, which may

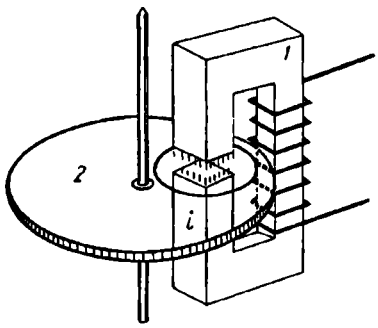


Fig. 333. Eddy currents induced in a disc by an alternating magnetic field

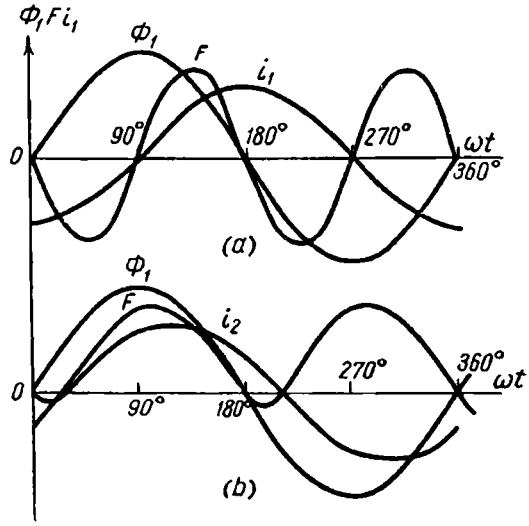


Fig. 334. Waveforms of magnetic fluxes and eddy currents induced in a rotating disc

be either rotating or travelling, induces currents in the moving system, thereby causing it to rotate. The operating principle of induction instruments is illustrated in Fig. 333.

Fig. 335 is a diagrammatic representation of an induction instrument utilizing a travelling magnetic field. In this type, the magnetic circuit *1* carries a coil *2* wound with a large number of turns of fine wire. For the most part the magnetic flux of this coil links through a magnetic shunt *3*, while the balance threads an aluminium disc *4*. Placed beneath the disc is another U-shaped magnetic circuit *5*, which carries a coil *6* divided into two parts and wound with a few turns of heavy wire. The magnetic flux of this coil threads through the disc twice. The two magnetic fluxes, out of phase with each other, induce eddy currents in the disc. The eddy currents interact with the fluxes to produce a torque which rotates the disc. The disc is damped by an U-shaped magnet *7*.

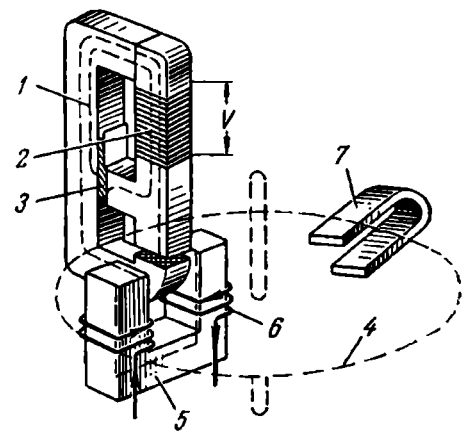


Fig. 335. Diagram of a travelling-field induction instrument

Fig. 336 gives a diagrammatic representation of an induction instrument using a revolving magnetic field. The magnetic circuit *1*, assembled of silicon-steel punchings, carries two windings. One of them, *2*, is carried by a pair of opposing pole pieces, and the other, *3*, is wound on the other pair of opposing pole pieces. The space enclosed by the magnetic circuit is occupied by an aluminium cylinder *4* mounted on a shaft carrying a pointer *5* and a spiral spring *6*. The aluminium cylinder houses a steel core *7* that serves to reduce the reluctance of the magnetic circuit.

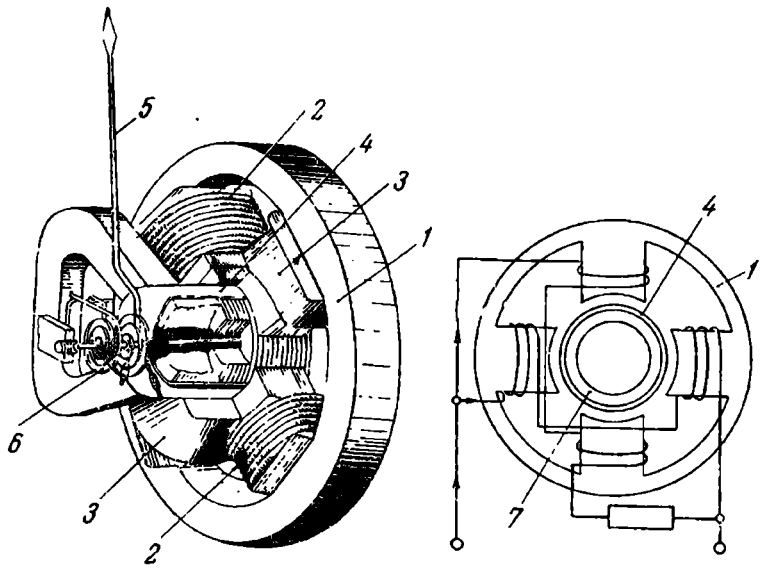


Fig. 336. Diagram of a revolving-field induction instrument

When the two windings, *2* and *3*, are energized, they produce two magnetic fluxes. To obtain the largest possible torque, a phase displacement of 90° is provided between the two fluxes by winding one pair of the coils with a few turns of heavy wire. This winding is a resistance, and the current flowing through it will be in phase with the voltage. The other pair of coils, wound with a large number of turns of fine wire, has a high inductive reactance, and the current flowing through it is nearly in quadrature with the voltage. The requisite phase difference between the two magnetic fluxes can also be obtained by inserting suitable resistances and inductances in the instrument circuit.

The magnetic field rotating in the air gap relative to the moving system will induce eddy currents in the aluminium cylinder. The eddy currents will react with the field to produce a torque which will cause the cylinder to rotate with the field. In effect, induction instruments operate on the same principle as two-phase induction squirrel-cage motors.

Damping is by eddy currents induced in the top of the aluminium cylinder when it moves in the field of two permanent magnets (one of them is

omitted in the drawing). Stray magnetic fields do not affect operation of induction instruments since they possess a strong magnetic field of their own. Other advantages of this type of instrument are robustness, high overload capacity, and reliability. Their disadvantages are applicability to a-c work only, a square-law scale, temperature and frequency errors, and low accuracy (the accuracy limit is 1 to 1.5 per cent). Power consumption is from 2 to 4 watts.

It should be noted that induction wattmeters have now given way to iron-cored electrodynamic wattmeters.

164. Thermocouple Instruments

For their operation, thermocouple (or thermo-emf) instruments depend on the emf generated at a junction of two dissimilar metals when the junction is maintained at a temperature differing from that of the remaining circuit.

A diagram of a thermocouple instrument is shown in Fig. 337. The current to be measured is passed through a heater *1* on which are soldered or welded two dissimilar conductors *2*, say an iron and a constantan wire. The other ends of the conductors are made fast to metal blocks *3* of good thermal conductivity. The blocks receive connections from a moving-coil measuring instrument *4*.

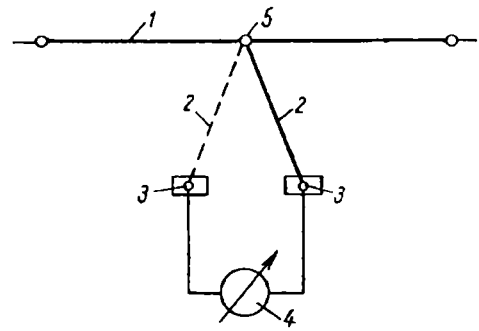


Fig. 337. Diagram of a thermocouple (thermal emf) instrument

When current flows through the heater, it is heated, as is the junction of the two dissimilar conductors (at *5* in Fig. 337). This is the hot junction of the thermocouple; the metal blocks *3* make up the cold junction. The difference in temperature between the two junctions gives rise to a thermo-emf. When the circuit is closed, a current is established, which flows around the circuit in one and the same direction, irrespective of the direction of the current being measured.

By Joule's Law, the quantity of heat generated at the hot junction of the thermocouple is proportional to the square of the current. Therefore thermocouple-type instruments use a square-law scale. To make the scale linear, the magnetic field of the associated moving-coil instrument is made nonuniform.

The thermo-emf of a single thermocouple does not usually exceed 15 mV, and so a very sensitive moving-coil instrument has to be employed. To improve this, several thermocouples are connected in series, making up a thermopile.

For higher sensitivity, the thermocouple may be placed in a vacuum.

Thermocouple-type instruments are very sensitive to overloads: even a momentary overload of 10 per cent can melt the heater. The accuracy is fairly high, and instruments can be manufactured in accuracy classes 0.5 and 1. Thermocouple-type instruments are especially employed in radio-frequency circuits for low ranges of milliamperes.

165. Rectifier Instruments

A rectifier instrument is a combination of a moving-coil instrument and one or several rectifiers, usually of the copper-oxide type. The connections are as shown in Fig. 338. Rectifier instruments are used to measure low

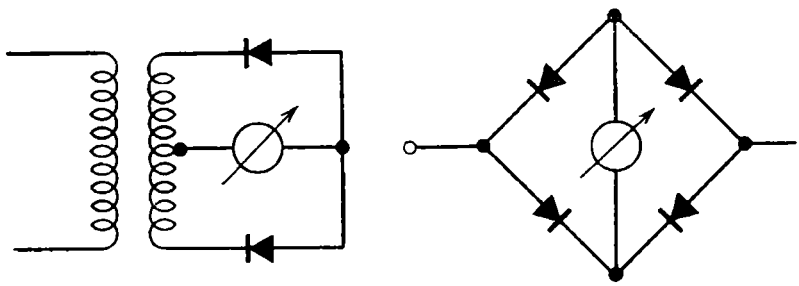


Fig. 338. Connection of a rectifier to a moving-coil instrument

ranges of alternating current and a-c voltage (from a few tenths of a milliampere and a few tenths of a volt upwards), and also in circuits operating at frequencies in the range 50-2000 c/s. There are also multi-range a-c/d-c voltammeters of the rectifier type. Their accuracy is rather low (they are available in accuracy class 2.5).

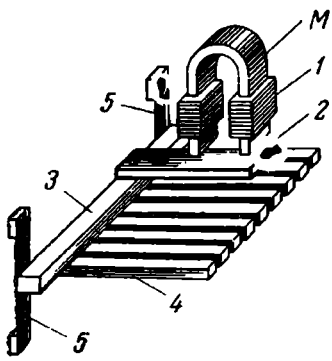


Fig. 339. Diagram of a frequency meter

166. Vibration Instruments

At present, vibration-type instruments are only made for frequency measurements. A diagrammatic representation of such a frequency meter is shown in Fig. 339.

In this instrument, an electromagnet *1* energized from the a-c source to be measured is placed above a steel armature *2* fastened to a metal bar *3*. On the bar there are numerous steel strips, or reeds *4*, of different lengths, each rigidly fastened at one end and free to vibrate at the other. The strips are adjusted, or tuned, to different natural periods of vibration. The bar carrying the reeds is screwed to leaf springs *5*. The ends of the reeds are

turned up and painted white. When the electromagnet is energized, an alternating magnetic flux is produced, which causes the armature to vibrate. The vibration is transmitted via the bar to the reeds. The reed, with a natural period corresponding to the alternations of the magnetic flux, will be set in vibration (it is said to be in resonance with the current being measured). The painted ends of the reeds can be seen through a square cut in the dial of the instrument, and above each reed there is a number indicating its resonance frequency (in cycles per second). During a measurement, the vibrating reed will be seen as a white band or blur. A portion of the scale of a reed-type frequency indicator is shown in Fig. 340; a general view of the instrument is given in Fig. 341.

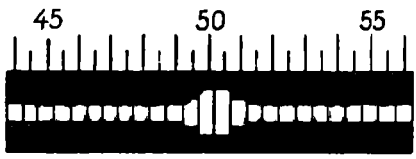


Fig. 340. View of the scale of a reed-type frequency indicator

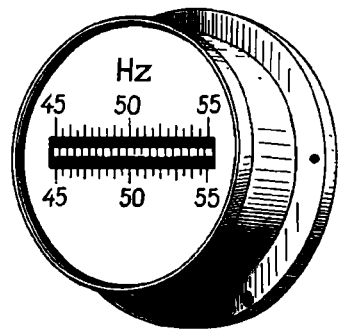


Fig. 341. General view of a reed-type frequency indicator

The electromagnet of the reed-type frequency indicator is wound with a large number of turns of fine wire and is connected in parallel in the circuit to be measured, as a voltmeter coil.

It should be noted that reed-type frequency indicators are falling out of use, giving way to deflectional indicators of various types with a continuous scale.

The various types of electrical instruments discussed above bear identification symbols which are summarized in Fig. 342.

167. Instrument Transformers

When measurements are taken in a-c circuits, it is essential for safety reasons to isolate high-voltage wires from instruments. Furthermore, it is often required to extend the range of instruments, since the magnitude of current, voltage or power in practical circuits may be many times the instrument range. Both purposes are served by instrument transformers.

There are current instrument transformers and potential (or voltage) instrument transformers. The former are used for increasing the current range of instruments and the latter for increasing the voltage range.

For accurate measurements, instrument transformers should have a constant current or voltage ratio (as the case may be) and a phase angle of 180° between the primary and secondary current or voltage, respectively.

Type of instrument	Type symbol: without screen	Type symbol: with a magnetic screen
Moving-coil with mechanical controlling force		
Same, without controlling force		
Moving-iron, with controlling force		
Same, without controlling force		
Electrodynamic, with controlling force		
Same, without controlling force		
Iron-cored electrodynamic with controlling force		
Same, without controlling force		
Induction, with controlling force		—
Same, without controlling force		—
Hot-wire		—
Thermocouple (thermo-emf)		—
Rectifier		—
Valve		—
Electrostatic		—
Vibration		—

Item designated	Symbol
d-c	
a-c	
d-c/a-c	
Three-phase circuits:	
balanced	
unbalanced	
four-wire	
50 c/s	
Insulation-test voltage of 2 kV	
Vertical scale	
Horizontal scale	
Inclined scale (about 60°)	
Accuracy class (for example 0.2)	

Fig. 342. Symbols used on the dials of electrical instruments

In practice, however, an instrument transformer has a *ratio error* and a *phase-angle error.*

The ratio error, as defined by the relevant U.S.S.R. State Standard, is the difference between the rms secondary current (or voltage) multiplied by the marked transformer ratio and the rms primary current (or voltage) divided by the rms primary current (or voltage):

$$\gamma_I = \frac{k_c I_2 - I_1}{I_1};$$

$$\gamma_v = \frac{k_v V_2 - V_1}{V_1},$$

where k_c = marked ratio of a current transformer, defined as the ratio of the primary to the secondary current as given on the nameplate;

k_v = marked ratio of a voltage transformer, defined as the ratio of the primary voltage to the secondary voltage as given on the nameplate.

The phase-angle error of instrument transformers is the angle by which the reversed secondary vector of current or voltage leads the corresponding primary vector.

Both the ratio error and the phase-angle error increase with the burden of the transformer, the burden being defined as the instruments connected in series in the secondary circuit. Therefore, transformers should not be connected to carry a burden greater than given on the nameplate.

Soviet-made instrument transformers have a ratio error of ± 1 per cent and a phase-angle error of $\pm 45'$.

168. Voltage Transformers

The primary and secondary of a voltage instrument transformer are wound with insulated copper wire on a closed magnetic circuit consisting of silicon-steel punchings. In fact, voltage instrument transformers do not differ materially in construction from power transformers which supply incandescent lamps, electric motors, etc.

The rated ratio of a voltage transformer is usually marked on its nameplate (hence it is called the marked ratio) as a fraction, the numerator giving the primary voltage and the denominator the secondary voltage. For example: "6000/100", which gives a marked ratio of 60.

The relevant U.S.S.R. Standard specifies a secondary voltage of 100 V when the rated voltage is applied to the transformer primary.

Where a voltage transformer is used in conjunction with one and the same instrument repeatedly, the scale is calibrated in terms of primary voltage adjusted for the transformer ratio.

The rated power is given also on the nameplate of the transformer. Soviet-made voltage transformers are available in ratings from 200 to 2000 VA. To avoid measurement errors, the burden should be such that the rated power of the transformer is not exceeded.

Voltage transformers may be single-phase and three-phase. Fig. 343 shows a circuit diagram for a single-phase voltage instrument transformer. The secondary circuit contains a low-voltage fuse as a protection against overloads and short-circuits in the instruments. The secondary and all metal parts of the transformer are earthed as a precaution against a high potential building up on them, should the insulation of the H.V. winding break down. No fuse is connected on the earthed side of the secondary. The primary circuit is protected by high-voltage fuses, which give it protection against short-circuit surges. The high-voltage fuses are connected in series with current-limiting resistors that reduce the short-circuit currents and improve the reliability of the fuses. Disconnection of voltage transformers from the line is by means of isolators.

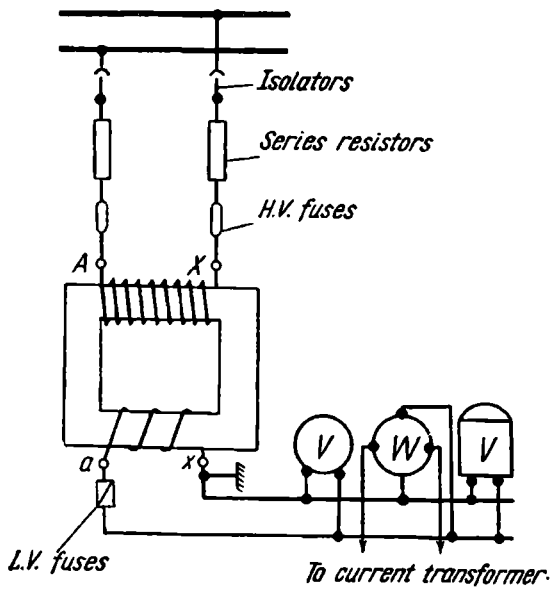


Fig. 343. Connection of a single-phase voltage transformer into a circuit

Soviet-made voltage transformers come in four accuracy classes: 0.2, 0.5, 1 and 3, the number giving the limit of ratio error in per cent of the full-scale value.

Voltage transformers for up to 3 kV are made air-cooled, for 6 kV and more, oil-immersed.

Voltage transformers find use in voltage measurements on H.V. lines; for connecting voltage relays, the zero-voltage coils of manual and automatic operating mechanisms in circuit-breakers, frequency indicators, pilot lamps, etc.; for supplying the potential coils of wattmeters, watthourmeters and power-factor meters; and for insulation testing. Some of the applications of voltage transformers are illustrated in Fig. 344.

In three-phase lines, use is made of three-phase voltage transformers. Their connection into a circuit is shown in Fig. 345.

Fig. 347 shows voltage transformers used for insulation testing. At *a*, the insulation of a single-phase circuit relative to earth is being tested. If the insulation is good, each of the voltmeters will read half the circuit voltage. In the case of a line-to-earth fault, the voltmeter connected be-

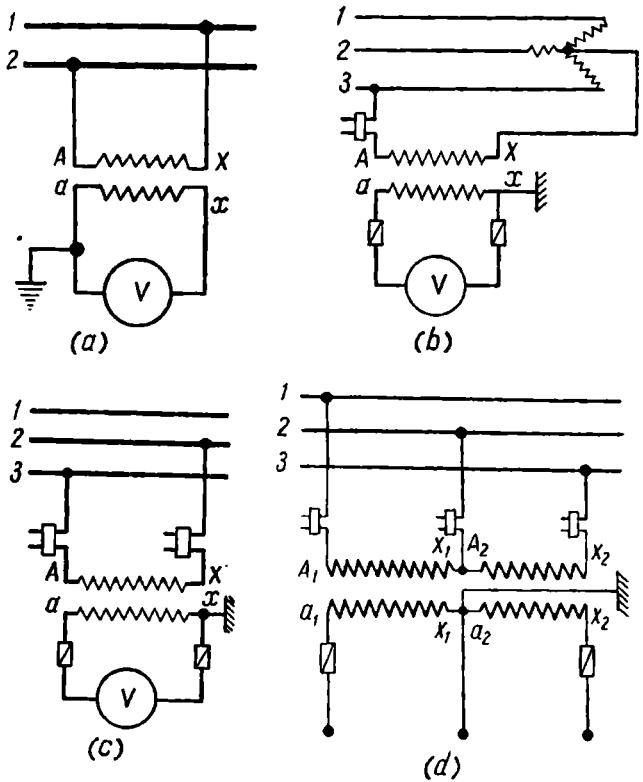


Fig. 344. Application of single-phase voltage transformers:

a—voltage measurement in a single-phase circuit; *b* phase-voltage measurement in a three-phase circuit, *c*—line voltage measurement in a three-phase circuit; *d*—two single-phase voltage transformers connected in an open delta

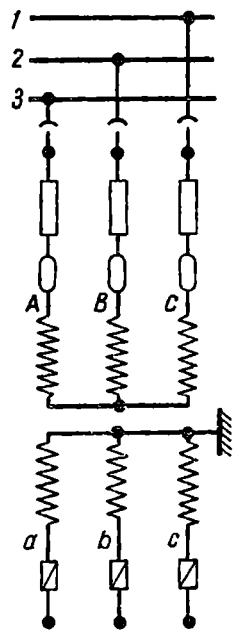


Fig. 345. Connection of a three-phase voltage transformer

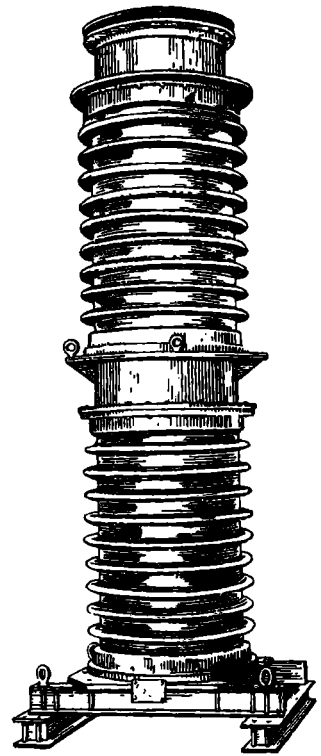


Fig. 346. Type НКФ-220 voltage transformer

tween the faulty line and earth will read zero, and the other voltmeter will give the full circuit voltage. At *b*, the insulation of a three-phase circuit is being tested. When the insulation is good, each of the voltmeters will read the phase voltage. In the case of a line-to-earth fault, the voltmeter connected to the faulty phase will read zero, while the other voltmeters will indicate the line voltage. Three-phase three-limb transformers used for the

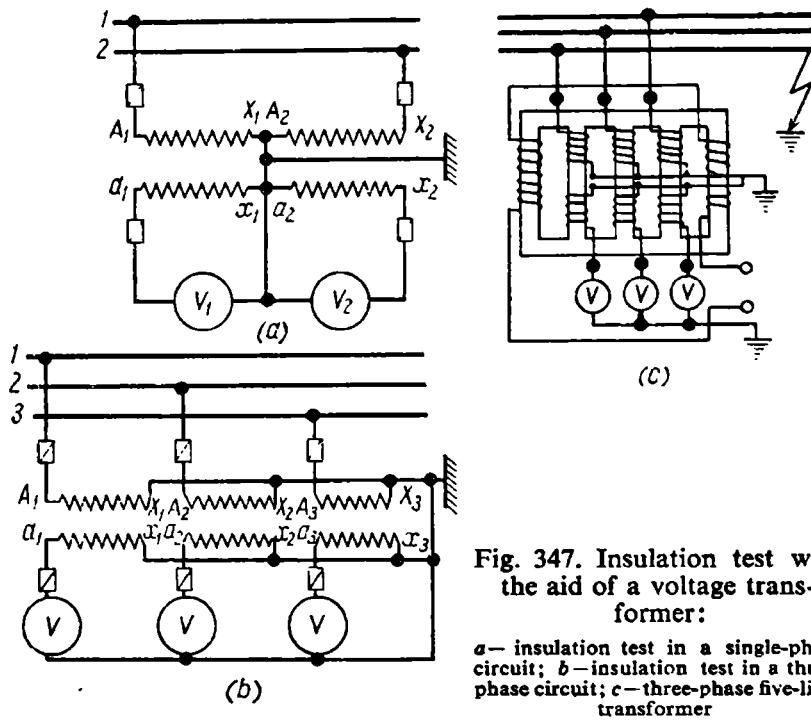


Fig. 347. Insulation test with the aid of a voltage transformer:

***a*— insulation test in a single-phase circuit; *b*—insulation test in a three-phase circuit; *c*—three-phase five-limb transformer**

purpose are made without the neutral point on the H.V. side being earthed. The reason for this is that in the case of a line-to-earth fault, one of the windings will be short-circuited and the remaining two windings will induce a heavy current in it, causing damage to the transformer. At *c* is shown a three-phase five-limb transformer used for insulation testing in three-phase circuits. If the circuit is good, the vector sum of the magnetic fluxes will be zero and no flux will link through the outer limbs. Should there be a line-to-earth fault, the respective winding of the transformer will be short-circuited, and the magnetic fluxes of the remaining phases will link through the outer limbs, inducing an emf in an auxiliary winding.

Voltage transformers made in the Soviet Union are classed into three groups as follows:

1. Dry transformers (Types HOC-0.5, HOC-3, HTC-0.5 and HTCH-0.5).
2. Oil-immersed transformers (Types HOM-10, HOM-35, HTMK-6, HTMK-10, HTMИ-6 and HTMИ-10).

3. Cascade oil-immersed transformers (Types НКФ-110, НКФ-220).

The type designations are read thus: "НОС" stands for a dry single-phase transformer; "НТС" for a dry three-phase transformer; "НТСИ" for an insulation-testing dry three-phase three-winding five-limb transformer; "НОМ" for an oil-immersed single-phase voltage transformer; "НТМК" for a compensating-winding oil-immersed three-phase voltage transformer; "НТМИ" for an insulation-testing oil-immersed three-phase three-winding voltage transformer; "НКФ" for a cascade porcelain-shrouded single-phase voltage transformer. The numeral following the letter designation gives the rated high voltage of the transformer in kilovolts.

Types НОМ-35, НКФ-110 and НКФ-220 are intended for outdoor service, the remaining types are designed for indoor installation.

In Type НТМК transformers the main turns of a phase in the primary are connected with some of the turns in the compensating winding of another phase. Such an arrangement reduces the phase angle error and improves the accuracy of the transformer.

A cascade transformer consists of several transformers so connected that the secondary of a preceding unit energizes the primary of the next one.

169. Current Transformers

Current transformers reduce heavy currents to values convenient and safe for measurement. The design and circuit connection of a current transformer are shown in Fig. 348. The magnetic circuit assembled of silicon-steel punchings carries two windings: the primary with a small number of turns and connected in series with the circuit to be measured; and the secondary wound with a large number of turns, across which the instrument (or instruments) are connected. The secondary is usually wound to produce a current of 5 (sometimes 10) amperes, while the primary current may be anywhere between 5 and 15,000 amperes. In this way, the instruments are isolated from high-voltage wires.

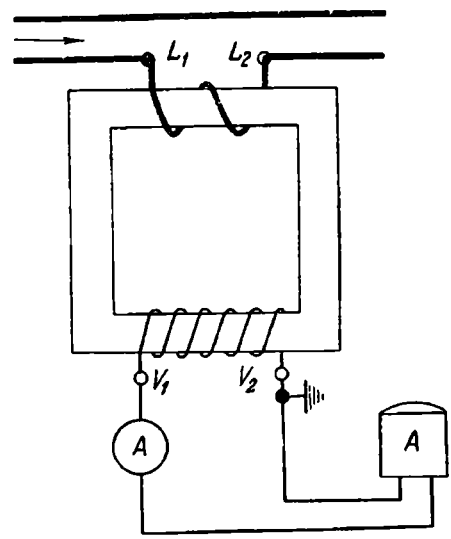


Fig. 348. Connection of a current transformer

Current transformers are characterized by the current *transformation ratio*, which is approximately equal to the reciprocal of the turns ratio. The rated transformation ratio is usually marked on the nameplate (hence it is called the marked ratio). For example, 150/5 A gives a transformation ratio of 30.

Associated instruments are calibrated for the primary current on the basis of the transformation ratio. The error of measurement due to current transformers is within ± 1 per cent.

As distinct from a voltage transformer, an increase in the resistance of the secondary circuit, with the primary current kept unchanged, will reduce the secondary current I_2 and the secondary flux Φ_2. Consequently, the resultant flux I will increase. When the secondary is opened ($I_2 = 0$; $\Phi_2 = 0$), the resultant flux threading the transformer core will build up to a value Φ_1. Designed for a flux Φ, the core can overheat, and the transformer will be damaged. Furthermore, an increased magnetic flux will induce a considerable emf (500 to 1000 V) in the secondary, making it dangerous to attending personnel. This is why the secondary must not be opened when the primary is energized; it must be either connected across instruments or short-circuited.

An increase in the resistance of the secondary circuit also increases the errors of measurement due to the transformer. Therefore, the secondary burden of a transformer should not exceed its standard burden. If, for example, a current transformer is rated for a burden of 20 VA, the resistance of the secondary circuit for a secondary current of 5 A must be

$$R = P/I_2^2 = 20 \div 25 = 0.8 \text{ ohm.}$$

Calculations should always take into account the resistance of connecting wires. Therefore the permissible burden of current transformers lies anywhere between 15 and 75 VA. One end of the secondary, in current transformers, is earthed for the same reason as in voltage transformers.

Current transformers may be classed in several ways as follows:

(a) *according to service*, into indoor and outdoor;

(b) *according to insulation*, into dry porcelain or paper insulated, oil-filled, compound-filled;

(c) *according to construction*, into coil-type, bushing-type, bar-type, window-type, pot-type, and pole-type;

(d) *according to the number of secondary circuits*, into single-winding and two-winding. In the latter case, the two windings are distributed on two separate cores, and the transformer can be used to power instruments from one winding and relays from the other;

(e) *according to the number of primary turns*, into single-turn and multi-turn;

(f) *according to accuracy*, into classes 0.2, 0.5, 1, 3 and 10 (the numbers give the ratio error in per cent of the full-scale value);

(g) *according to service*, into stationary and portable. The latter may have up to 28 transformation ratios.

An example of a portable bar-type current transformer is shown in Fig. 349, where it is associated with an ammeter. The split core can be opened by means of handles and slipped over a cable or a busbar. The ammeter mounted on the transformer is connected across the secondary wound on

the core. The wire or busbar carrying the current to be measured serves as the primary.

Current transformers serve to connect ammeters, the current (series) coils of wattmeters, watthour-meters, frequency indicators, current relays, etc.

The application of a current transformer is shown in Fig. 350.

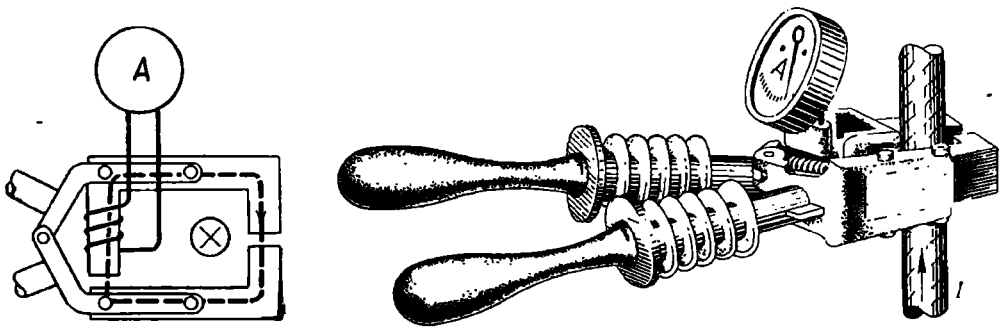


Fig. 349. Current transformer combined with ammeter

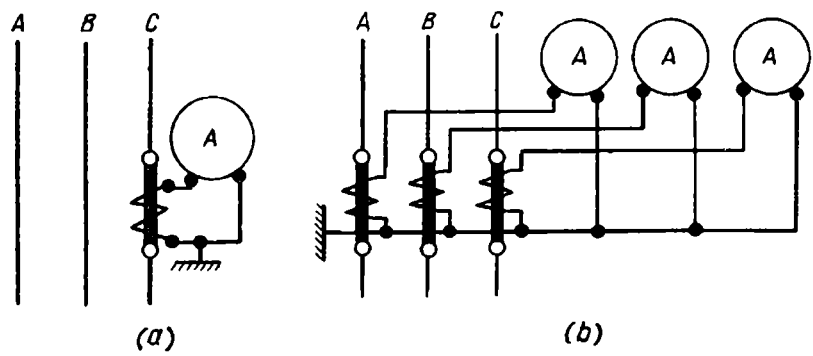


Fig. 350. Current measurement with a current transformer:
a—in a balanced circuit; *b*—in an unbalanced circuit

The type designations of Soviet-made current transformers are read as follows:

Т —current transformer;
П —through-type;
Ш—bar-type;
У —ruggedized;
Ф —porcelain insulation between the primary and secondary;
Д —with a core for differential protection;
З —with a core protected against earth faults;
Р —with a split core;
О —with a single turn in the primary (consisting of a copper bar);
К —coil-type;

В —built into a circuit-breaker;
М —modernized;
Б —quickly saturated.

The numeral following the type designation gives the voltage in kilovolts, the next numeral (or numerals) indicate the number of secondary windings and their accuracy.

Example. "ТПФУД-10-0.5/Д" identifies a porcelain-insulated ruggedized through-type current transformer for 10 kV with two secondary windings, one of accuracy class 0.5 and the other for differential protection.

Fig. 351 shows a Type ТФНД-220 current transformer for a rated voltage of 220 kV (height 3770 mm, weight, 4400 kg, including oil).

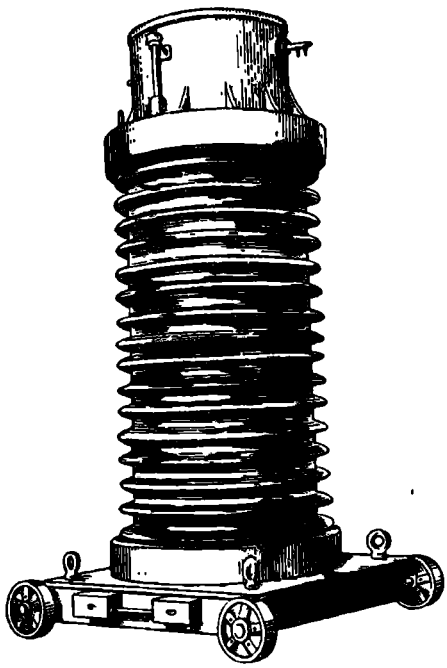


Fig. 351. Type ТФНД-220 current transformer

170. Current Measurements

Currents are measured with ammeters connected in series with the load circuit. So that an ammeter will not affect the value of the current, the resistance of its coil should be made as small as possible. To meet this requirement, the coil of an ammeter is wound with a few turns of large-diameter wire. The range of ammeters can be extended with the aid of shunts.

Shunts are usually composed of one or more thin strips of manganin sheet or bar, the ends of which are soldered to two heavy copper blocks that carry two terminals each (Fig. 352). Shunts are connected in series with the circuit and in parallel with the ammeter (Fig. 353). The circuit current I divides at the point A in inverse proportion to the resistance of the ammeter coil R_a and of the shunt R_{sh}:

$$\frac{I_a}{I_{sh}} = \frac{R_{sh}}{R_a}; \quad I_{sh} = I - I_a.$$

Hence, the resistance of the shunt will be equal to

$$R_{sh} = \frac{I_a R_a}{I - I_a}.$$

The ratio I/I_a is designated by the letter N, which is termed the *multiplying power of the shunt*. Then we have

$$R_{sh} = \frac{R_a}{N-1}.$$

Example 1. Calculate the resistance of a shunt for a 5-A ammeter with an internal resistance of 0.006 ohm, necessary to measure a current of 20 A.

$I_a = 5$ A
$R_a = 0.006$ ohm
$I = 20$ A

$N = 20 \div 5 = 4$

$R_{sh} = \frac{R_a}{N-1} = 0.006 \div 3 = 0.002$ ohm.

Shunts for currents up to 100 A are placed within instruments (internal shunts). For larger currents, external shunts are employed. They are connected to ammeters by calibrated leads for correct measurements. Sometimes, use is made of universal shunts for several ranges. As a rule,

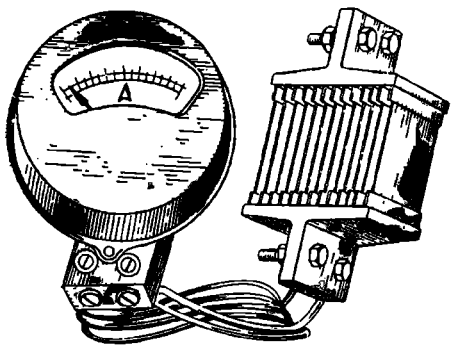


Fig. 352. Ammeter with a shunt

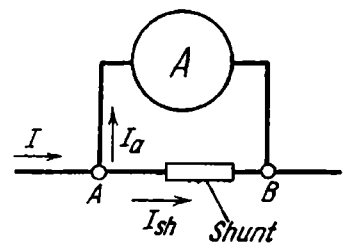


Fig. 353. Connection of an ammeter and shunt

shunts are calibrated together with the associated instrument, which bears an inscription to that effect. Alternatively, standard shunts may be employed in conjunction with instruments having the same voltage drop. In practice, only moving-coil instruments for d-c measurements use shunts.

The range of ammeters used in a-c circuits is extended by means of current transformers.

171. Voltage Measurements

Voltages are measured with voltmeters connected across the load circuit. So that a voltmeter should not take excessive current (this would affect the circuit voltage), its coil is made of high resistance. The higher the internal resistance of a voltmeter, the more accurate it is. This is the reason why voltmeter coils are wound with a large number of turns of fine wire.

The range of voltmeters can be extended with the aid of multiplying resistances, or multipliers, which are connected in series with the associated voltmeter (Fig. 354). Then the circuit voltage will divide between the instrument and the multiplier. A multiplier should be so chosen that the same current will flow through the voltmeter coil as in the case of normal voltage. The current of the instrument can be determined by the formula

$$I_{in} = V/R_{in}.$$

In a circuit, the voltage across which is N times normal, the current through an instrument using a multiplier should remain unchanged:

$$I_{in} = NV/(R_{in}+R_m) \quad \text{or} \quad V/R_{in} = NV/(R_{in}+R_m),$$

whence the resistance of the multiplier will be

$$R_m = R_{in}(N-1).$$

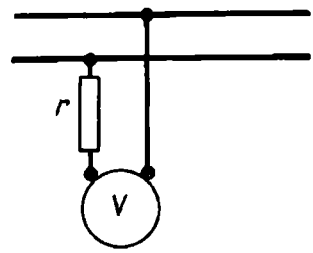


Fig. 354. Connection of multiplier to voltmeter

Example 2. A voltage of 150 V has to be measured with a 25-V voltmeter. Calculate the resistance of the required multiplier if the internal resistance of the instrument is 1000 ohms.

$R_{in} = 1000$ ohms
$N = 150 \div 25 = 6$

$R_m = R_{in}(N-1) = 1000(6-1) =$
$= 5000$ ohms.

Voltmeter multipliers are made of manganin wire wound on a porcelain or impregnated-paper bobbin. Multipliers may be internal or external.

Where high and extra-high voltages are involved, the range of voltmeters can be extended by means of voltage transformers.

172. Rheostats

A rheostat is a resistor provided with means for varying its resistance, thereby varying the current and voltage of the circuit in which it is connected. There are sliding-contact rheostats, dial-type rheostats, liquid-type rheostats, lamp rheostats, and plug-type rheostats.

A sliding-contact rheostat (Fig. 355) consists essentially of a porcelain tube *1* upon which is wound resistance wire *2* specially treated so as to produce a nonconducting film of oxide on it. A metal bar *3* running parallel to the axis of the tube supports a sliding contact *4*, which bears upon the resistance wire. For the sliding contact in the position shown in Fig. 355, the current path is shown by the arrows. As more resistance is brought into the circuit by moving the sliding contact to the left, the current flowing through the lamp *5* is reduced and so is the brightness of the lamp. Moving the sliding contact to the right will reduce the effective resistance of the circuit, and the brightness of the lamp will increase. Sliding-contact

rheostats are used where the circuit current has to be varied gradually and continuously.

A dial-type (or wire) rheostat (Fig. 356) consists of resistance wire wound in coils or stretched over an insulated frame. The coils (or wires) are

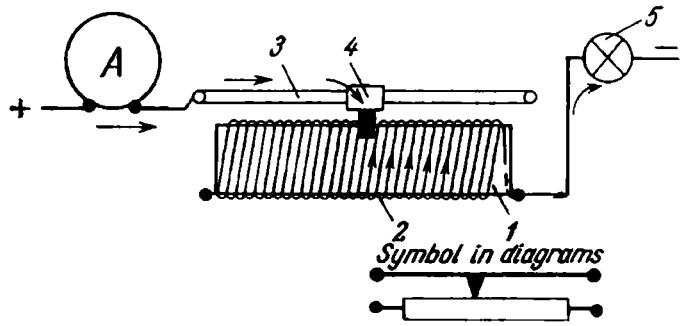


Fig. 355. Sliding-contact rheostat

connected in series. Leads are brought out from the beginning, end, and junctions of the coils to a dial over which a finger contact sweeps. The resistance of the rheostat and the circuit current can be varied by shifting the finger contact from terminal to terminal. Adjustment is in steps rather than continuous. Dial-type rheostats are mostly made of iron, nickeline, constantan, manganin or nichrome wire. A general view of a dial-type rheostat is shown in Fig. 357.

A liquid rheostat (Fig. 358) is a metal vessel *1* filled with a soda solution. A hinge *2* carries a bar *3* fitted with an iron or copper blade contact *4*. The bar and blade contact are insulated from the vessel by an insulator *5*. The circuit current can be varied by lifting or lowering the blade contact. When the blade contact is dipped into the solution, the contact area between the blade and electrolyte increases, and the current passing through the rheostat increases too. When the blade is fully immersed in the electrolyte, the contact *6* snaps into jaws *7*, and the rheostat is fully brought out of circuit. Liquid rheostats find use where large currents are involved.

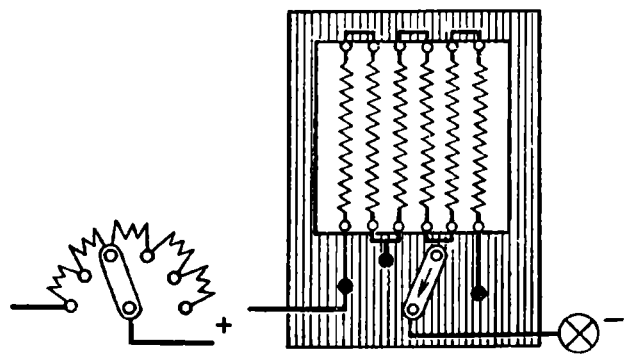


Fig. 356. Dial-type (wire) rheostat

A lamp rheostat (Fig. 359) consists of several lamps connected in parallel. If one lamp has a resistance of 150 ohms, two such lamps connected in parallel will have a total resistance of 75 ohms, three lamps, 50 ohms, etc.

In other words, the total resistance of several identical lamps connected in parallel will be equal to the resistance of one lamp divided by the number of lamps. The lamp rheostat shown in the figure is connected in the circuit of a storage battery and serves to control its charging current.

Plug-type rheostats, often called resistance boxes (Fig. 360), are sets of accurately calibrated resistance coils *1*, the ends of which are connected to

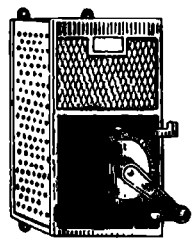


Fig. 357. General view of a dial-type (wire) rheostat

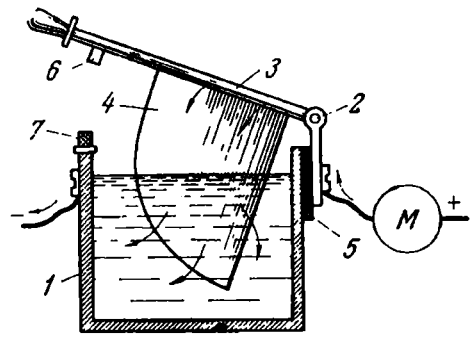


Fig. 358. Liquid rheostat

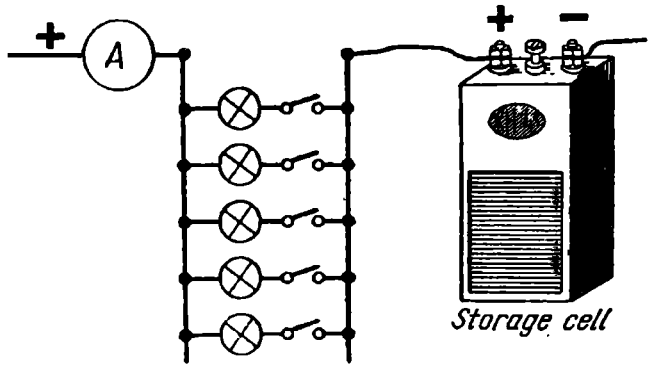


Fig. 359. Lamp rheostat

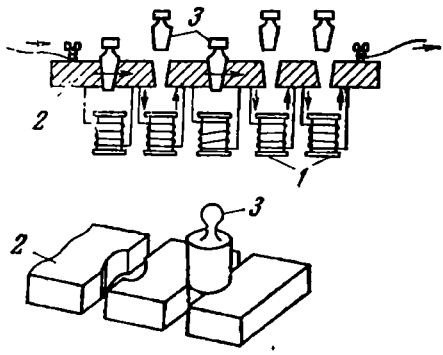


Fig. 360. Plug-type rheostat (resistance box)

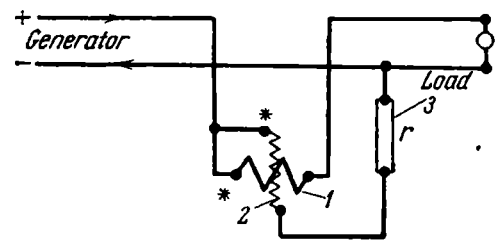


Fig. 361. Circuit diagram of an electrodynamic wattmeter

a split copper strip *2*. When a copper plug *3* is inserted into the split, two adjacent coils are shunted, and the resistance inserted in the circuit is reduced. When the plug is withdrawn from the split, the current has to pass through the respective coils, and more resistance is brought into the circuit. Thus, any value of resistance from the minimum to the total of the rheostat may be inserted in the circuit.

Resistance boxes make it possible to vary the resistance of circuits by accurately known amounts and are therefore used in measurements.

173. Power Measurements

1. Direct-current circuits. As follows from the formula $P = VI$, power in a direct-current circuit can be found by multiplying ammeter and voltmeter readings. In practice, however, this is done by instruments known as wattmeters. The construction of an electrodynamic-type wattmeter is shown diagrammatically in Fig. 361. It consists of a fixed coil *1* wound with a small number of turns of heavy wire and a movable coil *2* wound with a large number of turns of fine wire. The fixed coil is connected in series with the load circuit (hence it is called the series or current coil), while the movable coil is connected across the load circuit (for which reason it is called the shunt or potential coil). Connected in series with the shunt coil is a manganin multiplier *3*. This is done in order to reduce the weight of, and power loss in, the shunt coil. The interaction of the magnetic field of the two coils produces a torque proportional to the currents through both coils:

$$M = c_1 I_1 I_2.$$

When the shunt circuit has a constant resistance, the current through the shunt coil is proportional to the circuit voltage. Therefore,

$$M = c_2 I_1 V = c_2 P.$$

In other words, the torque of the instrument is proportional to the power taken by the circuit.

It is important to observe polarity in connecting the wattmeter in the circuit so that the pointer will deflect to the right. For this, the two terminals to which the starts of the coils are connected are marked with an asterisk (*) and are electrically connected. The dial of the instrument bears inscriptions indicating its rated current and voltage. If the dial of a wattmeter gives 5 A and 150 V, it can measure power up to 750 W.

The dials of some wattmeters have divisions. If a wattmeter for 5 A at 150 V has 150 divisions, it can read down to $750 \div 150 = 5$ watts. The power represented by one division is called the *wattmeter constant.*

In addition to electrodynamic-type wattmeters, there are also iron-cored wattmeters.

2. Single-phase a-c circuits. If an electrodynamic wattmeter is connected in an a-c circuit, the torque produced by the two coils at any given time

will be proportional to the product of the instantaneous values of current in both coils. The moving system of the instrument is not able to follow the rapid alternations of current, and the torque of the wattmeter will be proportional to the average or true power: $P = VI \cos \varphi$.

Power in a-c circuits can also be measured by induction-type wattmeters. Connection of an induction-type revolving-field wattmeter is shown in Fig. 362. The current coil *1-1*, wound with several turns of heavy wire, is distributed on two opposing pole pieces and is connected in series with the load circuit. The shunt coil, *2-2*, wound with a large number of turns of fine wire, is placed on the remaining two pole pieces. Connected in series with the shunt coil is an inductive reactance *3*, which produces a phase displacement of 90° between the current and voltage of the shunt coil. Then, in the case of a purely resistive load, there is a phase displacement of 90° between the currents of the two coils. This condition is necessary in order to produce a revolving field. When the instrument is energized the revolving field cuts an aluminium cylinder *4* and induces eddy currents in it. The eddy currents react with the field to produce a torque acting upon the moving system of the instrument. The moving system will deflect in proportion to the true power in the circuit:

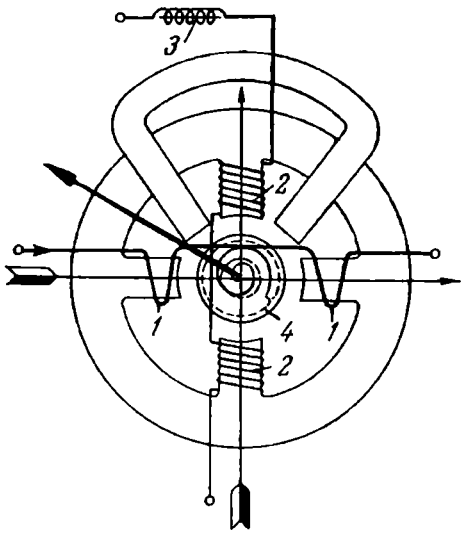


Fig. 362. Diagram of a revolving-field induction wattmeter

$$P = VI \cos \varphi.$$

Power in low-voltage heavy-current circuits can be measured with the aid of current transformers. In order to reduce the potential difference between the wattmeter coils, the primary and secondary of the current transformer have a common point. The secondary is not earthed, since otherwise one wire of the circuit would be earthed. The power in the circuit will then be equal to

$$P = P_w k_C,$$

where P = power in the circuit;

P_w = wattmeter reading;

k_C = transformation ratio of the current transformer.

Power in high-voltage circuits is measured with the aid of voltage and current transformers (Fig. 363). The power in the circuit will then be

$$P = P_w k_V k_C,$$

where P = power in the circuit;
P_w = wattmeter reading;
k_V = transformation ratio of the voltage transformer;
k_C = transformation ratio of the current transformer.

For example, with a wattmeter reading 80 watts and used in conjunction with a voltage transformer for 6000/100 V and a current transformer for 150/5 A, the power in the circuit will be

$$P = 80 \times (6000 \div 100) \times (150 \div 5) = \\ = 144 \text{ kW}.$$

When using instrument transformers, wattmeters must be connected so that the current will flow through their coils in the same direction as when connected directly in the circuit.

Power in single-phase a-c circuits can also be measured with an ammeter, voltmeter and power-factor meter according to the formula

$$P = VI \cos \varphi.$$

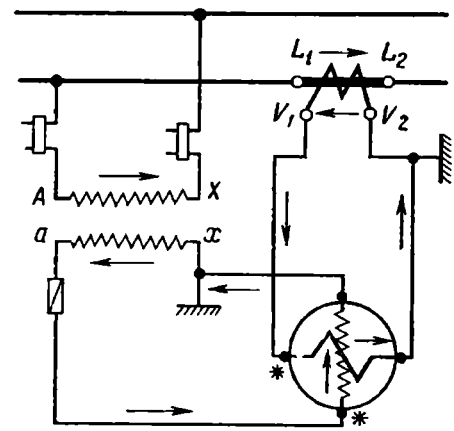


Fig. 363. Connection of a wattmeter via instrument transformers

3. Three-phase a-c circuits. In balanced three-phase systems, power can be measured with a single-phase wattmeter connected as shown in Fig. 364. The series coil of the wattmeter will then be traversed by the phase current while the shunt coil will receive the phase voltage. Consequently, the wattmeter will read power in one phase. The total power will be three times the reading of the wattmeter.

In unbalanced four-wire three-phase circuits, power is measured with three wattmeters connected as shown in Fig. 365. Each wattmeter reads power in one phase. The total power is the sum of the three readings.

In the case of a varying load it is difficult to take simultaneous readings on the three wattmeters. Furthermore, three single-phase wattmeters take up too much space. This is why use is often made of a single three-element three-phase wattmeter which combines three single-phase instruments in a common unit. In the electrodynamic type of three-element wattmeter there are three shunt coils mounted on the same shaft, and the total torque produced by the three coils is proportional to the total power in the cir-

cuit. In other types, the movable coils are located separately and linked up by flexible strips, which transmit the total torque onto a shaft and pointer.

Power in a balanced three-phase circuit can be measured with an ammeter, voltmeter and power-factor meter according to the formula

$$P = \sqrt{3}\, VI \cos \varphi,$$

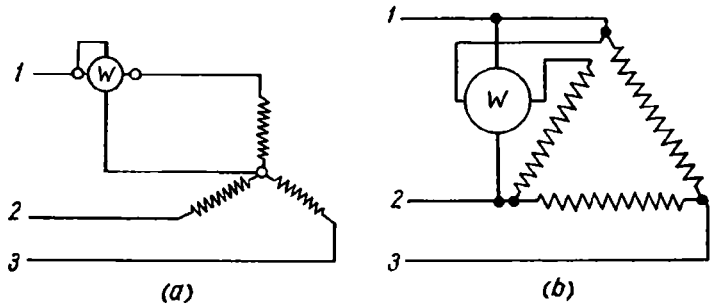


Fig. 364. Connection of a single-phase wattmeter in a balanced three-wire circuit:

a—star connection; *b*—delta connection

where V and I are line voltage and current, and φ is the phase angle between the phase voltage and current.

Power in a three-wire three-phase circuit (balanced or unbalanced), with the load connected either star or delta, can be measured by the two-wattmeter method.

According to Kirchhoff's first Law, the sum of the instantaneous values of current in the three phases is zero:

$$i_1 + i_2 + i_3 = 0,$$

whence

$$i_2 = -i_1 - i_3.$$

The instantaneous power in a three-phase circuit will be

$$p = i_1 v_1 + i_2 v_2 + i_3 v_3,$$

where v_1, v_2 and v_3 are the instantaneous values of phase voltages.

Substituting the value of the current i_2 in the last expression gives

$$p = i_1 v_1 - i_1 v_2 - i_3 v_2 + i_3 v_3$$

or

$$p = i_1 (v_1 - v_2) + i_3 (v_3 - v_2).$$

As follows from this equation, one of the two wattmeters should be so connected that its series coil carries the current of the first phase and its shunt coil receives the difference in voltage between the first and second phases; the other wattmeter should be connected so that its series coil carries the current of the third phase and the shunt coil receives the dif-

ference in voltage between the third and second phases. The total power will be the sum of the two readings. Three different arrangements for the two-wattmeter method are shown in Fig. 366.

Referring to Fig. 366, the series coils of the wattmeters are connected to any two line wires of the circuit. The beginnings of the shunt coils are connected to the line containing the current coil. The finishes of the shunt coils are connected to the third line.

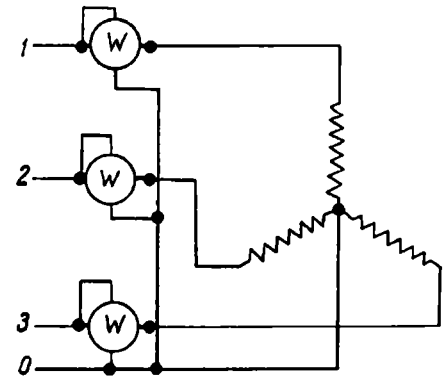


Fig. 365. Connection of three single-phase wattmeters for power measurement in a three-phase circuit

In the case of a balanced load (PF = 1), the wattmeters will give equal readings. At a power factor other than unity, the wattmeters will give different readings. At 0.5 power factor, one of the wattmeters will read zero; at less than 0.5 power factor, one instrument will read negative, and the connection of its series or shunt element will have to be reversed for correct indication. The total power will be the *algebraic* sum of the two readings. Fig. 367 shows an arrangement for the two-wattmeter method in conjunction with voltage and current instrument transformers.

A more convenient means for measuring power in three-phase circuits is provided by a three-phase wattmeter, in which two elements are arranged as if they were two instruments in the two-wattmeter method. In the electrodynamic type of instrument, two movable coils are either mounted on a common shaft or linked by flexible strips and rotate the same shaft. In the induction type of instrument, the two elements rotate either two discs on a common shaft or a single disc. The connection of a two-element

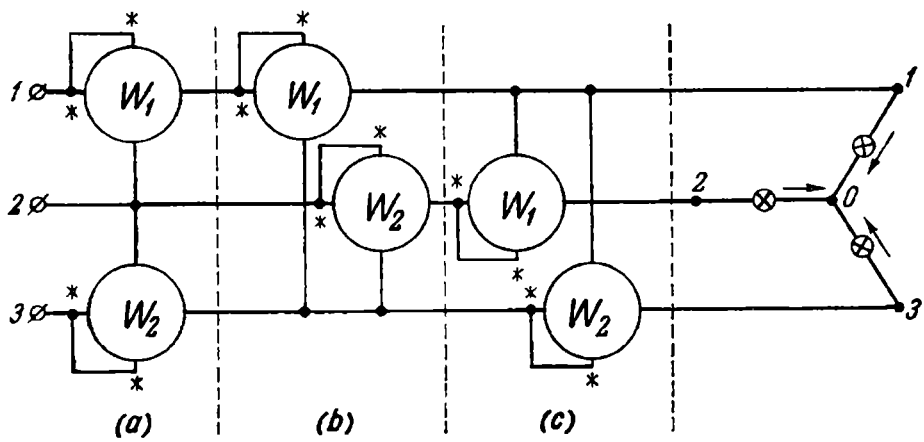


Fig. 366. Connection of two wattmeters

three-phase wattmeter in the circuit is shown in Fig. 368. In high-voltage circuits the connection of the instrument is via voltage and current instrument transformers.

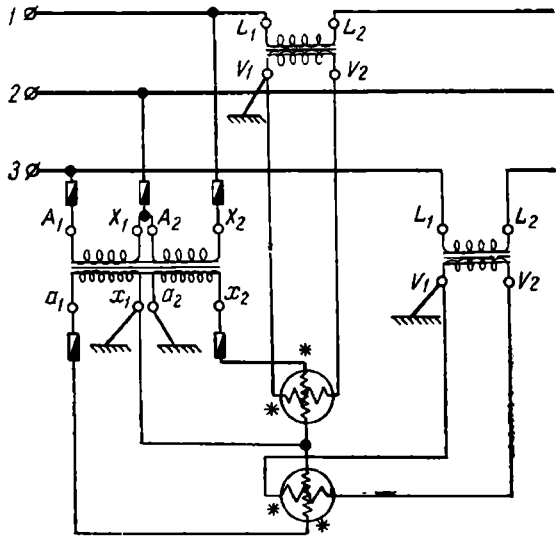


Fig. 367. Power measurement by the two-wattmeter method using instrument transformers

174. Energy Measurements

1. Direct-current circuits. Three types of instruments are used for the measurement of energy: electro-

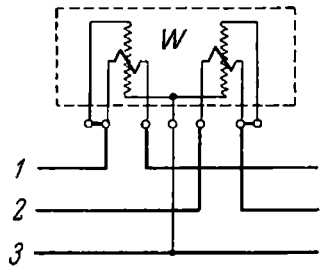


Fig. 368. Connection of a three-phase two-element wattmeter

dynamic, moving-coil and electrolytic watthour-meters. The most commonly used type is the electrodynamic watthour-meter (Fig. 369). The fixed current coils *1-1* wound with a few turns of heavy wire are connected in series with the external circuit. The moving coil, or armature *2*, is mounted on a shaft that rotates in jewel bearings. The armature is wound with a large number of turns of fine wire distributed into several sections. The ends of the sections are soldered to the segments of a commutator *3*, which is in contact with flat metal brushes *4*. In series with the armature is a resistance *7* and a friction-compensation coil 8.

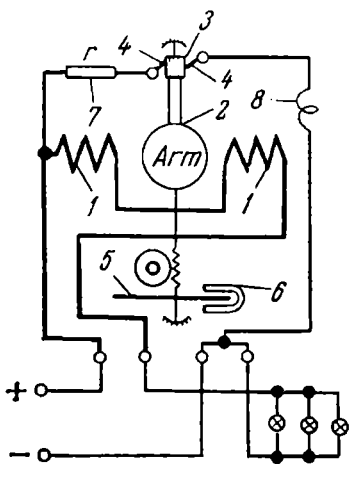


Fig. 369. Diagram of a d-c electrodynamic watthour-meter

The interaction of the magnetic fields produced by the armature and the fixed field coils produces an actuating torque, M_a, which drives the armature:

$$M_a = c_1 i_a \Phi.$$

Since $i_a \equiv V$ and $\Phi \equiv I$, $M_a = c_2 VI = c_2 P$. In other words, the torque of a watthour-meter is proportional to the power in the external circuit.

The shaft of the meter carries an aluminium disc *5* which rotates between the poles of a permanent magnet *6*. The eddy currents induced in the disc react with the field of the magnet and produce a counter torque, M_c, proportional to the speed of the disc:

$$M_c = c_3 n.$$

At constant speed the actuating and counter torques are equal:

$$M_a = M_c$$

or

$$c_2 P = c_3 n,$$

whence

$$P = (c_3/c_2)n = cn,$$

i.e., the speed of the meter is proportional to the power in the circuit. Each revolution represents a definite amount of energy. By connecting the shaft to a suitable registering mechanism by means of a worm or gear-type transmission, the total energy consumed is automatically registered. The transmission is so chosen that the meter reads hectowatt-hours or kilowatt-hours.

The amount of energy per revolution is termed the *wattsecond constant.* The number of revolutions per unit energy is called the *gear ratio.* Either the wattsecond constant or the gear ratio is marked on the nameplate of every watthour-meter for checking purposes.

Example 3. The nameplate of a meter reads "1 kWh = 12,000 revolutions". In a check-up the meter completed 120 revolutions during 50 sec. Calculate the power in the circuit. 1 kWh = 1000 Wh = 3,600,000 Wsec.

The wattsecond constant of the meter is

$$3{,}600{,}000 \div 12{,}000 \text{ Wsec/rev.}$$

The amount of energy represented by 120 revolutions is

$$3{,}600.000 \times 120 \div 12{,}000 \text{ Wsec.}$$

The power in the circuit will be

$$(3{,}600{,}000 \times 120) \div (12{,}000 \times 50) = 720 \text{ W.}$$

2. Single-phase a-c circuits. While d-c circuits are metered with electrodynamic watthour-meters like the one described above operating on the principle of d-c shunt-wound motors, energy in single-phase a-c circuits is metered with the induction-type instruments. The design of the induction-type watthour-meter is almost identical to that of the induction-type wattmeter. The only difference is that the meter has no springs supplying a counter force, and so the disc is free to rotate continuously. Instead of a pointer and dial, the watthour-meter has a registering mechanism. The permanent magnet which provides the damping force in the wattmeter

supplies the retarding force in the watthour-meter. A general view of a single-phase induction-type watthour-meter is shown in Fig. 370; its connection in the circuit is given in Fig. 371.

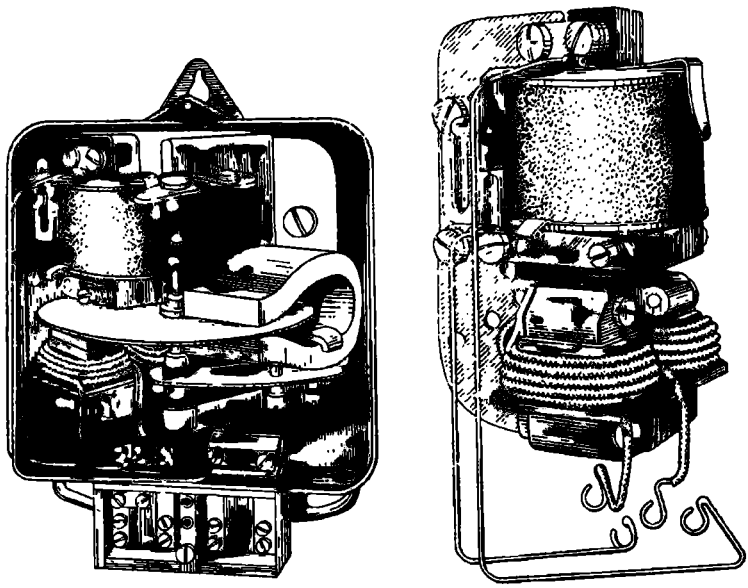


Fig. 370. Inside and general view of an induction single-phase watthour-meter

3. Three-phase a-c circuits. Energy in three-phase a-c circuits can be metered with two single-phase instruments connected exactly as in the power measurement by two wattmeters. A more convenient means is provided by a three-phase watthour-meter which combines two single-phase instruments in a single unit. Such a two-element three-phase watthour-meter is connected in the circuit exactly like the respective wattmeter.

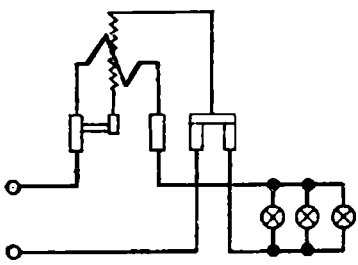


Fig. 371. Connection of a single-phase watthour-meter

In four-wire three-phase circuits, energy is measured by a method similar to the three-wattmeter technique. As an alternative, use may be made of a three-element three-phase watthour-meter. The total energy is found similarly to the total power for the respective arrangements.

In high-voltage circuits watthour-meters are connected via voltage and current instrument transformers.

175. Reactive Energy Measurements

1. Single-phase a-c circuits. Reactive energy in single-phase a-c circuits can be found as the product of the readings of an ammeter, voltmeter, power-factor meter and stop-watch. If the value of the power factor ($\cos \varphi$) is known, the value of $\sin \varphi$ can be taken from trigonometric tables. Substituting the numerical values in the formula

$$W_r = VIt \sin \varphi$$

gives the value of reactive energy.

Special-purpose single-phase var-hour meters (var-volt-ampere-reactive) known as "sine" meters, have not found commercial use.

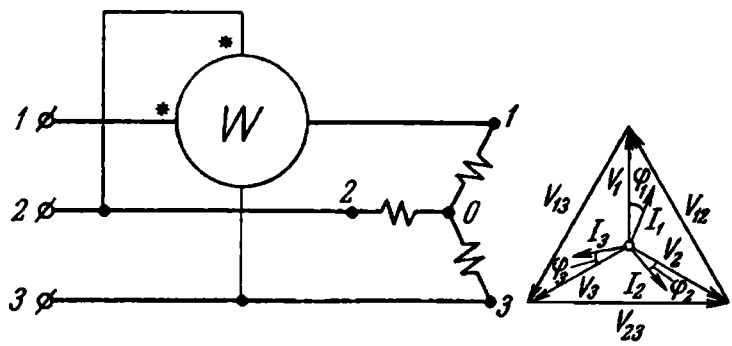


Fig. 372. Measurement of reactive energy with a single-phase watthour-meter

2. Three-phase a-c circuits. Reactive energy in three-phase circuits can be conveniently metered with normal "active" meters or special var-hour meters.

In balanced three-phase circuits, reactive energy can be measured with an ordinary active meter connected as shown in Fig. 372. In this arrangement the series element of the meter carries the current I_1 of the first phase, while the shunt element is connected to receive the line voltage V_{23} lagging 90° behind the phase voltage V_1, the line voltage being $\sqrt{3}$ times the phase voltage (see the vector diagram of Fig. 372). Then

$$W_r = V_{23} I_1 t \cos (90° - \varphi_1) = \sqrt{3}\, V_1 I_1 t \sin \varphi_1.$$

The reading will be equal to $\sqrt{3}$ times the reactive energy in the first phase. The total reactive energy in a three-phase circuit will be equal to $\sqrt{3}$ times the reading of the meter.

Special reactive energy meters, or var-meters, are similar in construction to a two-element three-phase wattmeter. The potential circuits of both

elements are connected in the load circuit as shown in Fig. 373. Instead of two current coils, the U-shaped cores carry four current coils. One current coil (at *1* in the figure) is wound on one tine and the second coil on the other tine of the core in the first element. The third and fourth coils are wound on the two tines of the core in the other element. Particular care must be exercised in connecting a var-hour meter so as to observe the correct rotation of phases.

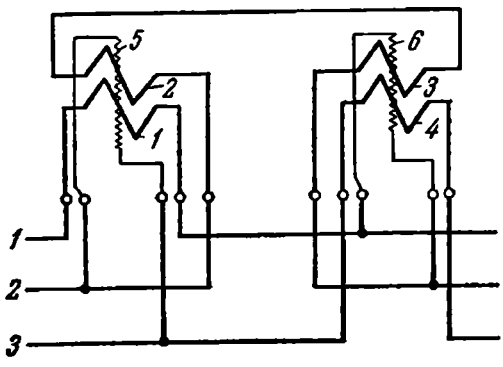


Fig. 373. Circuit diagram of Type ИР electrodynamic var-hour-meter

176. Power Factor Measurements

The power factor of single-phase circuits can be obtained from the readings of a voltmeter, ammeter and wattmeter from the relation

$$\cos \varphi = P/VI.$$

Using the same instruments, it is possible to determine the power factor of balanced three-phase circuits:

$$\cos \varphi = P/\sqrt{3}\, VI,$$

where V and I are the line voltage and current, and φ is the phase displacement between the phase voltage and current.

The average power factor over a period of time can be determined from the readings of an active and a reactive meter during the same period of time according to the formula

$$\cos \varphi_{avr} = W_a/\sqrt{W_a^2 + W_r^2},$$

where W_a is the active energy and W_r is the reactive energy in the circuit.

As an alternative we can find $\tan \varphi_{avr}$,

$$\tan \varphi_{avr} = W_r/W_a,$$

and take the respective value of $\cos \varphi_{avr}$ from trigonometric tables.

More often, however, the power factor is determined by means of power-factor meters (at *a* in Fig. 374). The electrodynamic type of single-phase power-factor meter (at *b* and *c* in Fig. 374) consists of a fixed coil *1* connected in series with the load circuit and two movable coils, *2* and *3*, mounted on a common shaft and spaced 90° apart. The movable coils are wound with a large number of turns of fine wire and are connected in parallel with the external circuit. The coil *2* is connected in series with a resistance R, and the current i_2 traversing it is in phase with the circuit voltage. The coil *3* is connected in series with a large inductive reactance X_L,

and the current i_3 through it lags 90° behind the circuit voltage. The torque produced by coil *2* is proportional to the true power in the circuit (exactly as in an electrodynamic wattmeter), while the torque produced

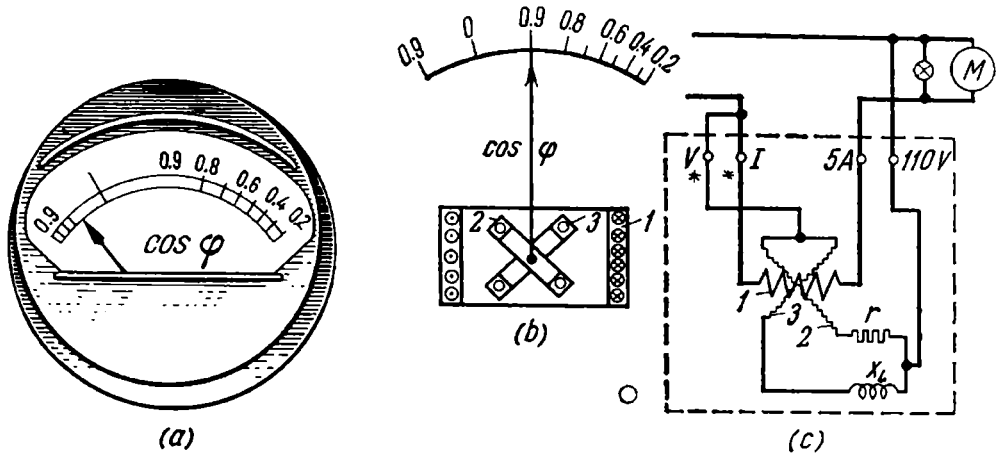


Fig. 374. Power-factor meter:

a—general view; *b*—construction; *c*—internal and external connections

by coil *3* is proportional to the reactive power in the circuit. The coils are so connected that their torques oppose each other. The position of the moving system depends on the deflection of coils *2* and *3*, i.e., on the phase displacement between the voltage and current. Therefore, the scale can be calibrated to read the power factor of the circuit directly.

Power-factor meters have no springs to provide the counter force, and so the pointer takes up an indifferent position when the instrument is de-energized. The scale is of the centre-zero type, with the right-hand portion reading the power factor in the case of an inductive load and the left-hand portion giving the power factor in the case of a capacitive load.

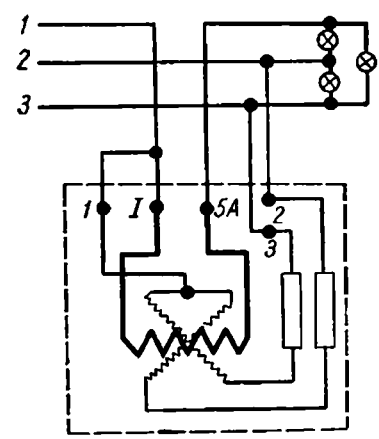
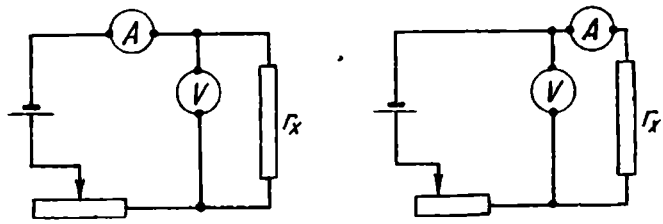


Fig. 375. Internal and external connections of a three-phase power-factor meter

A schematic diagram of an electrodynamic power-factor meter is shown in Fig. 375. The phase angle between the currents in the movable coils is obtained by appropriately connecting the coils in the three-phase circuit rather than by means of resistances and inductances.

177. Resistance Measurements

1. The ammeter-voltmeter (fall-of-potential) method. In d-c circuits, resistance may be measured by using the arrangements shown in Fig. 376. If the voltage drop across, and the current in, a part of the circuit are known, the resistance of this part can be readily calculated. In the arrangement at



(a) *(b)*

Fig. 376. Resistance measurement by the ammeter-voltmeter (fall-of-potential) method

a, the ammeter will read the sum of the currents flowing through the unknown resistance and the voltmeter, and the resistance can be obtained by the relation

$$R_x = V_x/I_x = V_x/(I-I_v) = V_x(I-V_x/R_v),$$

where I_v and R_v are the current through, and the resistance of, the voltmeter.

In the arrangement at *b*, the voltmeter will read the voltage drop across the unknown resistance and the ammeter winding:

$$V = V_x+V_a.$$

The unknown resistance can be then obtained by the relation

$$R_x = V_x/I_x = (V-V_a)/I_x = (V-I_xR_a)/I_x.$$

The arrangement at *a* in Fig. 376 is employed for resistances appreciably smaller than the resistance of the voltmeter. The arrangement at *b* in Fig. 376 is resorted to where high resistances are involved, since the resistance of the ammeter may be then neglected.

In a-c circuits, the impedance of the load can be obtained from ammeter and voltmeter readings by the relations

$$Z = V/I.$$

If the same load is connected in a d-c circuit, the resistance can be obtained by the relation

$$R = V/I$$

if the skin effect is neglected.

The resistance of a-c circuits may also be obtained directly from wattmeter and ammeter readings by the relation

$$R = P/I^2.$$

The reactance of the load can be found from the relation

$$X = \sqrt{Z^2 - R^2}.$$

It should be noted, however, that the sign of the inductance cannot be determined from ammeter, voltmeter and wattmeter readings.

2. Bridge methods. Bridge methods are the most accurate for resistance measurements because they are null methods and comparison is made with standard resistances, which can be made to a high level of accuracy. The principal types of bridges for direct-current measurements are (a) a four-arm (Wheatstone) bridge, (b) a slide-wire bridge and (c) a double (Kelvin) bridge.

(a) *The Wheatstone bridge.* A schematic diagram of the Wheatstone bridge is shown in Fig. 377. A storage battery *1* is connected to points *A* and *C* of the bridge via a switch *2*. A galvanometer *4* is connected to another pair of terminals *B* and *D* via another switch *3*. The unknown resistance r_x is inserted between terminals *A* and *B*. The values of r_a, r_b and r are so adjusted that the deflection of the galvanometer is nill when the switch *3* is closed. Then the potential at point *B* is equal to the potential at point *D* and

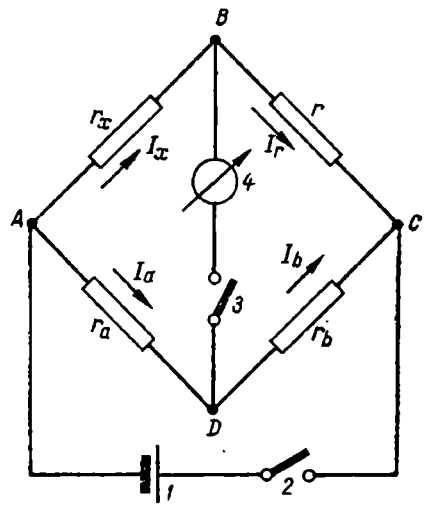


Fig. 377. A four-arm (Wheatstone) bridge

$$V_{AD} = V_{AB} = I_a r_a = I_x r_x;$$

$$V_{DC} = V_{BC} = I_b r_b = I_r r$$

Dividing the two relations gives

$$I_a r_a / I_b r_b = I_x r_x / I_r r.$$

Since $I_a = I_b$ and $I_x = I_r$ (no current flows through the galvanometer),

$$r_a / r_b = r_x / r,$$

whence

$$r_x = r_a / r_b r.$$

The resistances r_a and r_b are commonly called the ratio arms, and r the rheostat arm. They are all resistance boxes.

(b) *A slide-wire bridge* is shown diagrammatically in Fig. 378. A calibrated slide-wire is stretched between terminals *A* and *C*, with a sliding

contact D bearing upon it. The unknown resistance is inserted between points A and B, and a resistance box is connected between points C and B. The unknown resistance r_x is equal to the value of r adjusted on the resistance box multiplied by the ratio r_a/r_b. This ratio is given by the ratio of the lengths of the slide wire, l_1/l_2 which is marked on the scale of the bridge.

Electrolyte resistances are best measured with a-c bridges to avoid the effect of the emf of polarization. The alternating current is supplied by an inductance coil connected to a d-c source and fitted with an interrupter.

Both the Wheatstone and slide-wire bridges are not suitable for the measurement of low resistances (under 1 ohm), since the resistance of the connecting leads and contacts affect the accuracy of measurement. Instead, use is made of double (Kelvin) bridges.

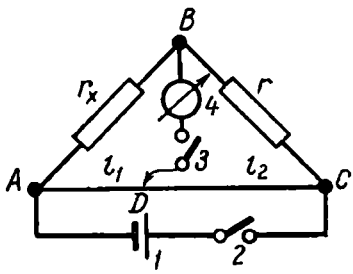


Fig. 378. Slide-wire bridge

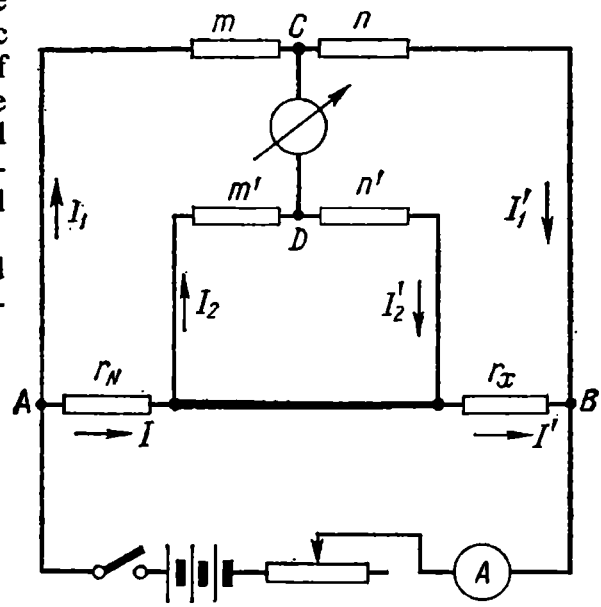


Fig. 379. Double (Kelvin) bridge

(c) *A Kelvin bridge* is shown diagrammatically in Fig. 379, where r_x is the unknown resistance, r_N is a known standard resistance, m, n, m', and n' are resistance boxes. Usually, $m = m'$ and $n = n'$. Connected to terminals C and D is a galvanometer. The supply circuit contains a knife switch, and the supply voltage is applied to terminals A and B. The values of m, m', n, and n' are adjusted (while keeping $m = m'$ and $n = n'$) until the galvanometer reads zero. Then $I_1 = I_1'$; $I = I'$ and $I_2 = I_2'$.

By Kirchhoff's second Law:

for the circuit $ACDA$

$$I_1 m - I_2 m' - I r_N = 0$$

or

$$I r_N = I_1 m - I_2 m';$$

for the circuit $CBDC$

$$I_1' n - I' r_x - I_2' n' = 0$$

or

$$I'r_x = I'_1 n - I'_2 n'.$$

Dividing these relations term by term gives

$$Ir_N/I'r_x = \frac{I_1 m - I_2 m'}{I'_1 n' - I'_2 n'}.$$

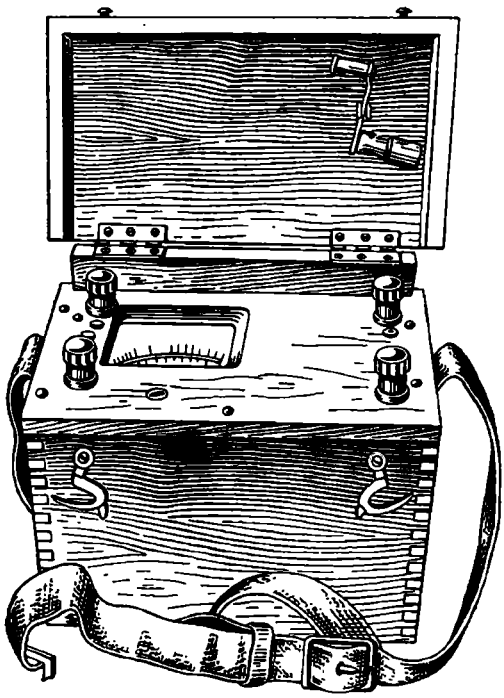
Since $m = m'$ and $n = n'$, we have

$$r_N/r_x = \frac{(I_1 - I_2)m}{(I'_1 - I'_2)n} = m/n.$$

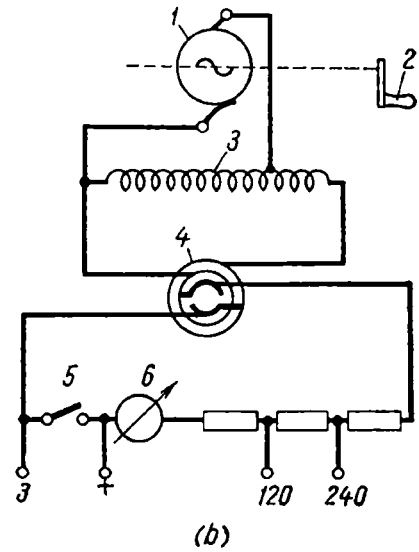
Hence,

$$r_x = r_N(n/m).$$

For higher accuracy, the wire connecting the resistances r_N and r_x must be of a very low resistance.



(*a*)



(*b*)

Fig. 380. Type МПИ megohmmeter:
a—general view; *b*—circuit diagram

3. Megohmmeters. Megohmmeters (or meggers) are instruments which measure the insulation resistance of electric circuits relative to earth and one another.

According to the regulations in force in the Soviet Union, the insulation resistance of conductors must be not less than 1000 ohms per volt of working voltage. For a network operating at 220 V, the insulation resistance must be not less than 220,000 ohms, or 0.22 megohm.

Insulation resistance should be measured with a voltage equal to the working voltage or not less than 100 V, whichever is the greater.

A megohmmeter consists of an emf source and a voltmeter. The scale of the voltmeter is calibrated in ohms (kiloohms or megohms, as the case may be). In measurements, the emf of the self-contained source must be equal to that of the source used in calibration.

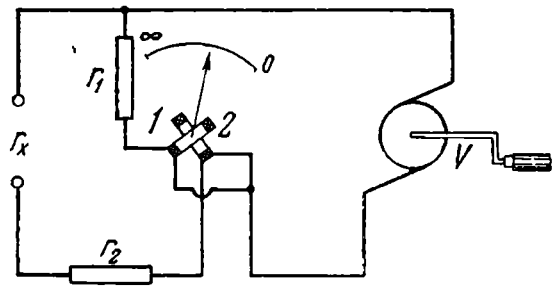


Fig. 381. Circuit diagram of TypeMOM megohmmeter

Fig. 380 shows a general view and a schematic diagram of a Type МПИ megohmmeter. An a-c generator *1* is turned by hand through a crank *2*. The generator impresses voltage upon a step-up autotransformer *3*, which raises the voltage to 400 V. A rectifier *4* converts the a-c voltage into d-c voltage. In insulation measurements, the terminal *3* is earthed, the "+" terminal is connected to the copper conductor of the wire under test, the button *5* is pressed, and the crank is rotated at a speed sufficient to cause the pointer *6* to give a full-scale deflection. Then the button is released, and the insulation resistance is read on the scale.

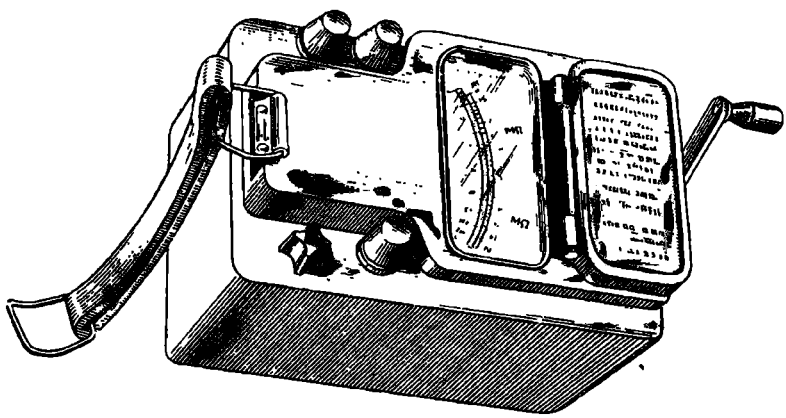


Fig. 382. General view of Type M-1101 megohmmeter

The Type МПИ megohmmeter is suitable for insulation resistance measurements at d-c voltages of 120 and 240 V. Its major drawback is that its readings depend on the speed of the generator.

Fig. 381 shows, diagrammatically, a Type MOM megohmmeter whose readings are independent of the speed of the self-contained generator. The moving system incorporates two coils *1* and *2* mounted on the same shaft and placed in the field of a parmanent magnet 90° apart. The generator

(supplying 500 V at 120 rpm) energizes the two coils over separate wires. Connected in series with one coil is a fixed resistance (or several different resistances in order to extend the range of the instrument). The unknown resistance is connected in series with the other coil. The currents in the coils interact with the magnetic field and produce opposing torques.

The deflection of the moving system depends on the ratio of the currents in the coils and is independent of the applied voltage. The unknown resistance is read directly from the scale of the instrument. The accuracy of measurement is unaffected by variations in the speed of the generator between 60 and 180 rpm.

Fig. 382 shows a general view of a Type M-1101 moving-coil megohmmeter.

Exercises

1. An ammeter reads 9.9 A. The true value of current is 10 A. Calculate the error and accuracy limit of the instrument. The range of the instrument is 20 A.
2. Calculate the resistance of a shunt required to extend five times the range of an ammeter with an internal resistance of 0.016 ohm.
3. An ammeter uses a shunt with a resistance 4 per cent of the instrument's resistance. Calculate the current in the circuit if the ammeter reads 3 A.
4. A voltage of 120 V has to be measured with a voltmeter for 15 V. Calculate the multiplier required, if the internal resistance of the voltmeter is 2000 ohms.
5. A frequency meter for 127 V has a resistance of 8000 ohms. Calculate the multiplier required for the instrument to be connected in a 220-V circuit.
6. What is the error of a single-phase wattmeter reading 60 W at 120 V, 0.6 A and 0.83 power factor?
7. A voltmeter is connected in a circuit via an instrument transformer with a marked ratio of 3000/100 V. Determine the voltage on the H.V. side if the voltmeter reads 95 V.
8. An ammeter connected in a circuit via an instrument transformer with a marked ratio of 150/5 A reads 4 A. Determine the current in the primary circuit.
9. An ammeter for 10 A, the scale of which has 100 divisions, is connected in a circuit via an instrument transformer with a marked ratio of 500/5 A. The ammeter reading is 42 divisions. Determine the current in the primary circuit of the transformer.
10. A three-phase wattmeter is connected in a circuit via a voltage transformer with a marked ratio of 3000/100 V and a current transformer with a marked ratio of 50/5 A. Calculate the power in the primary circuit if the wattmeter reads 150 W.
11. A wattmeter for 150 V and 5 A has a 150-division scale. It is connected in a circuit via a voltage transformer with a marked ratio of 3300/100 V and a current transformer with a marked ratio of 600/5 A. Calculate the power in the primary circuit if the wattmeter reading is 72 divisions.
12. A single-phase wattmeter for 150 V at 5 A has a 250-division scale. It is connected in a balanced three-phase circuit via a voltage transformer with a marked ratio of 500/100 V and a current transformer with a marked ratio of 40/5 A. Calculate the power in the three-phase circuit if the wattmeter reading is 50 divisions.

Review Questions

1. What are the conditions for a state of equilibrium of the moving system of an electrical measuring instrument?
2. How are electrical instruments classed according to the quantity measured, the kind of current, the physical principle used, and accuracy?

3. What requirements should be met by electrical instruments?
4. Describe the construction, merits, demerits, and applications of moving-coil instruments.
5. Same about moving-iron instruments.
6. Same about electrodynamic instruments.
7. Same about hot-wire instruments.
8. Same about induction-type instruments.
9. Same about thermo-emf instruments.
10. Same about rectifier-type instruments.
11. Same about vibration-type instruments.
12. What symbols are used on the dials of electrical instruments?
13. What instruments can be used for the measurement of power in d-c circuits?
14. Draw a connection diagram for a d-c watthour-meter.
15. How is power in a-c circuits measured?
16. Draw a connection diagram for a single-phase a-c watthour-meter.
17. Draw a connection diagram for a three-element wattmeter in a three-phase circuit.
18. Draw a connection diagram for a two-element three-phase wattmeter in a high-voltage circuit.
19. How is reactive energy measured?
20. How can the power factor of a circuit be determined?
21. Draw a connection diagram for a three-phase var-hour meter.
22. Draw a connection diagram for an electrodynamic power-factor meter in a high-voltage three-phase circuit.
23. How is resistance measured in d-c and a-c circuits?
24. How is insulation resistance measured?

Chapter Fifteen

ELECTRIC MOTOR DRIVES

178. General

Where a machine is actuated by an electric motor, it is said to be electric driven, and the motor doing this is called the *drive motor*.

The first practical application of electric drive in Russia was by Yakobi in 1838 when he installed an electric motor powered from a voltaic battery on board a boat which was capable of carrying 11 passengers at a speed of 4 kph. The spread of the electric drive in industry was especially rapid after Dobrowolsky invented his three-phase induction motor in 1890.

There are two methods that can be employed for driving the machines of a shop: in groups or individually. Group driving is from a line shaft carried in bearings. The line shaft is fitted with a multi-step pulley which serves to vary the speed of the driven machinery. The driven machinery can be stopped by switching off the drive motor. A serious drawback in this type of driving is that the line shaft has to be run at times when only a few or even a single machine is in operation. In individual driving, each machine is equipped with its own motor which may be arranged internally or externally to the driven machine. Group drive at one time was widely used. Individual drive, however, has so many advantages that it is used in practically all modern machines.

In some cases individually driven machines may have several motors, each executing a particular motion. A typical radial boring machine will, for example, have four motors, one for actuating the spindle, another for lowering and lifting the spindle arm, a third for slewing the spindle arm, and a fourth for supplying cutting fluid to the drill. In travelling cranes there are a hoist motor, a cross traverse motor for the cab, and a traction motor for the bridge.

179. Characteristics and Performance of Drive Motors

The nominal duty of a drive motor is the duty corresponding to the service conditions and performance marked on its nameplate.

The nominal power of a drive motor is the mechanical power developed on the shaft in nominal duty and marked on its nameplate in watts or kilowatts (in the Soviet Union).

The nominal speed of a motor is the speed marked on its nameplate for nominal duty.

The nominal torque is the torque developed on the shaft at nominal power and nominal speed under normal working conditions.

As has been stated elsewhere, the losses in an electric machine are converted into heat. Part of this heat is dissipated by the machine, and the remainder is absorbed causing the temperature to increase. The temperature becomes stationary when heat absorption is zero, that is, when the rate at which heat is generated in the machine equals the rate at which it is dissipated. Accordingly, there are three duties:

1. *Continuous duty* is that duty when the on-period is so long that the motor reaches a steady-state temperature.

2. *Short-time duty* is that duty when the motor does not attain a steady-state temperature during an on-period. The off-period is long enough for the motor to cool down to ambient temperature.

3. *Intermittent duty* is that duty in which an on-period and an off-period are not longer than 10 minutes each and recur repeatedly. Intermittent duty is characterized by the so-called *duty factor*, which is the ratio of the on-period to the sum of the on- and off-periods. If, for example, a motor operates for 5 hours during an 8-hour day, the duty factor will be

$$t_{on}/T = 5 \div 8 \times 100 = 62.5 \text{ per cent},$$

where t_{on} is the duration of the on-period and T is the sum of the on- and off-periods. Standard duty factors are 15, 25 and 40 per cent. They are given in the nameplate and certificate of each motor.

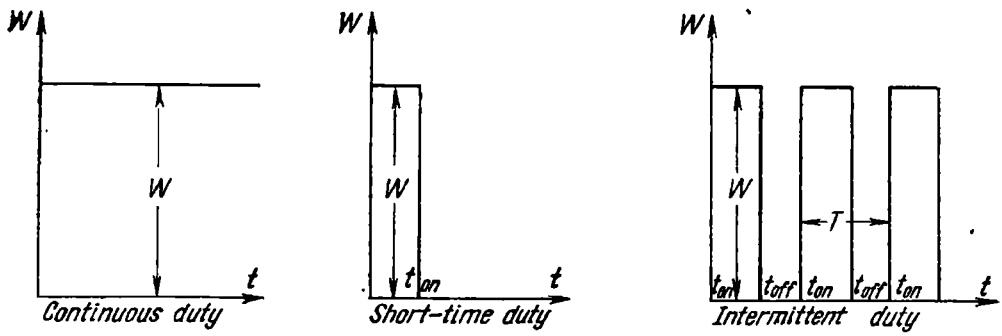


Fig. 383. Duty graphs of electric motors

Continuous-duty motors are employed to drive fans, compressors, generators, etc. They may be in operation for many hours and even days in succession. Short-time duty motors are at work in shipping locks, bascule bridges, and the like. Intermittent-duty motors are used in cranes, hoists, lifts, rolling mills, some metal-working machines, etc.

Fig. 383 shows graphs of the various motor duties.

180. Insulation of Drive Motors

The maximum power that can be developed by an electric motor is limited by its temperature rise. The permissible temperature rise is in turn dependent on the heat resistance of the motor insulation and the cooling of the motor.

In Soviet practice, motor insulation is grouped into five classes.

Class A insulation consists of materials or combinations of materials such as cotton, silk, and paper when suitably impregnated or coated with various oils, and also enamels and varnishes.

Class B insulation comprises materials or combinations of materials such as mica, asbestos and similar inorganic materials with suitable organic bonding, impregnating or coating substances.

Class BC insulation consists of mica, fibre glass and asbestos bonded, impregnated or coated with heat-resistant varnishes.

Class CB insulation embraces inorganic materials bonded, impregnated or coated with heat-resistant varnishes other than those listed in Class A.

Class C insulation consists of materials or combinations of materials such as mica, porcelain, glass, quartz and other inorganic substances without a binder.

The maximum safe temperature for Class A insulation is 105°, for Class B insulation 120°C, for Class BC insulation 135°, a somewhat higher temperature for Class CB insulation, depending on the varnish used, and no limit of temperature is fixed for Class C insulation.

The average ambient temperature is taken to be +35°C.

181. Types of Motor Enclosures

Different types of enclosures giving various degrees of protection to the operating parts and windings can be built. The different types as specified by relevant U.S.S.R. Standards are as follows:

1. *An open machine* is one having ventilation openings which give free access to live and rotating parts without protecting them from accidental contact or ingress of foreign objects.

2. *A guarded machine* is an open machine in which access to live and rotating parts is so limited as to prevent accidental contact with such parts or ingress of foreign objects.

3. *A drip-proof machine* is an open machine in which the ventilating openings are so constructed that drops of liquid falling on the machine vertically cannot enter the machine.

4. *A splash-proof machine* is an open machine in which the ventilating openings are so constructed that drops of liquid falling on the machine at an angle of 45° from the vertical cannot enter the machine from any side.

5. *A totally enclosed machine* is one so enclosed as to prevent the free exchange of air between the inside and the outside of the case, but not sufficiently enclosed to be termed airtight.

6. *A water-proof machine* is a totally enclosed machine so constructed that it will exclude water applied as a stream from a hose. This type of machine is used on ships.

7. *An explosion-proof (flame-proof) machine* is a totally enclosed machine whose enclosure is designed and constructed to withstand an explosion of the gas or vapour which may occur within it and to prevent the ignition of the gas or vapour surrounding the machine. The ventilating openings in the machine are made long enough for the flame to be contained within the frame. Explosion-proof machines are employed in collieries and certain chemical plants.

8. *An airtight machine* is a totally enclosed machine in which all the openings are tightly closed so that at a given pressure of the surrounding atmosphere there is no circulation of air between the inside and the outside of the machine.

182. Types of Motor Cooling

The following types of cooling are employed in drive motors:

1. *Natural ventilation* where no fans, either integral with or external to the machine are used. Cooling air is made to circulate within the machine by the fan action of rotating parts and convection. This type of cooling is used in open machines.

2. *Forced ventilation* where cooling air is made to circulate over the hot parts of the machine by a fan which may be: either (a) integral with the machine and mounted on its rotor; or (b) external to the machine and driven by a separate motor. In the former case the machine is said to be *self-ventilated*, and in the latter, *separately-ventilated.*

Self-ventilation is employed in both guarded and enclosed types, while separate ventilation is ordinarily confined to enclosed machines.

Low- and medium-power machines operating in sufficiently large rooms with a clean cooling atmosphere draw and discharge the air from and into the surrounding atmosphere.

Enclosed machines installed in locations with a contaminated atmosphere suck in cooling air through a pipe or pipes (for which reason such machines are termed pipe-ventilated) from the outside and discharge it into the room where the machine is located.

To avoid an excessive temperature rise in the room, the hot air may be discharged outdoors.

A further type of cooling is closed-circuit ventilation, in which after the ventilating air has been heated by the various parts of the machine, it is cooled by passing it through a cooler and then returning it to the system.

183. Speed Classification of Drive Motors

According to their speed-load characteristics, drive motors may be classified in one of several groups as follows:

1. *A constant-speed motor* is one whose speed is practically constant from no-load to full load. It has a flat speed-load characteristic curve. Motors in this class may have an absolutely constant speed regardless of the load (synchronous motors), or the speed may vary a few per cent from no-load to full load (shunt-wound d-c motors, a-c induction motors).

2. *A varying-speed motor* is one whose speed varies with the load, usually decreasing as the load increases. It has a drooping speed-load characteristic (series-wound d-c motors).

3. *An adjustable-speed motor* is one in which the speed can be varied gradually over a considerable range but, when once adjusted, remains practically constant regardless of the load (field-controlled d-c and slip-ring induction motors).

4. *A multi-speed motor* is one which can be operated at any of sever a definite speeds but, when once adjusted, remains practically constant regardless of the load. It differs from the adjustable-speed motor in that it can be operated only at certain definite speeds without any gradual adjustment of the speed between these definite values (pole-changing induction and synchronous motors).

184. Calculation of Motor Power

Several factors enter into the calculation of the power required to operate a machine. These factors are:

1. Force, F, necessary to perform a given operation.

2. Linear speed u, revolutions per minute n, or angular velocity ω of the moving parts of the driven machinery.

3. Output or useful power, P_{out}, and input power, P_{in}.

When the driven machine operates at constant speed, only static forces of resistance, consisting of useful and parasitic resistance (friction in the mechanisms, resistance of the material, etc.), have to be overcome. In the case of nonuniform motion, the total resistance is made up of static resistance and dynamic resistance due to the inertia of the moving mass.

In rotary motion, the forces of resistance may be replaced by the respective turning moments

$$M = M_s + M_d,$$

where M is the total moment of resistance, M_s is the static moment of resistance, M_d is the dynamic moment of resistance. The moments are often expressed in kg-m (kilogram-metres).

The static moment of resistance is made up of the moment of useful resistance M_u and the moment of friction M_f:

$$M_s = M_u + M_f \text{ kg-m.}$$

As is known from physics, in the case of uniform rotary motion, the power is equal to the turning moment $M_r (= M_s)$ times the angular velocity ω:

$$P_r = M_r \omega \text{ kg-m/sec,}$$

where

$$\omega = 2\pi n/60;$$

$$n = \text{rpm}$$

Since 1 kW = 1.36 hp or 1 kW = 1.36 × 75 = 102 kg-m/sec,

$$P_r = M_r 2\pi n/60 \times 102 = M_r n/975 \text{ kW (approximately).}$$

When the rotational speed of the driven machine is being increased, some additional power has to be spent in order to accelerate the rotating parts. Conversely, when the speed is being decreased, some power will be given up by the rotating parts. In some cases this is utilized to handle peak loads (by means of flywheels) or is returned to the supply line.

It is difficult to overestimate the importance of adequately selecting the power rating of a drive motor. In the case of an underrated motor the driven machinery will not be able to operate normally, the production rate will decrease, and the motor may grow dangerously hot. In the case of an overrated motor, on the other hand, the efficiency of the plant will be reduced, the power factor decreased (if an a-c motor is used), losses of energy will become prohibitive, and the total cost of the plant will increase materially.

Here are several examples of power calculation for drive motors employed in various applications. The calculation does not take into account the inertia of rotating parts. It is furthermore assumed that the driven machinery operates at constant speed, that starting is infrequent and is effected at no-load.

1. *Lathes.* The kW rating of motors for lathes can be obtained from the formula

$$P = F_c u_c / 60 \times 102\eta \text{ kW,}$$

where F_c = cutting force, kgf;
u_c = cutting speed, m/min;
η = efficiency of the lathe.

Sometimes the power of a motor drive may be calculated by means of the torque (double torques are taken for convenience in calculation):

$$2M = F_c d \text{ kg-m},$$

where F_c is the cutting force in kg and d is the diameter of the workpiece in metres. In this case the power of the motor will be

$$P = 2M_c n/2 \times 975\eta \text{ kW}.$$

Lathes are driven by three-phase induction motors with push-button control.

2. *Drilling machines.* The power of motors for drilling machines may be calculated from the formula

$$P = 2M_c n/2 \times 975\eta \text{ kW},$$

where $2M_c$ is twice the cutting torque in kg-m, n is the rpm of the drill, and η is the efficiency of the drilling machine.

Drilling machines are driven by three-phase induction motors.

3. *Planers.* There are several types of planing machines,[1] which also include shapers and slotters. The power of planer-type motors and the cutting force can be calculated from the same formula as for lathes.

4. *Liquid pumps.* The power of motors for liquid pumps can be determined from the formula

$$P = \frac{Q\gamma H}{102\eta_p\eta_{tr}} \text{ kW},$$

where Q = delivery of the pump, m^3/sec;
γ = specific weight of the liquid, kg/m^3;
H = total head, m;
η_p = efficiency of the pump;
η_{tr} = efficiency of transmission between pump and motor.

The delivery of reciprocating pumps can be found thus:

$$Q = k\frac{\pi D^2 ln}{4 \times 60} \text{ m}^3/\text{sec},$$

where k = volumetric efficiency (the ratio of the actual volume discharged to the piston displacement; it is usually 0.9);
l = length of the piston, m;
D = diameter of the piston, m;
n = strokes per minute in single-acting pumps.

The total design head is

$$H = h_s + h_d + h_f,$$

where h_s = suction lift, m w.g.;
h_d = delivery head, m w.g.;
h_f = friction head, m w.g. (it is taken from tables per 100 linear metres of piping).

Table 18

Summary of Drive Motor Applications

Type of motor	Speed control	Applications
Induction, squirrel-cage:		Low- and medium-power drives where speed control is not essential; infrequent startings.
with normal slip	None	
with high slip	None	Same, except for frequent startings; also in conjunction with flywheels.
multi-speed	Speed control by changing number of poles of stator winding; speed range of 6 : 1; no control between steps; max 1/6th of top speed	Adjustable-speed service (low-power metal-working machines and presses).
Induction, slip-ring	Speed can be reduced continuously to 50 per cent below normal by constant-torque resistor	Cranes, auxiliary fans, smoke exhausters.
Same	Speed can be reduced to 25 per cent or less of normal by fan-duty resistor	Same
Direct current, shunt, powered by d-c	Continuous speed range of 4 : 1 max	Mainly for metal-cutting machines.
Direct current, shunt	Continuous speed range of 100 : 1 and more by means of Ward-Leonard control	For service requiring broad speed range or frequent reversals (metal-cutting machines, reversible rolling mills, paper-making machines, etc.).
Direct current, shunt, powered from thermionic converters	Continuous speed range of not over 60 : 1	Nonreversible rolling mills and some metal-cutting machines with electric speed control.
Direct current, series	Continuous speed range of 3 : 1 max	Crane service with close speed regulation, low speed and a flat speed-load characteristic, also widely used as traction motors.
Synchronous	None	Large compressors, pumps, motor-generator sets, main drives of continuous rolling mills; medium and large drives in the paper and cement industries.

5. *Compressors.* The power of motors for compressors can be found from the formula

$$P = \frac{WQ}{102\eta_c\eta_{tr}},$$

where W is the work required to compress 1 m^3 of air to the desired pressure, in kg-m; and Q is the delivery of the compressor in m^3/sec.

Table 18 summarizes the principal types of drive motors, their characteristics and applications.

185. Protection of Motors

To save the motor insulation, windings and wiring fittings from being damaged beyond repair in the case of abnormal motor operation, protective devices must be incorporated in the control circuit (or circuits) to interrupt the flow of current to the motor.

The most frequent causes of abnormal operation of electric motors are overloads, short-circuits, low voltage, or voltage failure.

An overload is an increase in motor current above its safe limit. Overloads which are of short duration and limited magnitude should not cause the protective device to operate, since they are not dangerous to the motor. On the other hand, the protective device must respond to overloads of long duration and of high magnitude which are dangerous to the motor in that they may burn the windings and insulation. This also applies to short-circuits in the windings which are equally dangerous. The effect of a protective device, operating in response to excessive currents or short-circuits, to interrupt current flow to the motor is termed *overload* or *overcurrent protection.*

Overload or overcurrent protection may be effected by fuses, electromagnetic and thermal overload relays. The selection of a particular protective device depends on the kW rating, type and purpose of the motor, starting duty and nature of the overload.

186. Protection of Motors by Fuses

A fuse is a wire or strip of low-melting-point material (copper, zinc or lead) mounted on an insulating base. The section of wire or strip is chosen so that it will melt when the current reaches a predetermined value in a specified time, thereby disconnecting the equipment from the supply.

Although widely employed, fuses suffer from many disadvantages. One of them is a relatively low *rupturing capacity* (or maximum current carried). Also the blowing of a fuse produces flames and hot gases, which may burn attending personnel and start fire.

Ordinarily, fuses are capable of withstanding an overload of 25 per cent for a long time, 60 per cent for one hour, and 80 per cent for about two

minutes. This implies that fuses cannot give protection to the motor against sustained but small overloads which may well cause damage to the motor while the fuse remains intact. It is only in the case of sudden current inrushes or short-circuits that fuses will disconnect the motor rapidly.

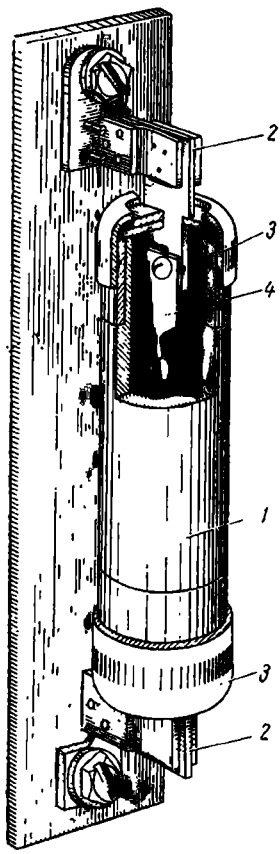


Fig. 384. Gas-generating fuse

It would seem that choosing a fuse for a current below the rated current of the motor would solve the problem. In such a case, however, the fuse would blow at starting, since the starting current is many times the rated current of the motor (for squirrel-cage induction motors the starting current is 5 to 7 times the running current). In practice, the maximum current of a fuse is determined from the relation

$$I_f \geqslant I_s/2.5,$$

where I_f is the current of the fuse link and I_s is the starting current of the motor.

The above relation is valid for motors which come up to speed within 5 to 10 seconds (normal starting duty). If a motor comes up to speed in 30 to 40 seconds (heavy starting duty), the current of a fuse must be determined from the relation

$$I_f \geqslant \frac{I_s}{1.6/2}.$$

There are open and enclosed fuses, the latter including plug-type and cartridge-type fuses.

Open-link fuses have the disadvantage of sputtering and are difficult to replace, for which reason they are used on a limited scale.

Plug-type fuses are manufactured for voltages up to 500 V and currents from 2 to 60 A. They are used in lighting circuits and for motor protection.

Low-voltage cartridge fuses are made for voltages up to 500 V and currents from 6 to 1000 A. The fuse link of a cartridge fuse can be placed either in an open porcelain container or in a closed glass, hard-fibre or porcelain container. Some makes of cartridge fuses are filled with quartz sand. At blowing, the sand breaks up the arc, thus cooling and extinguishing it.

Mention should also be made of gas-generating or deionizing fuses (Fig 384). A gas-generating fuse consists of a vulcanized fibre container *1* having brass caps *3* on both ends. The container houses a zinc-alloy fuse-link *4*. The fuse is mounted in the jaws *2* of a fuse holder. When the fuse blows, considerable heat is generated by an electric arc, which decomposes the surface layer of the vulcanized-fibre container. A gas high in hydrogen

is evolved under a high pressure; this quenches the arc due to a deionizing effect. Gas-generating fuses have a high rupturing capacity and are safe in service.

187. Protection of Motors by Circuit-breakers

D-c and a-c circuits operating on voltages up to 500 V use automatic air circuit-breakers.

The purpose of an automatic circuit-breaker is to open an electric circuit in the case of an overload or a short-circuit. In the Soviet Union, a-c

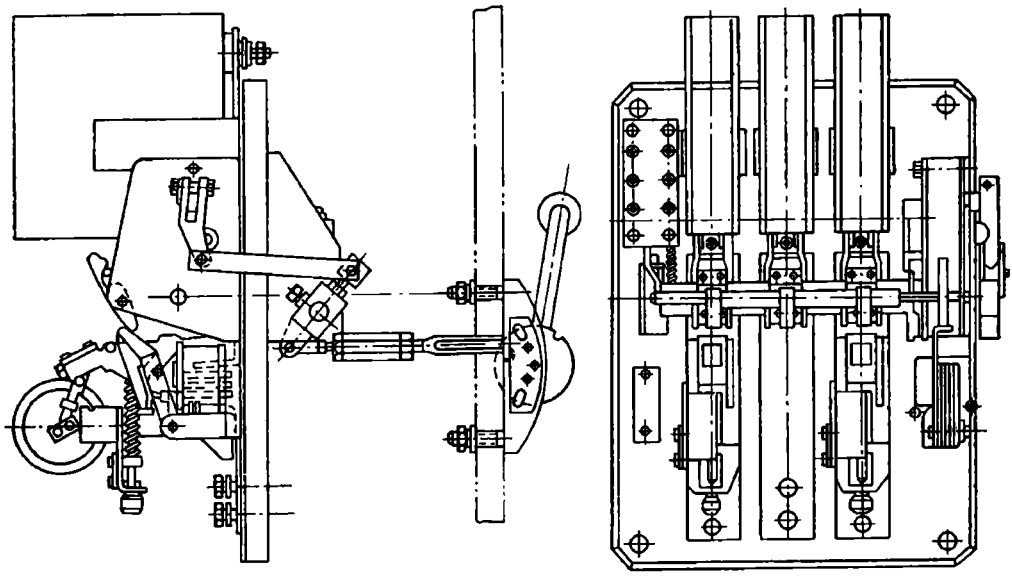


Fig. 385. General view of Type A-2050 air circuit-breaker

circuits use Series A-2000 circuit-breakers and d-c circuits, Series AB-45-1 circuit-breakers. Within each series the various types differ only in current ratings.

Circuit-breakers have one, two or three poles or phases and are locally or remotely operated. A general view of a Type A-2050 circuit-breaker is shown in Fig. 385.

188. Protection of Motors by Thermal Overload Relays

The operating principle of a thermal relay using a bimetal strip is discussed in Section 53. Diagrammatically, the construction of a thermal overload relay used in magnetic starters (see Sec. 194) is shown in Fig. 386.

The key element of a thermal overload relay is a bimetal strip *1*. Placed next to the bimetal strip is a heater coil *2*. When an overload causes suf-

ficient heating of the coil, the bimetal strip flexes upwards at its outer end and releases a catch *3*. A spring *4* rotates the catch about its axis to actuate the tripping member *5*, which opens the normally closed contacts *6*. The relay is hand-reset after tripping by means of a reset push-button.

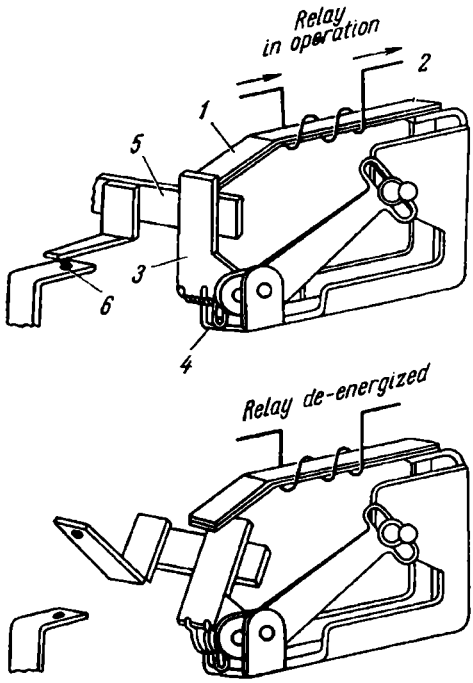


Fig. 386. Diagram of a thermal overload relay

There are thermal overload relays in which the bimetal strip is arranged to carry the motor current.

Thermal overload relays using a bimetal strip are manufactured for currents from 0.5 to 600 A. The heater coil is chosen to protect the current rating of the motor.

Thermal overload relays furnish protection for the motor only in the case of an overload and do not provide protection against short-circuits. This is because a short-circuit must be cleared instantaneously, while the heater coil takes some time to heat the metal strip to the set temperature.

Since the ambient temperature affects the operation of thermal overload relays, they must be mounted next to the motors they protect.

189. Electric Motor Control

Electric motors operating on voltages up to 500 V may be controlled manually, semi-automatically or automatically.

Manual control is effected by means of various switches. Semi-automatic control makes use of control apparatus which is energized by pressure on a push-button. In the case of automatic control, the driven machinery itself may start up the drive motor, or this can be done by suitable controllers.

190. Switches

The switches employed for manual motor control are of the knife-blade type and of the packet type.

Knife-blade switches are made single-, double- and triple-pole. They can be operated either by a handle or through a toggle leverage. In the former case thtk nife-blade switch is mounted on the front side of a switchboard, while in the latter case it is mounted on the rear side.

A handle-operated double- or triple-pole knife-blade switch without arc-breaking blades consists of two or three blades held together by a cross-bar. Each blade is free to rotate in a hinge jaw. When a knife-blade switch

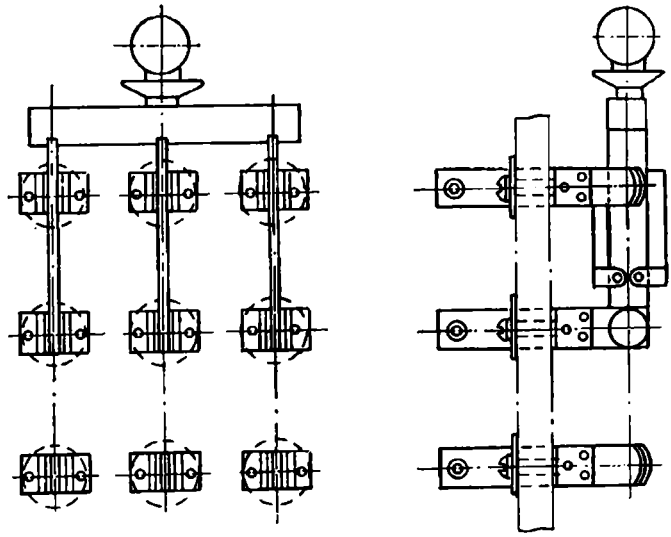


Fig. 387. Knife-blade switch with arc-breaking blades

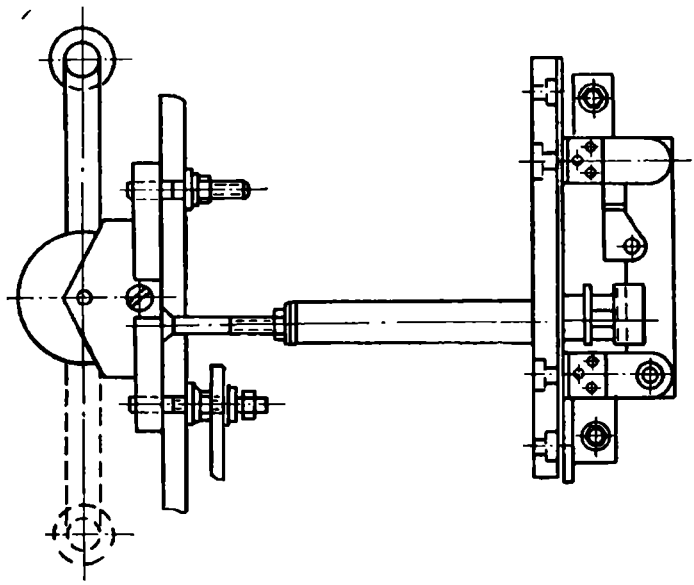


Fig. 388. Knife-blade switch operated through a lever and toggle mechanism

is closed, the blades are forced between forked contact- or break-jaws. When a switch is open, the blades are held locked in a horizontal position by a catch.

A knife-blade switch may be fitted with additional blades, which serve to extinguish the arc. When such a switch is being opened, the main blades break contact first while the arc-breaking blades remain in their jaws. Then springs rapidly force the auxiliary blades out of their jaws, and the arc that strikes at this instant has no chance of burning the main blades.

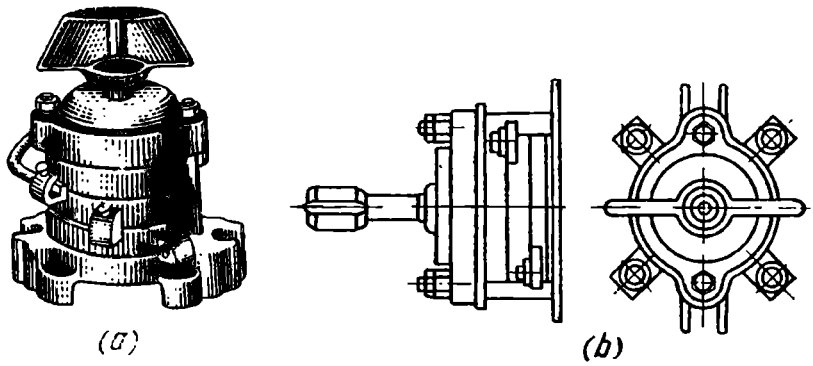


Fig. 389. Packet switch:
a—general view; *b*—drawing

Safety regulations require that knife-blade switches used on circuits operating at 380 and 500 V be enclosed in a protective casing or be operated by a lever through a toggle mechanism.

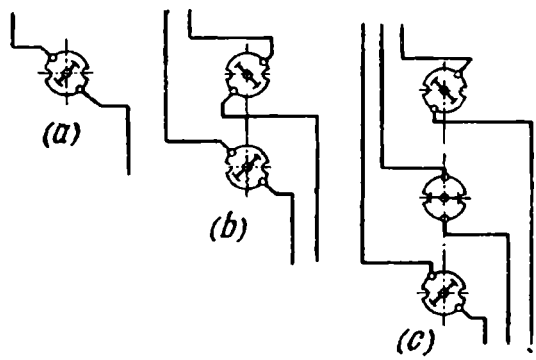


Fig. 390. Connection of packet switches into a circuit

Fig. 387 shows a knife-blade switch with arc-breaking blades, and Fig. 388 one with a lever and toggle mechanism.

Packet switches are used for manual closing and opening of electric circuits operating at up to 250 V direct current and up to 380 V alternating current (Fig. 389). A packet switch consists of several units, each mounted on an insulating base. The finger contact rotating within each unit makes and breaks the respective circuit. Packet switches are made single-, double- and triple-pole for currents from 6 to 100 A. The respective connection diagrams are shown in Fig. 390.

191. Contactors

Remote and automatic control of electric motors is effected by means of contactors which may be manufactured for either alternating or direct current.

Diagrammatically, a d-c contactor is shown in Fig. 391. The power circuit controlled by the contactor is connected to terminals *1* and *2* mounted on an insulating baseplate *3*. At *4* and *5* are contacts of the contactor, and

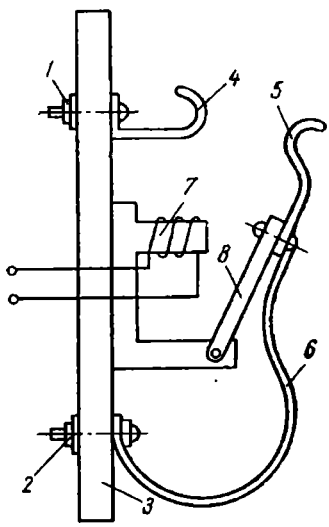


Fig. 391. Diagram of a d-c contactor

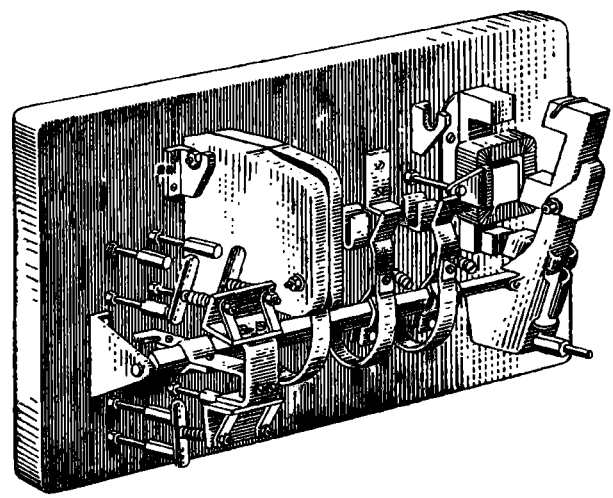


Fig. 392. A-c contactor

at *6* is a flexible conductor. The contactor is closed by a stationary electromagnet *7* energized from an auxiliary control circuit. When the control circuit is closed, the electromagnet attracts a movable armature *8*, thereby completing the circuit. The contactor will be held closed as long as the operating coil of the electromagnet remains energized.

Soviet-made d-c contactors are manufactured with one, two or three pairs of main contacts for voltages 220, 440 and 600 V, and for currents from 20 to 250 A. The operating coils of d-c contactors are rated for voltages of 48, 110 and 220 V.

Apart from the main contacts, which make and break the circuit being controlled, contactors also have auxiliary contacts for signalling and other purposes. D-c contactors can perform from 240 to 1200 switching operations an hour.

Fig. 392 shows a diagram of an a-c contactor. The operating coils of a-c contactors are made for voltages of 127, 220, 380 and 500 V, 50 c/s. They can perform up to 120 switching operations an hour.

192. Controllers

An electric controller is a device which serves to alter the electric power delivered to the apparatus to which it is connected.

In the case of motor control, the duty of a controller includes starting, speed control, reversing and stopping of the motor.

There are a-c and d-c controllers. If intended for service in power circuits, they are called *power controllers*. Where controllers serve to govern the operation of contactors and auxiliary devices of an electric controller, which then makes or breaks the power circuit of a motor, they are called *master switches*.

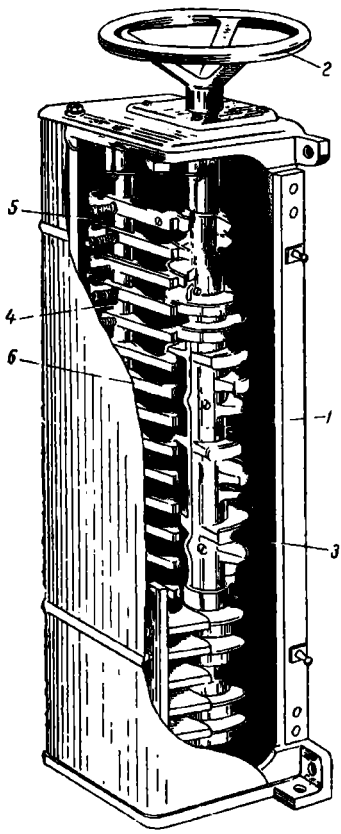


Fig. 393. Drum-type controller:

1—shaft; *2*—handwheel; *3*—copper contact segments; *4*—fixed finger-contacts; *5*—insulating bars; *6*—renewable copper tips

Controllers may be further subdivided into the *drum type* and the *cam type*.

A drum-type controller is shown in Fig. 393. The shaft *1* of the controller can be rotated by means of a handwheel *2*. The shaft carries copper contact segments *3* on its periphery. The segments are insulated from the shaft and serve as moving contacts. They differ in length and in relative angular position on the shaft. Some of them are connected electrically. An insulating bar *5* carries fixed finger-contacts *4* with renewable tips *6*. When the shaft is rotated, the segments come in contact with the respective fingers, thereby completing the requisite circuit.

The fixed fingers are separated from one another by asbestos cement partitions. This is done to prevent an arc-over. Controllers are also fitted with a blowout coil to quench the arc. When the coil is energized, the current sets up a magnetic field through the blowout structure and across the contact tips. When an arc is formed, the magnetic field of the arc and the magnetic field of the blowout coil repel each other, and the arc is forced upward and away from the contacts. The arc is lengthened, cooled and rapidly extinguished.

A cam-type controller (Fig. 394) consists of a series of contact pairs which are made and broken by cams mounted on the controller shaft. When the shaft rotates, cam *1* turns through an angle and bears upon a roller *2* hinged on a lever *3*. Rotation of the lever through an angle (on the right of Fig. 394) closes contacts *4* and *5*. As the contacts touch each other, the movable contact rolls over the fixed contact and removes the oxide film from the latter in a wiping action. The operating spring *6* holds the

roller tight against the surface of the cam. Contact pressure is maintained by another spring 7. Each pair of contacts has an arc-breaking device of its own, thus improving arc control.

Cam-type controllers have a greater rupturing capacity than have drum-type units and are capable of performing a greater number of switching operations (up to 600 per hour).

Fig. 395 shows a Soviet-made Series KA-5000 master switch. It operates on the same principle as does a cam-type controller. Master switches can control from 2 to 14 circuits and have from 1 to 7 working positions.

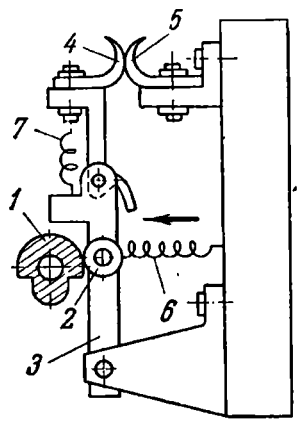


Fig. 394. Diagram of a cam-type controller:

1—cam; *2*—roller; *3*—lever; *4* and *5*—contacts; *6*—operating spring; *7*—pressure spring

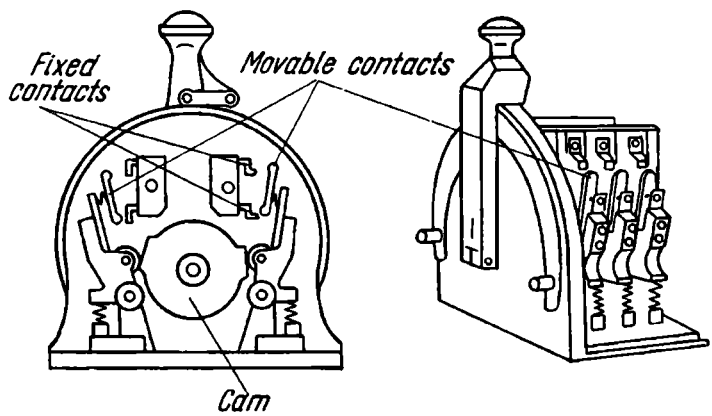


Fig. 395. Master switch

193. Push-button Switches

Remotely operated electromagnetic apparatus and signalling circuits make use of push-button switches. These switches are arranged as push-button stations consisting of from one to three push-button elements. Each element has one normally open and one normally closed contact pair. The contacts of a push-button switch are operated by a rocking action imparted to them by means of the push-button through a spring-and-cam mechanism. Fig. 396 shows a two-element push-button station.

Soviet-made push-button switches are manufactured for up to 440 V d-c and up to 500 V a-c.

194. Magnetic Starters

A magnetic starter is an electromagnetic apparatus intended for remote control of three-phase induction motors. It has a power circuit and a control circuit. The power circuit delivers the supply voltage to the stator

winding of the controlled motor; the control circuit handles the operation of the starter itself.

The power circuit contains fuses, line contacts and the heater coils of thermal overload relays. The control circuit incorporates a start-stop two-element push-button station, the operating coil of the starter, an auxiliary contact pair, and the contacts of the thermal overload relays.

Fig. 397 shows, diagrammatically, a nonreversing magnetic starter operated from a push-button station. Pushing the "Start" button closes a circuit from a phase of the external circuit through the "Start" button, a

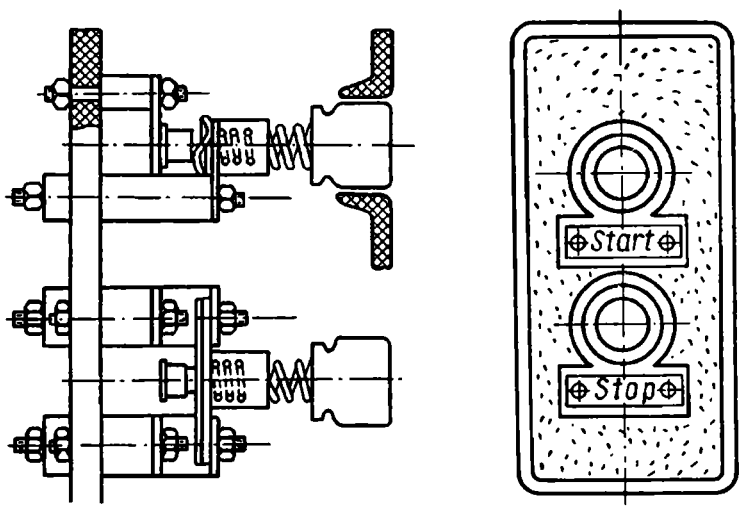


Fig. 396. Two-element push-button station

link, the "Stop" button, the operating coil *OC*, the contacts of the thermal overload relay *TR*, and back to the third phase. The operating coil pulls in the armature, the main line contacts *L* close, the motor is energized and begins to rotate. Just as the main line contacts close, the pair of auxiliary contacts *AC* close, thereby shunting the "Start" button, which now may be released. The path of the coil current will now be from the first phase through the auxiliary contacts *AC*, the "Stop" button, etc. The motor can be stopped by pressure on the "Stop" button, which breaks the line contacts. Overload protection for the motor is furnished by thermal overload relays whose heater coils are chosen in accordance with the current rating of the motor. When the current through the motor winding exceeds its rating, the contacts of the thermal relays break, the operating coil is de-energized, the line contacts break, and the motor is stopped automatically. Resetting of the thermal relays is by pressure on the "Reset" button.

After the motor has been stopped by a thermal overload relay, it may be again started after 0.5 to 3 minutes' time sufficient for the bimetal strip and the motor winding to cool.

The operating coil of the starter is designed to receive a voltage equal to 85 to 100 per cent of the rated voltage. The coil will hold the starter closed with the supply voltage falling to 50-60 per cent below the nominal voltage. Should the supply voltage drop suddenly or disappear altogether, the main contacts will break and the motor will be stopped automatically.

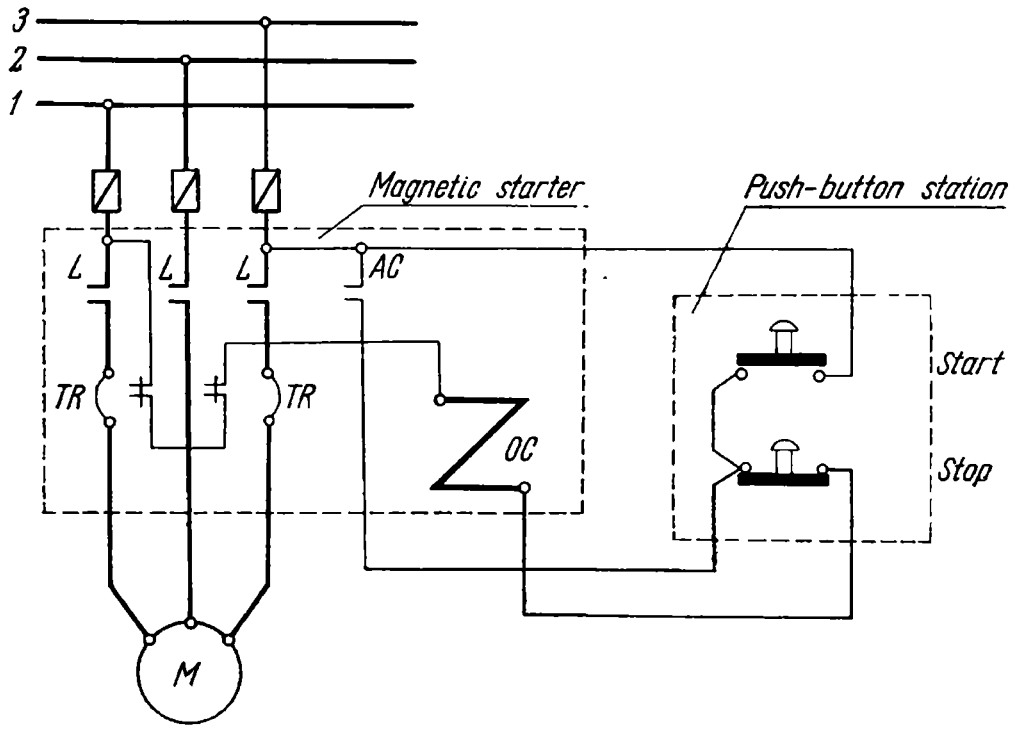


Fig. 397. Nonreversing magnetic starter:

L—main line contacts; *TR*—thermal relays; *AC*—auxiliary contacts; *OC*—operating coil; *M*—motor.

The fuses installed separately before the starter protect the circuit from short-circuits which may occur in the motor.

Where motors have to be reversed, reversing magnetic starters are employed. Diagrammatically, a reversing magnetic starter is shown in Fig. 398. It consists of two contactors, one for the forward and the other for the reverse direction of rotation. Accordingly, the starter has two operating coils, one for the forward direction (*OC*) and the other for the reverse direction (*RC*). The push-button station has three buttons marked "Forward", "Reverse" and "Stop", respectively.

The two contactors of the starter have both mechanical and electrical interlocks so as to prevent them from being closed together. The electrical interlock is effected by the contacts of the push-button station, which are normally closed by the "Forward" and "Reverse" buttons. The thermal

relays *TR*, the relay contacts and the auxiliary contacts serve the same purpose as their counterparts in nonreversing starters.

A reversing-magnetic starter operates as follows. Pushing, say, the "Forward" button closes a circuit from one phase through the "Stop" button, the "Reverse" button, the "Forward" operating coil *OC*,. through the contacts of the thermal relays *TR* and back to another phase.

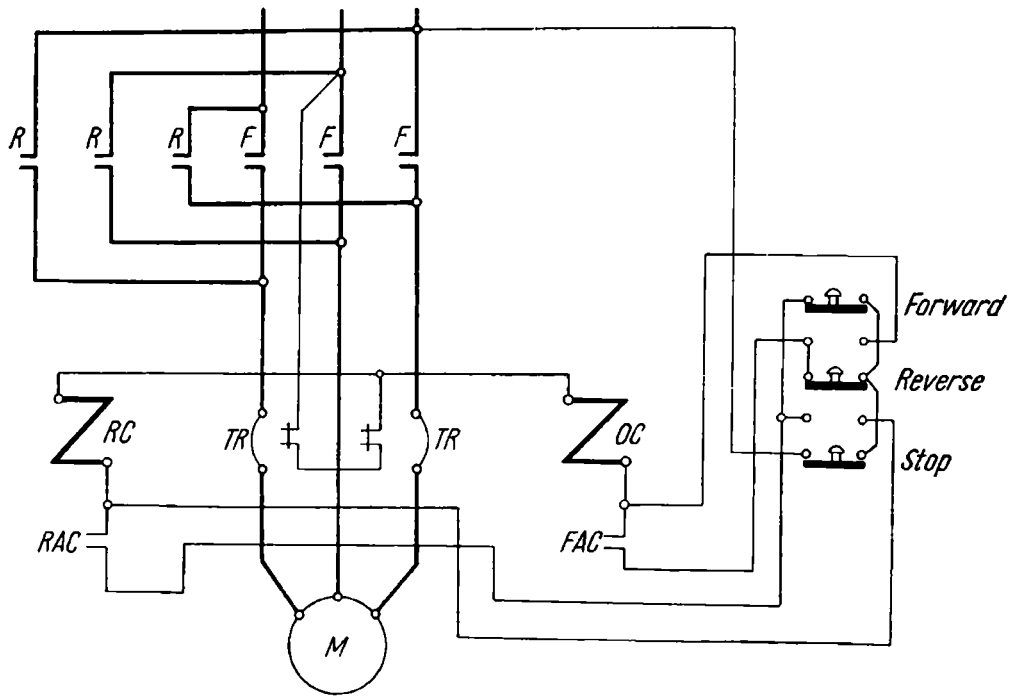


Fig. 398. Circuit diagram of reversing magnetic starter

The operating coil *OC* is energized to close the "Forward" line contacts. Simultaneously, the auxiliary contacts *FAC* are closed to shunt the "Forward" button, which may now be released. The motor is energized and begins to rotate. It should be noted that when pressure is on the "Forward" button, the current path is through the "Reverse" button, and vice versa. That is how an electrical interlock is effected. The motor is reversed by closing the "Reverse" contactor, which interchanges the outer phases.

195. Methods of Starting Induction Motors

Induction motors can be started by connecting them either to full line voltage (across-the-line, or direct-on, starting) or to a reduced line voltage.

Across-the-line starting may be effected by means of knife switches, packet switches, magnetic starters, contactors and controllers.

Since the motor is connected directly to the supply lines, the motor will draw an inrush current of from 2 to 7 times running current.

Across-the-line starting is most easily applied to squirrel-cage induction motors by closing a knife switch (a magnetic starter, etc.). Fig. 399 shows diagrammatically the across-the-line starting of a squirrel-cage induction motor. Slip-ring induction motors are started by means of a starting rheostat connected in the rotor circuit via the slip-rings and brushes. Before starting a slip-ring induction motor the operator should make sure that the starting rheostat is fully brought into circuit. As the motor comes

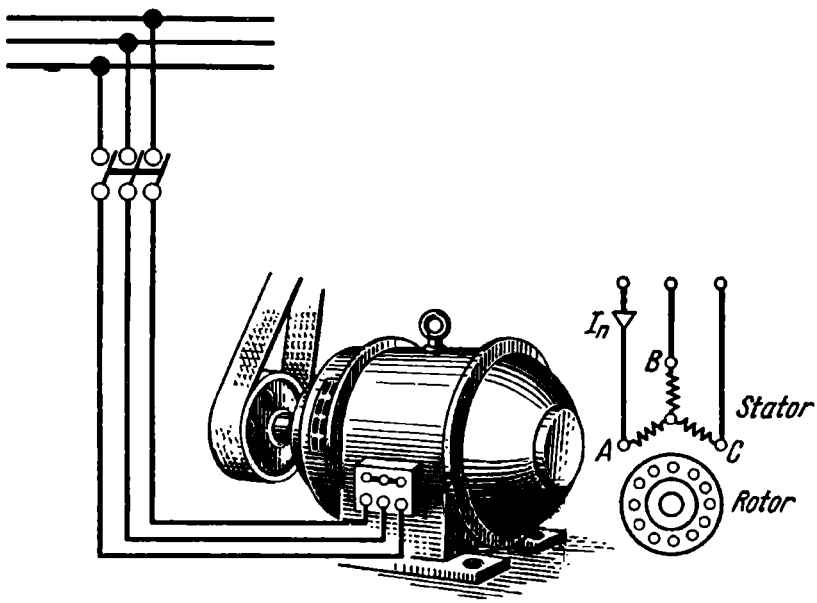


Fig. 399. Across-the-line starting of a squirrel-cage induction motor

up to speed, the rheostat is gradually brought out of circuit and finally short-circuited. The inclusion of a resistance in the rotor circuit reduces the starting current and increases the starting torque. The starting of a slip-ring induction motor is shown diagrammatically in Fig. 400.

Starting at a reduced voltage is resorted to where it is essential to limit the starting current of the motor. This can be done by means of (a) a star-delta starter and (b) an autotransformer starter.

A star-delta starter is shown diagrammatically in Fig. 401. At starting, the stator winding is star-connected by means of a knife-blade switch. As soon as the motor has picked up the maximum speed attainable with this type of winding connection, the knife switch is thrown over to the left, the stator winding is now delta-connected, and the motor is able to come up to maximum speed. This method of starting reduces the starting current to one third. The following is a practical example of star-delta starting.

Fig. 402*a* is the diagram of a stator winding connected in a star for starting. Let the voltage between the lines be 380 V. Then the phase voltage will be

$$V_{fY} = V_{lY}/\sqrt{3} = 380 \div 1.73 = 220 \text{ V}.$$

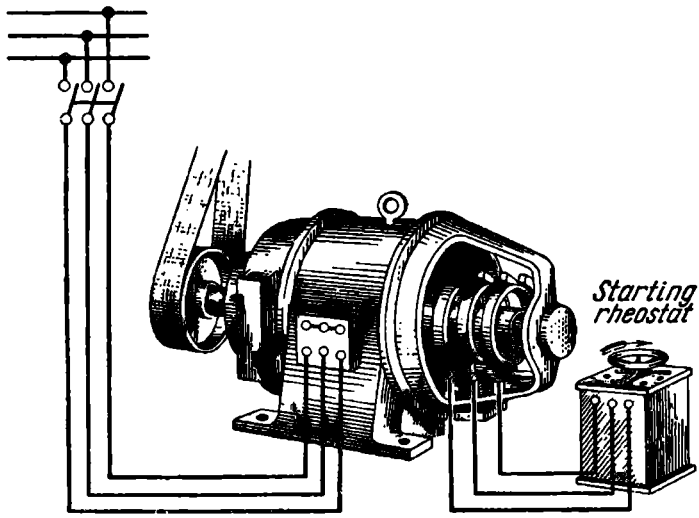


Fig. 400. Starting a slip-ring induction motor

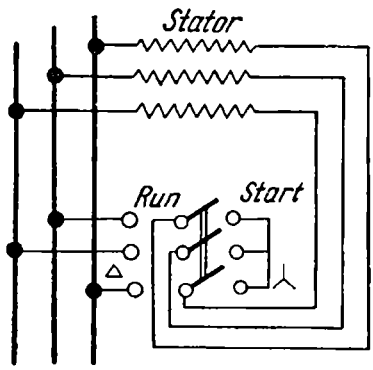


Fig. 401. Star/delta starting of a motor

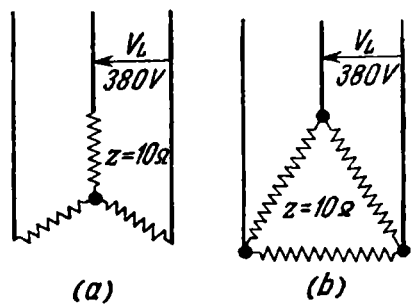


Fig. 402. Connection of the stator winding of a motor:
a—in star; *b*—in delta

Since the impedance of the phase winding of the motor is 10 ohms, the phase current will be

$$I_{fY} = V_{fY}/Z = 220 \div 10 = 22 \text{ A}.$$

In the star connection, $I_{lY} = I_{fY}$. Therefore, the current drawn by the motor at the end of the starting operation (with the starting inrush neglected) will be also 22 A.

In Fig. 402*b* is a diagram of the same motor with the stator winding connected in a delta and receiving a line voltage of 380 V. Then $V_{l\Delta} = V_{f\Delta}$, and the phase current will be equal to

$$I_{f\Delta} = V_{f\Delta}/Z = 380 \div 10 = 38 \text{ A}.$$

Since in the case of the delta connection $I_{l\Delta} = \sqrt{3} I_{f\Delta}$, the motor will draw a line current

$$I_{l\Delta} = \sqrt{3} \times 38 = 66 \text{ A}.$$

Thus, the line current of a motor with the stator winding star-connected is one third of the line current of the same motor when the stator winding is delta-connected.

Fig. 403 shows the connection of a star-delta change-over switch in the stator circuit of an induction motor.

In star-delta starting, the starting torque of the motor is also reduced to one third of the normal torque. For this reason, star-delta starting is only applicable to motors started at no-load or underloaded.

It stands to reason that star-delta starting can be employed only where the motor normally operates with the stator winding delta-connected.

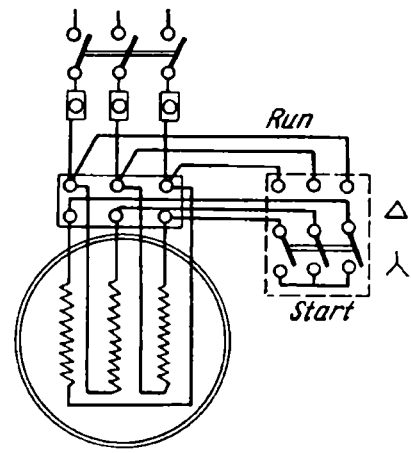


Fig. 403. Star-delta change-over switch connected into the stator circuit of an induction motor

An autotransformer starter is shown diagrammatically in Fig. 404 (at *a* for a low-voltage induction motor and at *b* for a high-voltage induction motor). The autotransformer *AT* is used for applying a reduced voltage to the motor windings at starting. At *b*, the autotransformer has a neutral which may be opened.

The motor *M* is started by closing an oil circuit-breaker *1*, which in turn closes the neutral of the autotransformer. This closes the main oil circuit-breaker *2*, and a reduced supply voltage is impressed on the motor terminals to set the motor into motion. After the motor has attained the maximum speed possible with this type of connection, the oil circuit-breaker *1* opens, and another oil circuit-breaker *3* closes. The motor is now connected to receive the full supply voltage and is able to come up to maximum speed.

In the case of autotransformer starting, the starting current can be reduced by 50 to 80 per cent.

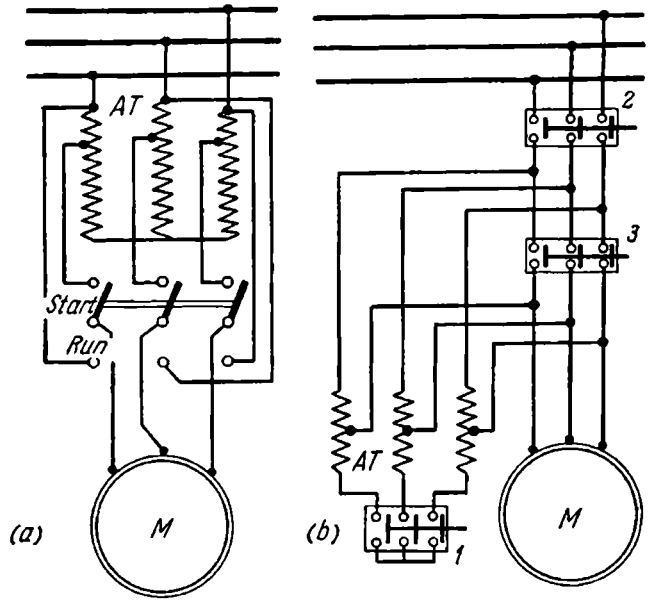


Fig. 404. Autotransformer starter:
a—for a low-voltage motor; *b*—for a high-voltage motor

196. Methods of Starting Synchronous Motors

Synchronous motors are generally started (1) by means of an auxiliary induction motor and (2) by induction starting.

The starting-motor method illustrated in Fig. 405 consists in that the auxiliary motor *2* is first set into motion by closing a knife-blade switch *3*.

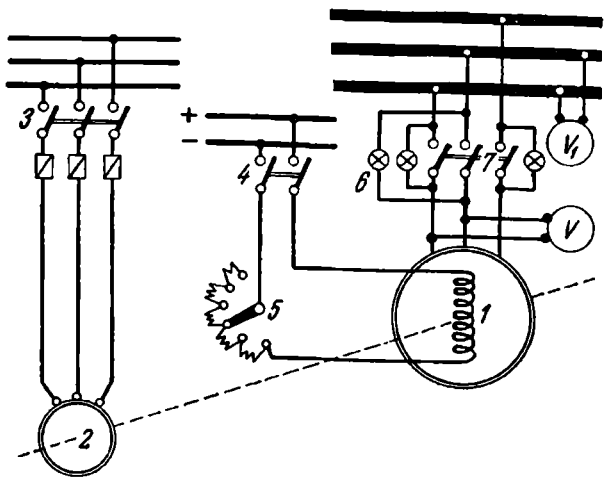


Fig. 405. Starting a synchronous motor with the aid of a starting motor

This brings the rotor of the synchronous motor *1* to the speed of the stator field. The speed of the starting motor is indicated by a tachometer. Then a d-c knife-switch *4* is closed to excite the rotor poles. Before the synchronous motor may be connected to a three-phase supply circuit, it must be synchronized in exactly the same way as for parallel operation of generators. To do so, a rheostat *5* is adjusted to obtain the same voltage on the voltmeter V (the stator voltage of the motor) as on the voltmeter V_1 (the supply voltage). The lamps *6* connected in parallel with the main knife-blade switch *7* will flicker at a rate which can be slowed down by varying the speed of the starting induction motor. The knife-blade switch may be closed when the lamps go dark simultaneously. Now the motor will pull into synchronism all by itself, and the starting motor *2* may be disconnected from the supply source by opening the switch *3*. It should be noted that the starting-motor method is employed on a limited scale since it calls for an additional induction motor and a complicated starting procedure.

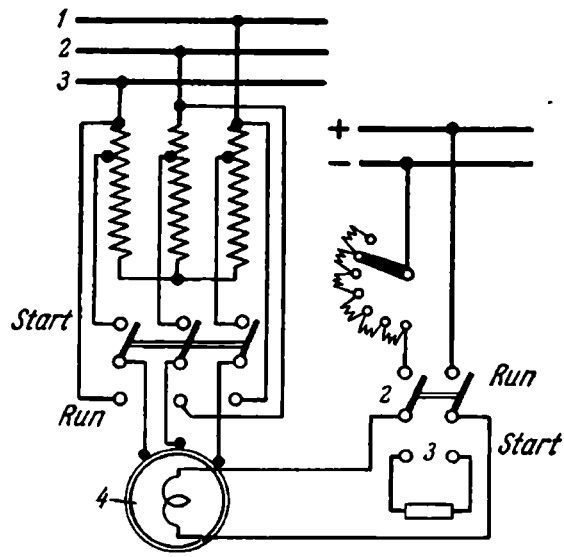


Fig. 406. Induction starting of a synchronous motor with the aid of an autotransformer

The induction-start method is accomplished by means of an auxiliary short-circuited winding placed on the rotor poles. It is connected to a resistance *3* (Fig. 406) by a knife-blade switch *2* for safety, since a large emf is induced in the field winding *1* of the motor at starting.

When three-phase supply voltage is impressed on the stator winding *4*, a revolving magnetic field is set up which cuts across the short-circuited (starting) winding and induces a current in it. The reaction of this current with the revolving field produces a torque which sets the rotor into motion. When the rotor comes to 95-97 per cent of synchronous speed, the knife-blade switch *2* is thrown over so that the rotor winding is connected to a d-c supply.

A serious disadvantage of the induction-start method is that the motor draws a large inrush current (which may be 5 to 7 times running current). The inrush current causes fluctuations in the supply line, which in turn affect operation of other loads. In order to reduce this inrush current, a modification of the induction-start method is used in which the motor is connected to receive a reduced voltage via a reactor or an autotransformer.

At the present time, synchronous motors are started by the induction-start method almost exclusively because of its simplicity and reliability. There are automatic types of induction starters for synchronous motors.

197. Reversal of Electric Motors

The direction of rotation of a d-c electric motor may be changed by reversing either the armature or the field connections, but not both. If both circuits are reversed, the motor will rotate in the same direction as before.

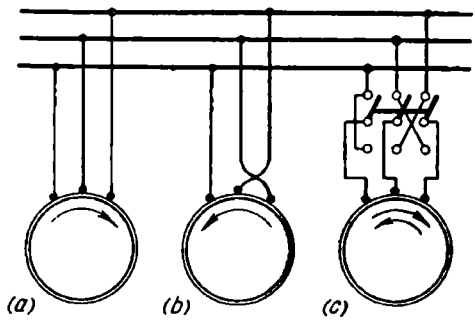


Fig. 407. Reversal of a synchronous motor: *a*, *b*—by interchanging connections of two of the three stator terminals; *c*—by means of a change-over switch

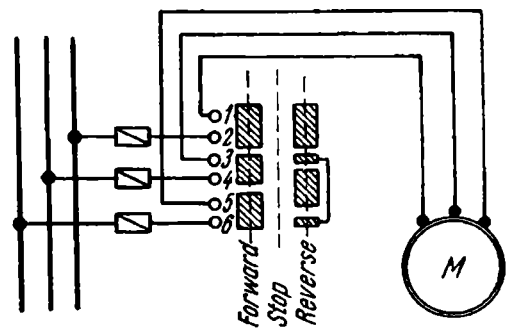


Fig. 408. Reversal of a motor by means of a controller

D-c series-wound motors can be reversed by either method, while shunt-wound and compound-wound motors can be reversed only by armature reversal.

An attempt to use field reversal on a shunt-wound motor would result in breakdown of the insulation due to the excessive emf of self-induction induced in it.

The direction of rotation of induction motors depends on the direction of rotation of the rotating field in the stator. Therefore, an induction motor can be reversed by changing over the connections of any two of the three stator terminals, as shown in Fig. 407*a* and *b*. This can be done by means of a change-over switch, as shown in Fig. 407*c*.

Where a motor is remotely controlled, its reversal can be accomplished by means of a reversing-type magnetic starter.

High-power motors are reversed by means of a controller (Fig. 408). Its drum carries copper segments which come in contact with fixed fingers *1* through *6*, thereby making the requisite switching.

198. Speed Control of Electric Motors

As follows from the formula $n \approx V/c\Phi$, the speed of a d-c motor is proportional to the applied voltage and inversely proportional to the magnetic flux. Therefore, the speed of a d-c motor can be controlled by varying the voltage applied to the motor or by varying the magnetic flux of the motor.

The applied voltage can be varied by inserting an adjustable resistance in series with the armature or by connecting the armature windings of several motors in series and parallel. The latter method has come into use in traction motors.

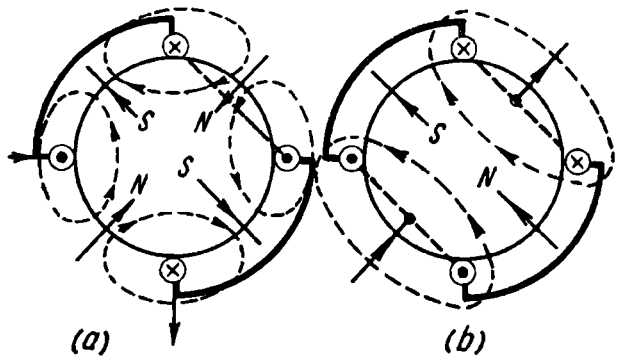


Fig. 409. Changing the number of poles on the stator of a motor

In most cases, however, the speed of a motor is controlled by varying the magnetic flux. To this end, a rheostat is connected in series with th field winding, and the speed can then be adjusted continuously over a broad range.

The speed of induction motors can be controlled in one of the following ways.

1. By changing the number of poles. The synchronous speed of an induction motor is given by the equation

$$n_s = 60f/p_2 \text{ rpm},$$

where f is the supply frequency and p_2 the number of pairs of stator poles. It is thus possible to change the speed by changing the number of poles, which can be done by suitably regrouping or reconnecting the stator winding. Gradual speed control cannot be effected by this method. Only a certain number of fixed speeds can be obtained.

By having two separate stator windings it is possible to obtain four speeds. A drum-type controller may be employed for making the necessary changes in the connections. In Fig. 409*a*, two coils of a phase are connected in series and give four poles. The same coils connected in parallel (Fig. 409*b*) will give two poles, or twice the speed obtained with the series con-

nection. Practical induction motors are made with four speeds as follows: 3000, 1500, 1000 and 750 rpm.

Two-speed operation may be obtained with a single winding arranged to give either alternate or consequent poles on the stator. In the former case adjoining poles are of different polarity, while in the latter case adjoining poles have the same polarity, thus giving half the number of effective poles and, therefore, twice the speed obtained with alternate poles.

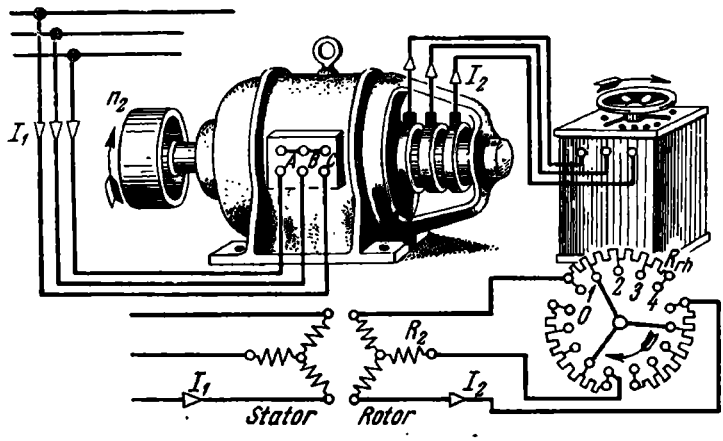


Fig. 410. Connection of an induction motor fitted with a combined starting-rotor rheostat

Speed control by pole changing is possible only in squirrel-cage induction motors, which can operate with any number of stator poles. On the other hand, slip-ring induction motors will normally operate with a definite number of poles. As a result, the application of the pole-changing method to slip-ring induction motors would require reconnection of the rotor winding, which is difficult to accomplish.

2. By changing frequency. The speed of an induction motor is proportional to the applied frequency so that by varying the frequency it is possible to obtain speed control. This may be done by supplying the motor from a frequency changer (or converter). Speed control by frequency changing is desirable where a large number of motors have to be controlled simultaneously (the motors of roller tables, looms, etc.).

3. By rotor resistance control. A three-phase resistance connected in the same way as a starting resistance may be used in the rotor circuit of a slip-ring motor to reduce the speed. As distinct from the starting resistance, the rotor resistance is designed to carry the full rotor current for a long time. In many cases, both resistances are made as a single unit.

This is how the rotor resistance effects speed control. The revolving field of the stator induces in the rotor winding an emf E_{2s}, which establishes a current I_2 in the rotor circuit. This current reacts with the stator field F_1 to produce a torque equal to the mechanical load. The insertion of a resist-

ance in the rotor circuit will reduce the rotor current I_2, the torque will become smaller than the mechanical load, and the rotor will slow down until a new increased emf, E_{2s}, restores the current and, consequently, the former torque. Thus, every increment in the rotor resistance reduces the speed of the motor.

A serious drawback to this method of speed control is that a considerable amount of energy is lost in the rotor resistance, the loss increasing as the speed range is extended. The connection of a combined starting-rotor resistance in a slip-ring induction motor is shown in Fig. 410.

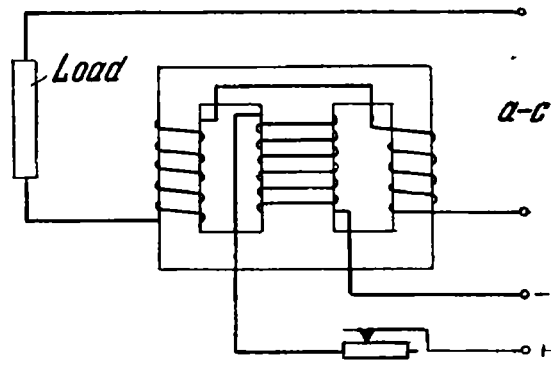


Fig. 411. Circuit diagram of a saturable reactor

4. By insertion of saturable reactors. In recent years, saturable reactors have come into use as speed regulators for electric motors. Fig. 411 is a diagram of a single-phase saturable reactor. It has one winding connected to carry an alternating current, and another winding (called the operating or magnetizing winding) connected across a d-c source. As the current through the magnetizing coil increases, the magnetic circuit of the reactor reaches saturation, and the inductive reactance of the a-c coil is reduced. Speed control is obtained by varying the current through the magnetizing coils of saturable reactors inserted in each phase of an induction motor, or by varying its impedance.

There are other methods of obtaining speed control: by the direct coupling of two motors connected to rotate either in the same direction (direct concatenation) or in opposite directions (differential concatenation); by shifting the stator; by using an induction slip clutch, etc.

199. Ward-Leonard Control of Direct-current Motors

In cases where wide ranges of speed control are required, especially with motors of large size (mine hoists, rolling mills, ships' steering gear, paper-making and cotton-printing machines), the power losses in resistances with the methods of starting and speed control already discussed may be prohibitively heavy. In order to avoid such losses, use may be made of a motor-generator set for supplying a variable voltage to the motor armature. One such arrangement is known as the Ward-Leonard system, shown diagrammatically in Fig. 412.

The Ward-Leonard system operates as follows. The work-motor *1* is powered by a dynamo *2*, which is driven by a synchronous or induction

motor *3*. The excitation current for the work motor and the dynamo is supplied by an exciter *4* mounted on the same shaft as the dynamo. The Ward-Leonard set is started by starting the driving motor. The field rheostat *5* of the dynamo is gradually brought out of circuit as the dynamo picks up speed, and the work-motor begins to rotate. The speed of the work-motor is controlled by means of the rheostat in the dynamo field.

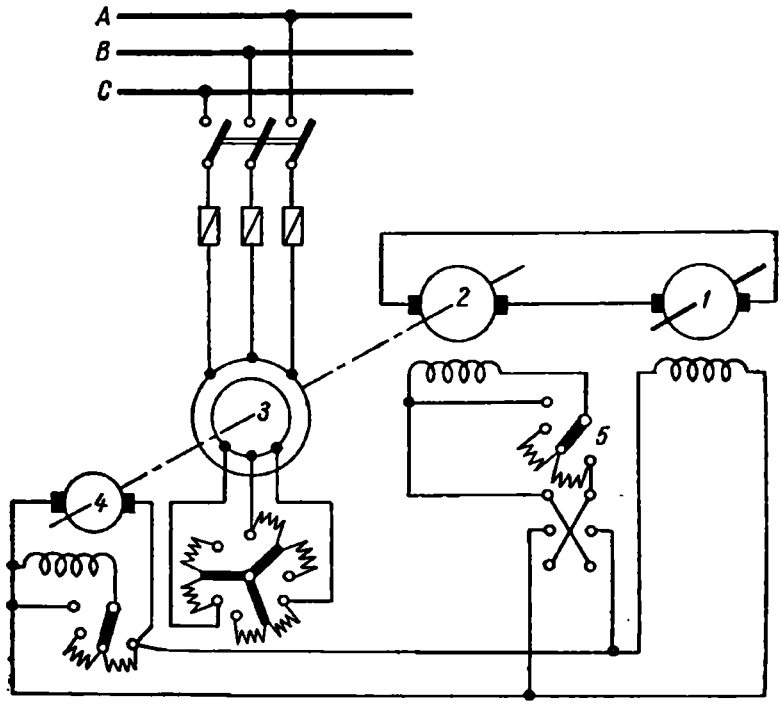


Fig. 412. Ward-Leonard control system

Gradual starting and speed ranges of the order of 100 to 1 are readily obtained with this type of arrangement. When reversal of the work-motor is required, a reversing-type potentiometer is used for the dynamo field control.

Where the load on the motor shaft sharply varies (such as in mine hoists, rolling mills, etc.), a modified Ward-Leonard arrangement, known as the Ilgner system, is used. It incorporates a heavy flywheel mounted on the shaft coupling the driving motor and dynamo. In operation with a flywheel, the driving motor has to have a drooping speed-load characteristic, i.e., its speed must decrease with the load on the shaft.

The purpose of a flywheel is this. An increase in the load on the shaft causes the motor to take more current from the dynamo, and more power is required to drive the latter. If there were no flywheel, the driving motor would take all the additional power from the supply line, thus causing

sharp fluctuations in it. The heavy flywheel, however, stores a large amount of kinetic energy. When an increase in the dynamo load causes the driving motor to slow down, some of the kinetic energy of the flywheel goes to sustain the peak load on the shaft of the work-motor. When the load on the work-motor decreases, the driving motor picks up speed, and the flywheel stores up kinetic energy.

200. Braking Systems for Electric Motors

If the load is removed from a motor and the motor is disconnected from the supply line, it will continue to run for some time due to inertia. The time elapsing before it stops will be especially long if the motor is massive and has run at high speed. It is essential, however, in many cases that the motor be stopped and reversed quickly (in machine tools, cranes, hoists, etc.). In fact, quick stopping of a motor is more essential than quick starting. Delay in starting up a motor only causes the machinery to stand idle; a delay in stopping a motor may result in heavy damage to equipment and even the loss of human life.

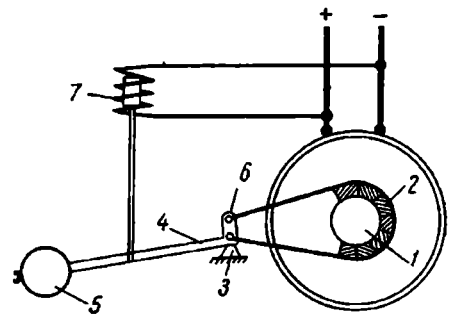


Fig. 413. Friction braking of a motor with an electromagnetic release

In general, electric motors can be braked mechanically or electrically.

Mechanical or friction braking. The usual type of friction brake consists of lined brake-shoes or a brake-band. Fig. 413 shows diagrammatically a band brake with an electromagnetic release. The pulley *1* of the motor is enveloped by a steel band *2* carrying wooden blocks. The ends of the band are made fast to the arms *6* of the braking lever *4*. The braking lever is hinged to a support *3* at one end and carries a weight *5* at the other. When the solenoid *7* is de-energized, the weight brings down the braking lever, the arms of the lever stretch the band taut, and the blocks close upon the pulley.

To start up the motor, the solenoid must be energized to pull in a steel plunger hinged to the braking lever. The lever will then rise, making the band slack, and the pulley will be released.

Friction braking is often employed in cranes. In this type of braking, the kinetic energy of the motor is converted to heat at the surface of the blocks or band.

Electric braking. Electric braking can be performed by (1) dynamic braking; (2) regenerative braking; and (3) plugging.

Dynamic braking for d-c motors. In this method the armature of the shunt-wound motor is disconnected from the supply and connected across a braking resistance, the motor field being kept energized. The machine

then acts as a generator loaded with a fixed resistance, and its armature emf is

$$E_{ret} = V - I_a R_a,$$

where V is the supply voltage and I_a, R_a, the armature current and resistance, respectively.

This emf is directed against the supply voltage, and so is the current established by it. Since the field current flows as before, the reversed armature current will react with it to produce a retarding torque. As the motor speed falls during dynamic braking, the retarding effect decreases. In order to maintain the retarding torque, some sections of the braking resistance may be then cut out of circuit. If, during dynamic braking, the field is also de-energized, the magnetic flux of the motor will collapse, and a very small retarding torque will be produced.

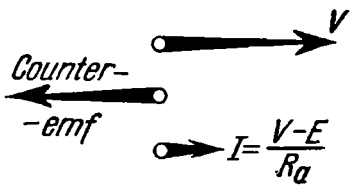


Fig. 414. Supply voltage, emf and current of a running motor

In the case of a series-wound d-c motor, when the armature is connected across a braking resistance the connections of the field winding should be changed over. If this is not done, the current through the field will also be reversed, and no braking will occur. To avoid this, the field connections are so changed that the current through the field winding will flow in the same direction as before. For compound-wound d-c motors, dynamic braking is performed in the same way as for shunt-wound motors.

Dynamic braking for a-c motors. It is possible to obtain dynamic braking of induction motors by disconnecting the stator winding from the a-c supply and exciting it from a d-c source so as to produce a stationary d-c field. This field is cut by the rotor conductor and the machine behaves as a generator. The braking effect is then produced as in the case of d-c motors.

Regenerative braking. A motor is said to regenerate when it is transformed into a generator and returns power to the supply line. For this action it is necessary for (1) the supply voltage to drop; (2) the motor to be over-excited; or (3) the motor to be running at higher than no-load speed. In all of these cases the armature current is reversed and a retarding torque is produced, slowing down the motor until the counter-emf of the armature becomes equal to the supply voltage. Regenerative braking will not stop the motor. It is effective only for braking overhauling loads. Overhauling loads are those which attain a speed greater than that of the motor speed-load characteristic and under which, therefore, the load tends to drive the motor. They may occur in a crane motor due to the action of the lowering weight or in the motor of an electric locomotive moving downgrade.

Regenerative braking is an inherent characteristic of shunt-wound motors and does not require any changing of connections. If a series-wound

d-c motor is overhauled by its load, the motor loses its excitation and no regenerative braking will occur unless the connections are changed. Therefore, for regenerative braking to be applicable in a series-wound d-c motor, the field and armature windings must be separated from each other and connected individually to the supply. Compound-wound d-c motors can be regeneratively retarded by an overhauling load only if their shunt field is stronger than their series field.

In the case of induction motors, regenerative braking occurs when the motor is driven at hypersynchronous speed.

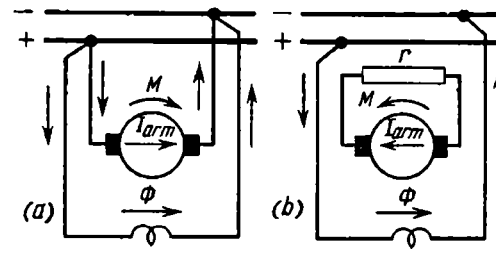


Fig. 415. Dynamic braking of a motor:
a—prior to braking; *b*—at braking

Plugging. Rapid braking of motors can be obtained by reconnecting the armature winding (in the case of a d-c motor) or any two of the three phases (in the case of an induction motor) in the order corresponding to reversed direction of rotation while the motor is running at full speed. Thus reconnected, the motor will develop a high retarding torque. When deceleration is completed and the motor stops, it must be disconnected from the supply, or it will restart in the opposite direction.

Plugging may be resorted to only in an emergency because of the shock it gives to the motor and power system. When the armature of a d-c motor is reconnected to the supply in the reverse direction, the counter-emf of the armature is added to the supply voltage, and the current through the armature will be 20 to 40 times running current. Apart from the mechanical shock, the heat dissipated may prove dangerous to the motor insulation and winding. In a-c motors, the shock due to plugging may also reach dangerous proportions. As a safety measure, d-c motors must use an additional resistance in series with the reversed power connections (the starting rheostat can be used for this purpose). The same is true of squirrel-cage induction motors. In slip-ring induction motors, the braking effect can be cushioned by adjusting the resistance in the motor secondary circuit.

201. Motor Control Systems Based on Electromagnetic Apparatus

1. Control of squirrel-cage induction motors (Fig. 416). In this arrangement, use is made of two contactors, one line *L* and the other retarding *RC*; two thermal relays *RR*1 and *RR*2; and a speed-control relay *SCR*. The supply circuit to the motor stator winding is called the main line. The main line is completed and interrupted by the line contacts *RC* and *LC* of the respective contactors. The contactor coils are controlled through a control circuit. The relay *SCR* is coupled to the shaft of the

motor and makes its normally-open contacts when a predetermined speed is attained by the motor.

Pressure on the start button energizes the contactor *L*, which closes the contacts *L* in the supply circuit to the motor. The auxiliary contact *L* shunts the start button, and the latter may now be released. At the same time, another (normally-closed) contact *L* interrupts the circuit

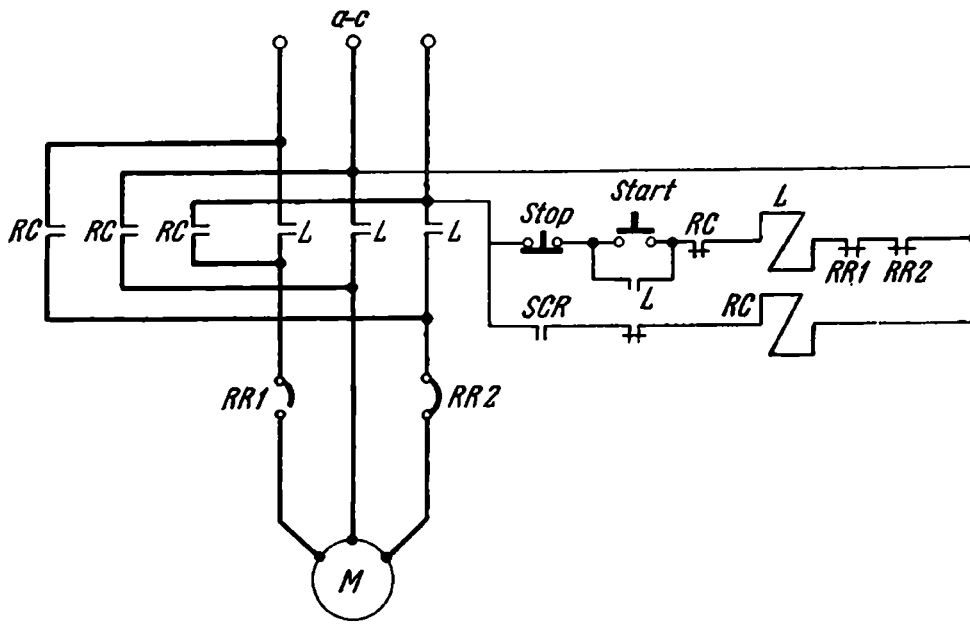


Fig. 416. Control of a squirrel-cage induction motor

to the contactor *RC*, and the latter cannot close, although the relay *SCR* closes its contacts.

Pressure on the stop button interrupts the circuit to the contactor *L*, the contacts *L* in the supply circuit of the motor break, and the motor is disconnected from the supply. The normally-closed contact *L* closes to energize the contactor *RC*, and the latter closes its contacts in the supply circuit to the motor. Referring to the diagram, this changes over the connection of two phases so that the motor, while running as before, develops a torque in the opposite direction. This is braking by plugging. As soon as the speed of the motor drops to 10 or 15 per cent of nominal, the relay *SCR* breaks its contacts, and the motor is disconnected from the supply. In the case of an overload, the thermal overload relays will break their contacts in the circuit of the contactor *L*, and the motor will be disconnected from the supply automatically.

2. Control of slip-ring induction motors. This arrangement (Fig. 417) makes use of a line contactor *L* and three accelerating contactors *AC*1, *AC*2 and *AC*3. The rotor circuit contains starting resistances and the coils

of the current accelerating relays $AR1$, $AR2$ and $AR3$. These relays are so adjusted that the drop-out current of $AR1$ is greater than that of $AR2$ and $AR3$, and the drop-out current of $AR2$ is greater than that of $AR3$.

Pressure on the start button energizes the line contactor, and its contacts complete the supply circuit to the stator winding of the motor. At the same time, the auxiliary contact L shunts the start button, and the latter may

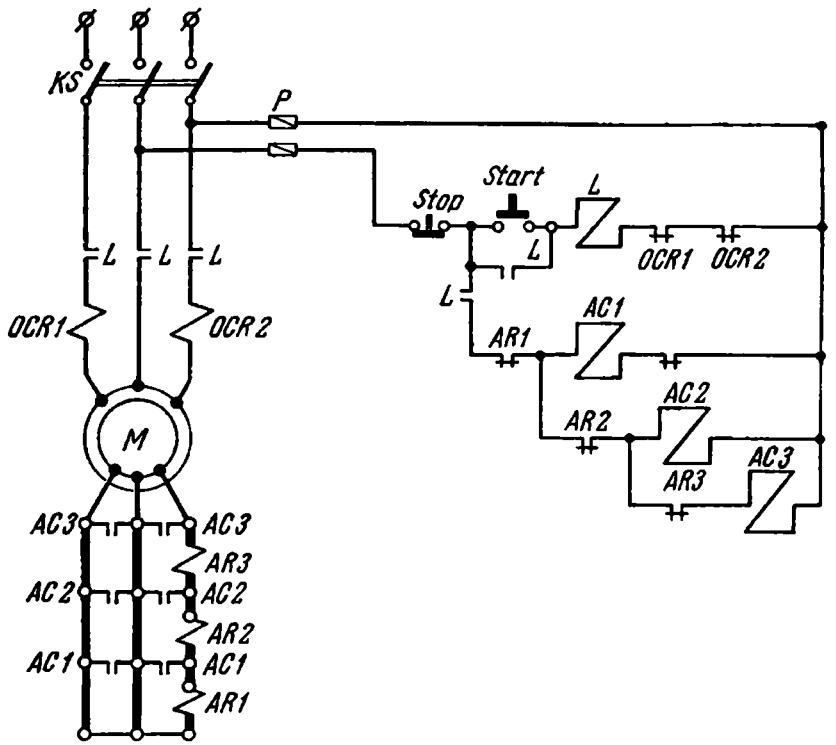


Fig. 417. Control of a slip-ring induction motor

now be released. Due to heavy current in the starting resistances, the coils of the three accelerating relays are energized at starting and they break their contacts in the circuits of the accelerating contactors $AC1$, $AC2$ and $AC3$.

As soon as the current in the rotor circuit drops to the value of the drop-out current of $AR1$, the latter is de-energized to complete the circuit to the contactor $AC1$, which will make its contacts $AC1$ in the rotor circuit, and the first step of the starting resistance will be short-circuited. As the current in the rotor circuit decreases, first $AR2$ and then $AR3$ ceases to function. In the control circuit the contactor $AC2$ is first energized to short-circuitanother step and then $AC3$, which short-circuits the last step of the starting resistance in the rotor circuit. Two overcurrent relays, $OCR1$ and $OCR2$, disconnect the motor from the supply in the case of a short-circuit in the motor or an overload.

3. Control of shunt-wound d-c motors. This principle is illustrated by the typical control diagram in Fig. 418. It performs starting, stopping and reversing of the motor.

The motor is controlled by means of a three-position master switch, the rest position being marked *MS*0, the forward-run position *MS*1, and the reverse-run position *MS*2. The contacts *MS*0 are normally closed.

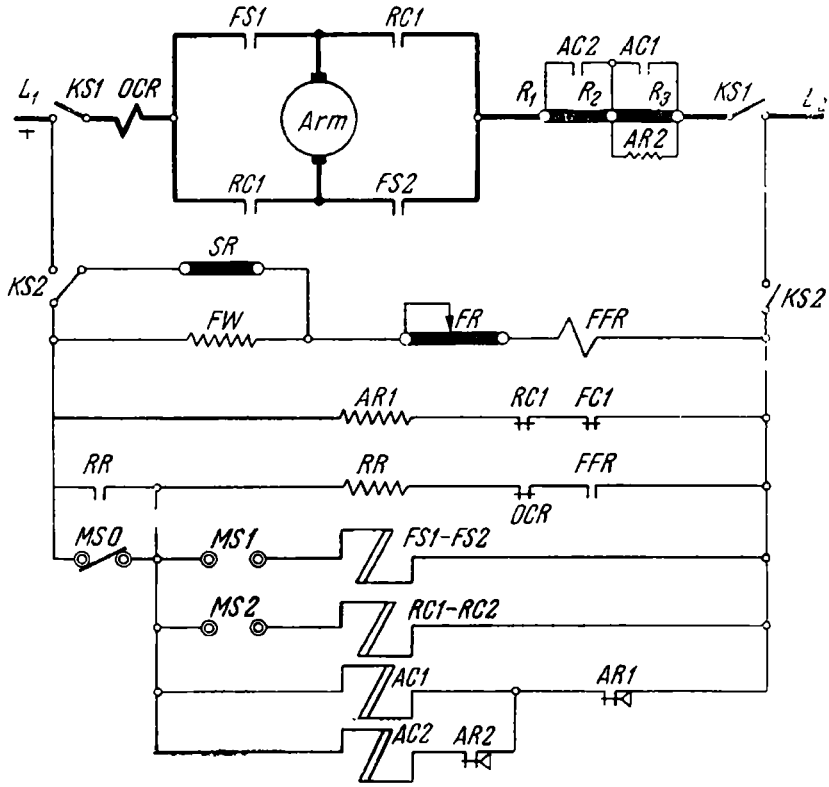


Fig. 418. Control of a shunt-wound d-c motor

The main line contains the coil of an overcurrent relay *QCR*, the armature winding of the motor and a starting resistance. The main line is connected to the supply by means of a knife-blade switch *KS*1. The control circuit is connected to the supply by means of another knife-blade switch, *KS*2. The control circuit contains the shunt field *SF* of the motor, the field resistance *FW*, and the coil of a field failure relay, *FFR*. The field winding is connected across a swamping resistance *SR* as a protection against overvoltage when the circuit is opened. The motor is started up by closing the knife-blade switches *KS*1 and *KS*2. This impresses the supply voltage on the field winding *FR* and energizes the relay *FFR*, which closes its contacts in the circuit of the relay *RR*. The latter is energized and closes its normally open contacts. The positive side of the supply is connected to the re-

lay *RR* via its *NO* contacts and the normally closed contact *MS*0 of the master switch.

Next, the relay *AR*1 breaks its normally closed contact in the circuits of the contactors *AC*1 and *AC*2. Pushing the handle of the master switch to the forward-run position *MS*1 energizes the contactor *FS*1-*FS*2, which makes its two contacts in the supply circuit to the motor, and the motor is set into motion. The voltage drop across the starting resistance energizes the coil of the relay *AR*2 connected in parallel with the first step of the starting resistance, and the relay breaks its contacts in the circuit of the contactor *AC*2. When the contactor *FS*1-*FS*2 is closed, its contacts in the circuit of *AR*1 break, and the relay *AR*1 ceases to function. As a result, the contact *AR*1 closes with a time delay, while the contactor *AC*1 closes to shunt the first step of the starting resistance.

Then the relay *AR*2 loses its excitation and closes its contacts in the circuit of the contactor *AC*2 with a time delay. The second step of the starting resistance is shorted out, and full line voltage is impressed on the armature brushes.

For areversal, the handle of the master switch is thrown into the reverse-run position *MS*2. This causes thecontactor *RC*1-*RC*2 to function, and it closes its contacts in the supply circuit to the motor. The current through the armature winding is reversed, and the motor runs backwards. The rest of the arrangement functions as before.

If the motor current exceeds the current setting of the overcurrent relay *OCR*, the latter will break its contacts in the circuit of the relay *RR*, and the motor will be disconnected from the supply.

Current failure in the field winding of a shunt-wound motor can cause the motor to run away, which may prove dangerous. To avoid this, the relay *FFR* will open its contacts in the circuit of the relay *RR*, and the motor will be disconnected from the supply line.

Speed control is effected by means of the rheostat *SR*.

4. Control of a nonreversing series-wound d-c motor. The typical control diagram is shown in Fig. 419. This arrangement uses one line contactor, *LC*, two accelerating contactors, *AC*1 and *AC*2 and one retarding contactor, *RC*; two overcurrent relays, *OCR*1 and *OCR*2, two accelerating relays, *AR*1 and *AR*2, and a retarding relay, *RR*.

Closure of a knife-blade switch energizes the relay *AR*1. Current now flows from the positive side of the supply through the coil of the relay *OCR*1, relay *AR*1, the armature winding, the series field *SW*, the starting resistance R_3-R_1, the coil of the relay *OCR*2 and back to the negative side of the supply. The resistance of the coil of relay *AR*1 is high. Therefore, the current flowing through the circuit at this moment cannot set the motor into motion. When energized, the relay *AR*1 breaks its contacts in the circuits of the contactors *AC*1 and *AC*2.

Pressure on the start button causes the line contactor *LC* to close its contact in the main line, thus completing the circuit to the motor. At the

same time, an auxiliary contact in the control circuit closes to shunt the start button, and the normally closed contact of contactor *LC* opens in the circuit of the contactor *RC*. The motor sets into motion, having the full starting resistance in series with its armature circuit.

The voltage drop across the starting resistance R_3-R_1 produces a potential difference between points P_2 and P_1; relay *AR*2 is energized and breaks its contacts in the circuit of contactor *AC*2. Closure of line contactor *LC* short-circuits the coil of relay *AR*1, which then closes its contact in the circuit of contactor *AC*1 with a time delay. The closing of *AC*1 in the motor circuit brings the first step of the starting resistance out of circuit. Additionally, the coil of *AR*2, having lost its excitation, closes its contacts in the circuit of contactor *AC*2 with a time delay, and the latter short-circuits the second step of the starting resistance.

While the motor is running, the relay *RR* is energized and its contacts in the circuit of contactor *RC* are closed. However, the contactor cannot

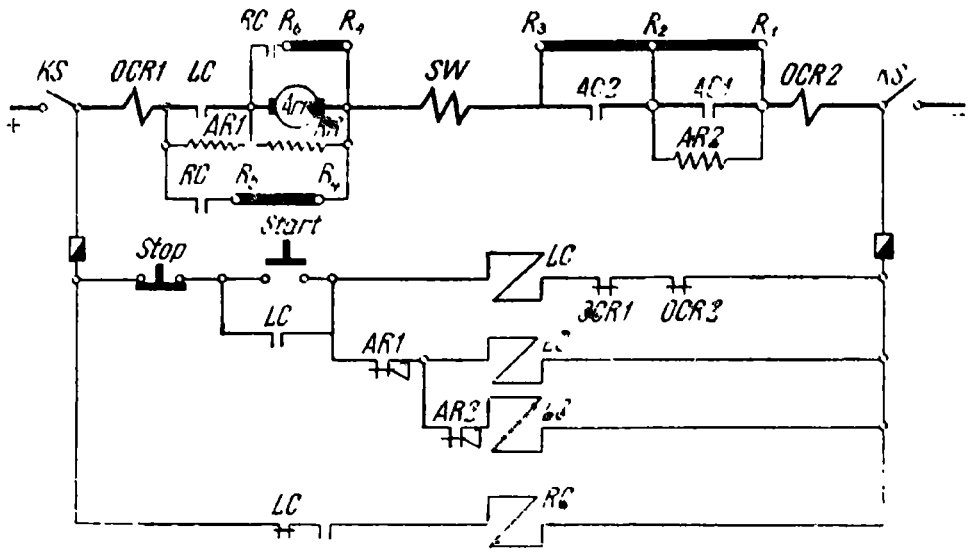


Fig. 419. Control of a nonreversing series-wound d-c motor

close, since the contactor *LC* in its circuit is open as long as the motor is running. The motor is stopped by pressure on the stop button. In this case, contactors *LC*, *AC*1 and *AC*2 are disabled, and the motor is disconnected from the supply. The relay *RR* is then energized by the counter-emf, and contactor *RC* is caused to function and closes its two pairs of contacts in the motor circuit. One pair of contacts connects the armature winding of the motor across resistance R_6-R_4, while the other pair of contacts and resistance R_5-R_6 complete the field circuit of the motor. This brings about dynamic braking of the motor. The slowing down of the motor reduces

the counter-emf. At about 20 per cent of nominal speed, relay *RR* breaks the circuit of contactor *RC* and the latter de-energizes the whole arrangement.

202. Control of Electric Motors by Means of Controllers

Schematically controllers are shown developed on a plane. The segments are depicted as rectangles, and the finger contacts as circles. The fingers and resistances of a controller bearing the same numeral are not usually connected in a diagram. The working positions of a controller are shown by vertical dotted lines.

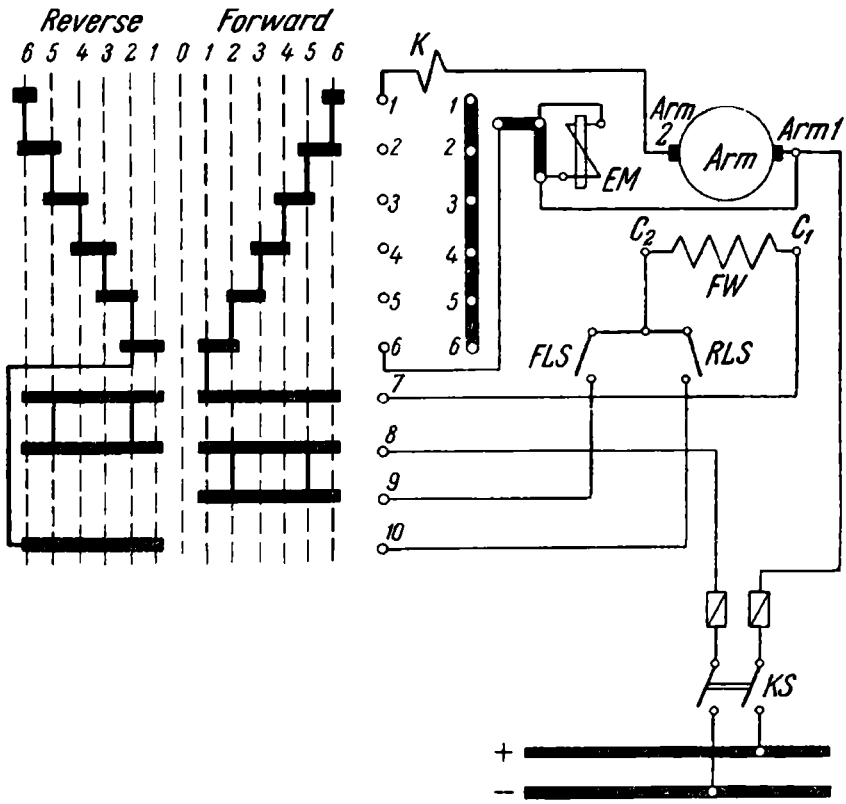


Fig. 420. Control of a d-c motor based on a drum-type controller

Fig. 420 is a typical control diagram of a series-wound d-c motor based on a drum-type controller. This arrangement may be used to control the motor of the cab or bridge of a travelling crane. The arrangement uses limit switches *FLS* and *RLS*, a braking electromagnet *EM*, and an arc-breaking coil *K*. The controller has six positions for the R.H. and L.H. rotation of the motor. The finger contacts *1* through *10* of the controller,

shown to the right of the segments, are in fact located on the vertical line corresponding to the rest position.

Suppose that the knife-blade switch is closed and the controller is set in the forward position *1*. Then the segments make contacts with fingers *6* and *7*, *8* and *9*. The current path is from the plus busbar, through the armature winding (Arm_1-Arm_2) of the motor, the arc-breaking coil *K*,

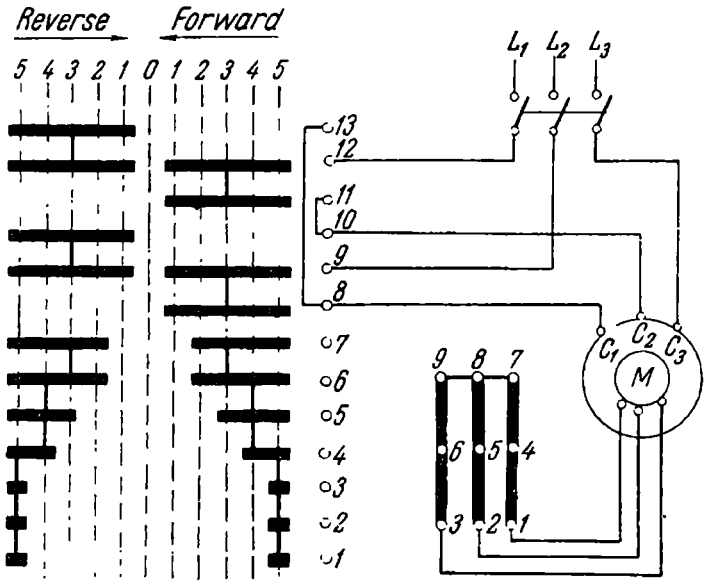


Fig. 421. Control of a slip-ring motor based on a drum-type controller

all the starting resistance *1-6*, the field winding *FW* of the motor (C_1 and C_2), the forward limit switch *FLS* and back to the minus busbar.

In the second forward position, the segments of the controller come in contact with fingers *5* and *6*, thereby bringing some of the starting resistance out of circuit.

In the subsequent forward positions, 3rd, 4th and 5th, additional steps of the starting resistance are brought out, and finally in the 6th position all the starting resistance is short-circuited.

As will be noted, in all the six forward positions, the current flows from Arm_1 to Amr_2 in the armature winding and from C_1 to C_2 in the field winding.

In the first reverse position, the current path is from the plus busbar, through the armature winding, the arc-breaking coil *K*, finger *1*, all the starting resistance *1-6*, finger *6*, finger *10*, the reverse limit switch *RLS*, the field winding of the motor (from C_2 to C_1), finger *7*, finger *8*, and back to the minus busbar. It can be readily seen that the flow of current through the armature winding remains unchanged, while through the field winding it has been reversed. Accordingly, the motor also reverses. The electro-

magnet *EM* is connected in parallel with the motor and releases the motor at starting. When the motor is disconnected, it is braked mechanically.

Fig. 421 shows the control diagram of a slip-ring induction motor based on a drum-type controller. In the first forward position, the segments of the controller make contact with fingers *11* and *12*, *8* and *9*, completing three current paths: (1) from line L_1, through finger *12*, finger *11*, finger *10* to the stator terminal C_2; (2) from line L_2 through finger *9*, finger *8* to stator terminal C_1; and (3) from line L_3 to stator terminal C_3. The rotor is connected to the supply in series with all the starting resistance.

In the second forward position, the segments make contact with fingers *6* and *7*, thereby short-circuiting some of the starting resistance. The subsequent positions bring out and short-circuit all the starting resistance.

When the controller is put in the reverse position, the control circuit operates as in the forward position, except that two of the three stator phases are interchanged.

Review Questions

1. Define electric drive.
2. Define group and individual drive. What is a multi-motor drive?
3. What data are given on the nameplate of a motor?
4. What types of duty can be specified for a motor?
6. What classes of insulation are used in motors and what is the maximum safe temperature for each of the classes?
7. How are motors classified according to types of enclosure?
8. How are electric motors classified according to types of cooling and ventilation?
9. How are electric motors classified according to their speed characteristics?
10. What braking methods do you know?
11. How is an electric motor reversed?
12. What are the methods of speed control?
13. What are the methods of starting electric motors?
14. List control equipment for electric motors.

Chapter Sixteen

STORAGE BATTERIES

203. General

Storage batteries are made up of lead-acid or alkaline cells, the lead-acid type being most common.

The subsequent discussion will be concerned with the battery types employed in the Soviet Union.

Stationary lead-acid batteries use either Type C cells (for low discharge rates) or Type CK cells (for high discharge rates). Type CK cells differ from Type C cells in that they have more robust collecting bars. The numerals following the letter designation give the ampere-hour capacity, discharge current and charging rate of the battery. Commercially available batteries are marked from C-1 (CK-1) to C-148 (CK-148).

Type C cells are intended for discharge during 3 to 10 hours; the maximum 3-hr rate is 9 A. Type CK cells can be discharged in one hour, the maximum 1-hr rate being 18.5 A.

The short-time (not over 5 sec) rate must not exceed 250 per cent of the 3-hr rate for Type C cells and 250 per cent of the 1-hr rate for Type CK cells.

The maximum charging rate is 9 A for Type C cells and 11 A for Type CK cells.

The ampere-hour capacity marked on a storage battery may vary within a broad range, depending on discharge current and time rate.

Leading particulars for Type C-1 and CK-1 batteries are summarized in Table 19.

On the basis of this table, it is possible to determine the ampere-hour capacity and discharge current of any battery. To do so, the table data must be multiplied by the numeral in the type designation of the battery. For example, for a CK-6 battery discharged at the 2-hr rate, the

Table 19

Leading Particulars of Type C-1 and CK-1 Batteries

Characteristic	CK-1		C-1 and CK-1			
Discharge time, hours	1	2	3	5	7.5	10
Capacity:						
in Ah	18.5	22	27	30	33	36
per cent of 10-hr rate	51.4	61.1	75	83.3	91.7	100
Discharge current:						
in amperes	18.5	11	9	6	4.5	3.6
per cent of 10-hr rate	514	305	250	166	125	100
Max. charging rate, A	11	11	9	9	9	9
Final voltage, V	1.75	1.75	1.8	1.8	1.8	1.8

capacity $Q = 22 \times 6 = 132$ Ah, and the discharge current $I_d = 11 \times 6 = 66$ A.

Commercially available lead-acid cells are manufactured with a capacity of 36 to 5328 Ah at the 10-hr rate of discharge. Leading particulars for Type C and CK lead-acid batteries are summarized in Table 20.

Stationary batteries also use Type СП and СПК (for "armoured type") lead-acid cells. Portable batteries are assembled of Type СТ (for "starter type") lead-acid cells.

Alkaline batteries are made up of Type ЖН or ТЖН iron-nickel cells. Their leading particulars are given in Table 21.

As will be seen from the table, the numeral in the type designation of a cell gives its nominal ampere-hour capacity. For Types ЖН-22, ЖН-45, ЖН-60 and ЖН-100, the ampere-hour capacity is given at the 8-hr rate of discharge to a final voltage of 1.1 V, while for Types ТЖН-250, ЖН-300, ЖН-350 and ЖН-500, at the 5-hr rate of discharge to a final voltage of 1 V.

At the normal charging rate, it takes six to seven hours to charge a cell. Cells may, however, be charged at a higher rate as follows: at twice the normal charging rate for 2.5 hours and then at the normal charging rate for 2 hours.

The time of discharge can be found by dividing the ampere-hour capacity of a cell by its discharge rate.

Table 20

Leading Particulars of Type C and CK Batteries

Cell size	Plate type	Plates per cell		Outside tank dimensions, mm			Electrolyte, litres	Total weight, kg	CK cells			C cells				
									1-hr charge		max, discharge A	3-hr charge		10-hr charge max,		
		+ve	−ve	length	width	height			Ah	charge A		Ah	discharge A	Ah	discharge A	charge A
1	И-1	1	2	80	215	270	4.0	14.2	18.5	18.5	11	27	9	36	3.6	9
2	И-1	2	3	130	215	270	6.6	22.5	37	37	22	54	18	72	7.2	18
3	И-1	3	4	180	215	270	9.0	30.0	55.5	55.5	33	81	27	108	10.8	27
4	И-1	4	5	230	215	270	11.0	36.2	74	74	44	108	36	144	14.4	36
5	И-1	5	6	230	215	270	10.5	41.0	92.5	92.5	55	135	45	180	18	45
6	И-2	3	4	195	220	485	16.0	51.8	111	111	66	162	54	216	21.6	54
8	И-2	4	5	195	220	485	15.5	60.0	148	148	88	216	72	288	28.8	72
10	И-2	5	6	260	220	485	20.0	76.3	185	185	110	270	90	360	36	90
12	И-2	6	7	270	220	485	19.5	84.2	222	222	132	324	108	432	43.2	108
14	И-2	7	8	295	220	485	21.0	93.7	259	259	154	378	126	504	50.4	126
16	И-2	8	9	345	220	485	27.0	112.0	296	296	176	432	144	576	57.6	144
18	И-2	9	10	395	220	485	30.0	126.0	333	333	198	486	162	648	64.8	162
20	И-2	10	11	425	220	485	31.0	133.0	370	370	220	540	180	720	72.0	180

Table 21

Leading Particulars of Iron-Nickel-Alkaline Cells

Cell type	Rated Ah capacity	Nominal current on charge at 7-hr rate, A	Time rates					Dimensions, mm			Weight, kg	
			8-hr	5-hr	3-hr	2-hr	1-hr	width	L, including trunnions	H, including terminal posts	without electrolyte	with electrolyte
			Final voltages, V									
			1.1	1.0	0.8	0.65	0.5					
			Discharge current									
ЖН-22	22	5.5	2.75	4.4	7.33	11	22	32	125	213	1.41	1.73
ЖН-45	45	11.25	5.65	9	15	22.5	45	53	125	213	2.31	2.85
ЖН-60	60	15.0	7.5	12	20	30	60	45	152	349	3.88	4.78
ЖН-100	100	25.0	12.5	20	33.3	50	100	70	152	349	5.40	6.8
ТЖН-250	250	62.5	31.3	50	83.3	125	250	118.5	176	375	12.6	15
ТЖН-300	300	57.0	37.5	60	100	150	300	118.5	176	440	14.3	18.5
ТЖН-350	350	60.0	43.7	70	115	175	350	141.5	176	520	19.5	25
ТЖН-500	500	125.0	62.5	100	166.5	250	500	141.5	176	550	21.0	28.2

Portable alkaline batteries are made up of Type 10-ЖН iron-nickel cells with a voltage of 12.5 V, Type 4-ЖН cells with a voltage of 5 V, and Type 5-ЖН cells with a voltage of 6.5 V each.

204. End-cell Switches

With time, the voltage at the terminals of the cells making up a battery falls, and so does the voltage at the terminals of the battery as a whole if no measures are taken to maintain it. Furthermore, there may be varia-

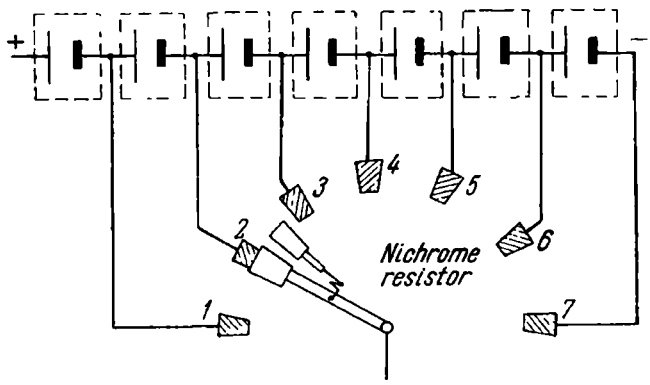


Fig. 422. End-cell switch

tions in the voltage to be taken from the battery at different times during the day. In both cases, the need arises for a means to adjust the battery voltage accordingly. This is done with *end cells*, i.e., cells which may be cut in or out of circuit, and an *end-cell switch.*

The end-cell switch, shown diagrammatically in Fig. 422 consists of a series of contact plates connected to different points in a group of end cells, over which a contact brush is caused to travel to vary the number of cells in circuit on the bus. Obviously, for the uninterrupted operation of the battery, the circuit should be kept completed while a cell is being taken out of, or put into, service. It would seem that this requirement might be met by making the contact arm of the switch wider than the spacing between the contact plates. This, however, would short-circuit the cell being switched in or out. And so the contact arm of the switch is made up of two metal brushes spaced some angle apart. The angle is so chosen that when one brush leaves a contact plate, the other brush comes onto an adjacent plate. The two brushes are connected by a high-resistance nichrome or nickeline resistor to keep the current in the circuit to a minimum when a cell is short-circuited.

205. Applications for Storage Batteries

Of the many applications to which storage batteries can be advantageously put, special mention should be made of their services at electric stations and substations. These services are:

1. As a normal power source for signal and pilot lamps on switchboards and control boards, and for some of the relays and protective devices.

2. Control-bus service in which the function of the battery is to carry the momentary peak loads due to the operation of remote-controlled circuit-breakers, of some of the relays and protective devices, and to ensure a source of current for such apparatus for starting up after an emergency shutdown.

3. Stand-by service in which the principal function of the battery is to ensure continuity of service of a substation, should there be an interruption in the normal source of supply. Apart from d-c motors, this service may include emergency lighting.

For the above applications, storage batteries supply power at voltages of 12, 24, 48, 110 and 220 V. In the case of small substations employing manually or spring operated circuit-breakers, use is made of batteries supplying power at 24 V or 48 V. Electric stations and large substations employ batteries as a source of power for house and auxiliary connections, operating at 110 V and 220 V.

206. Charging of Storage Batteries

Direct current (or unidirectional current) is required for charging a storage battery. It may be supplied by a motor-generator set, a mercury-arc or selenium rectifier.

A battery-charging motor-generator set consists of an induction three-phase motor and a shunt-wound d-c generator mounted on the same shaft.

The field circuit of the generator contains a field rheostat with which it is possible to vary the voltage within a broad range: between 230 and 320 V for 220-V lead-acid batteries; between 115 and 160 V for 110-V lead-acid batteries; and from 230 to 440 V and from 115 to 220 V, respectively, for iron-nickel-alkaline batteries.

Glass mercury-arc rectifiers have certain advantages over motor-generator sets as a source of direct current for battery charging. They have no rotating parts, they operate noiselessly and reliably, and have a high efficiency. On the other hand, they are unable to withstand sudden rises in load and their glass envelopes have a limited service life (2000 hours). This is why they are employed for battery charging only in special cases.

Selenium rectifiers are a much better choice as a low-power source of direct current for battery charging. They have a long service life (up to 25,000 hours), are reliable in operation and simple to service, have a high

efficiency (80 to 85 per cent), are light in weight and small in size. Furthermore, they have a high overload capacity, no fragile or wearing parts, and stand up to vibration well.

207. Operation of Storage Batteries

Storage batteries may be used either in *cycle service* or in *floating service*.

In *cycle service* a battery, after it has been fully charged, is disconnected from the charging unit to power the load. This service is used in connection with permanent loads (signal and pilot lamps), control-bus loads, and stand-by loads. When it has been discharged to the final voltage, it is again connected to a charging unit which both charges the battery and supplies power to the load circuit.

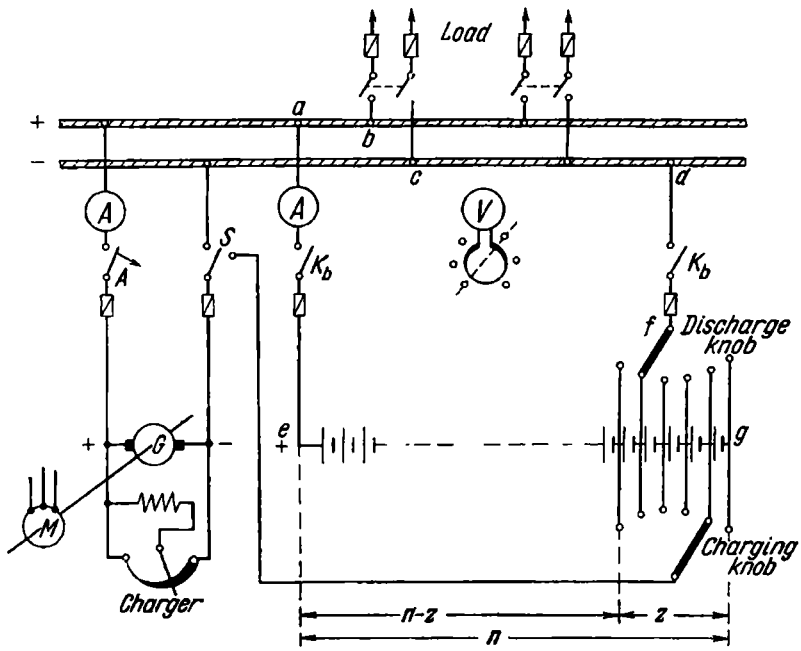


Fig. 423. Diagram of a storage battery fitted with a twin end-cell switch

In *floating service* the battery is given a continuous charge sufficient to maintain an approximately constant state of charge. As distinct from cycle service, a battery in floating service sustains short peak loads due to operation of high-power circuit-breakers. Once a month, batteries in floating service are given an *equalizing charge*, i.e., an extended charge given to the battery to ensure the complete restoration of the active material in all the plates of all the cells.

In comparison with cycle service, floating service offers a number of advantages. The plates undergo less wear and the battery is always in a

state of charge and ready for service. On the other hand, in the case of a battery in cycle service it may happen that a partially discharge battery is not able to deliver the requisite voltage. For these reasons, storage batteries are mostly used in floating service.

In cycle service, a storage battery is operated in conjunction with a twin end-cell switch (Fig. 423). Charging is done by a motor-generator set. The generator is connected to the bus via fuses, an overcurrent circuit-breaker combined with a reverse-current cutout relay, an ammeter, and a two-position switch. The overcurrent circuit-breaker protects the generator from overloads and the reverse-current cutout relay will disconnect the generator if its emf drops below that of the battery. This may occur when the speed of the generator falls or the supply voltage fails, etc. If the generator is not disconnected from the battery at this moment, it will begin to motor and become a load for the battery.

The end-cell switch has two knobs, one for charging and the other for discharging. The two knobs are mounted on the same shaft but can be rotated independently.

The total number of cells that should be assembled into a battery must be such that the bus voltage is 115 V or 230 V even though the cells may be discharged to the final voltage. As pointed out above, the final voltage is 1.75-1.8 V for lead-acid cells and 0.9-1 V for alkaline cells. The total number of cells in a battery is

$$n = V_b/V_f,$$

where V_b is the bus voltage, and V_f is the final voltage of the cells.

Example. $V_b = 115$ V, $V_f = 1.8$ V. The number of cells in the battery must be

$$n = 115 \div 1.8 = 64.$$

Usually, 66 cells are taken per battery.

The arrangement shown in Fig. 423 can function as follows:

1. The battery alone delivers power to the load circuit.
2. The charging unit alone delivers power to the load circuit.
3. The battery and charging set operate in parallel, delivering power to the load circuit.
4. The charging unit both charges the battery and powers the load.

Let us consider each of the cases in more detail.

(1) The battery operates alone. The charging unit is inoperative. The circuit-breaker A (Fig. 423) and the switch S are open. The discharge knob of the end-cell switch is set in the left-hand position. The knife-blade switches K_b are closed. As the battery discharges, the knob of the end-cell switch is shifted to the right to maintain the bus voltage.

(2) The charging unit delivers power to the load circuit. The circuit-breaker A is closed. The switch S is in the left-hand position. The knife-blade switches K_b are open.

(3) The battery and charging unit operate in parallel. The charging unit is operating. The circuit-breaker A is closed. The switch S is in the left-hand position. The knife-blade switches K_b are closed.

(4) The charging unit both charges the battery and powers the load. This is possible if the load is small. Towards the end of the charging operation,

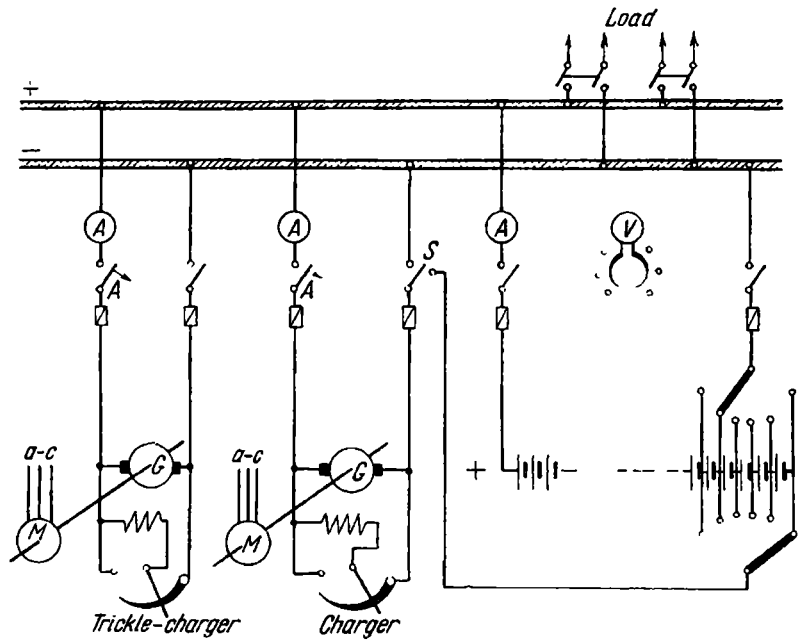


Fig. 424. Diagram of a storage battery in floating service

however, the generator will deliver a voltage exceeding the normal voltage of the load circuit. The voltage could be reduced by inserting a rheostat, but this is not economical. A simpler solution is provided by a twin end-cell switch which diverts the excessive voltage for charging a group of cells connected to the switch.

The number of cells, z, that may be connected to the end-cell switch can be found as follows. The number of cells in operation is $n-z$. When the charging unit both charges the battery and powers the load, this group of cells is connected in parallel with the busbar. Therefore, the bus voltage V_b, towards the end of the charging operation, must be equal to the voltage supplied by the group of cells in service. Thus, the number of cells that may be connected to the end-cell switch will be

$$z = n-(V_b/V_{ch}).$$

Example. $V_b = 115$ V; $n = 64$; $V_{ch} = 2.7$ V.

$$z = 64-(115 \div 2.7) = 21 \text{ cells.}$$

Twenty-two cells are usually taken, and so one third of all the cells in a battery are connected to the end-cell switch at any particular time.

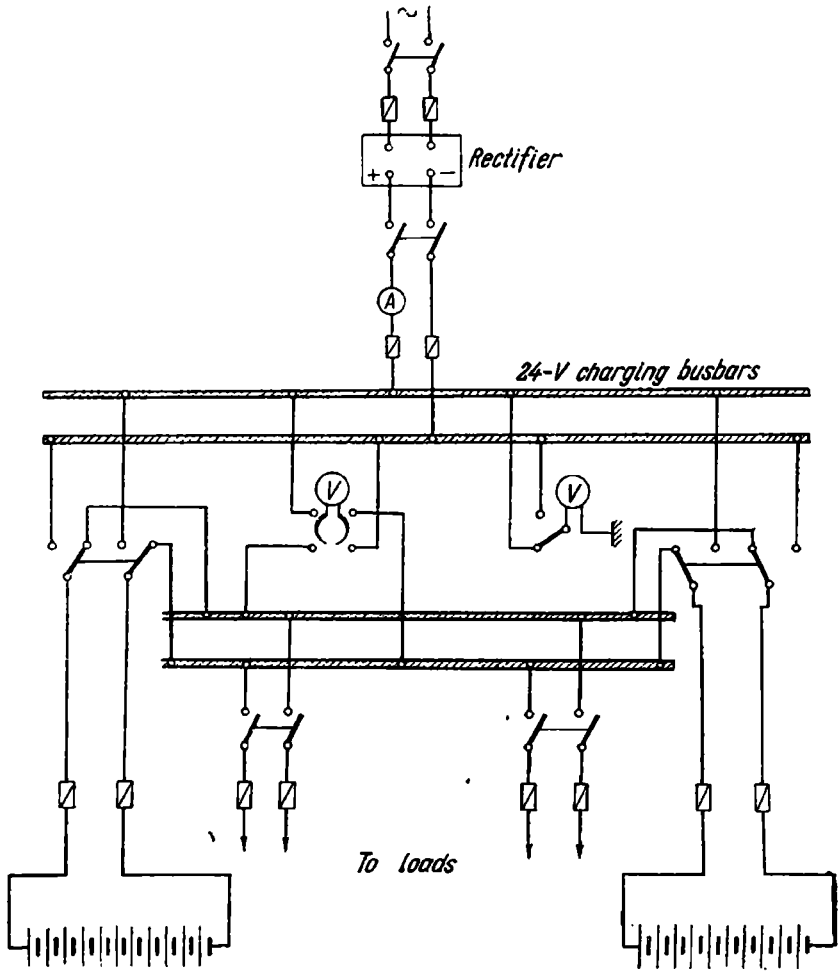


Fig. 425. Low-capacity storage battery

When the charging unit has to both charge the battery and deliver power to the load circuit, the induction motor of the set is started and the nominal voltage is adjusted at the bus by varying the excitation of the generator. Then the circuit-breaker A is closed, the switch S is put in the right-hand position, and the knife-blade switches K_b are closed. The charge knob of the end-cell switch is put in the rightmost position and the discharge knob a bit left of the charge knob.

The generator current I is the sum of two components: the load current I_l and the battery-charge current I_{ch}. From point *a*, the generator current divides between two paths: from point *b*, it flows into the load circuit to arrive at the discharge knob (point *f*) of the end-cell switch via points *c* and *d*. The charge current I_{ch} flows from point *a* through the battery to point *f*. The current flowing between points *f* and *g* (i.e., through the cells connected between the two knobs of the end-cell switch) is the total current of the generator. So that these cells will not be damaged, those with a heavy charging current are taken. As the battery charges, the charge and discharge knobs of the end-cell switch are shifted to the left.

Fig. 424 shows an arrangement for a battery used in floating service, although the battery may also operate in cycle service. A small trickle-charger is operative at all times, while the main charging unit is switched on only when the battery operates in cycle service. In effect, the trickle-charger is a reserve for the charging unit.

Low-capacity batteries of 24 to 48 V are divided into two sections, which are charged and discharged in turn. A schematic diagram of such an arrangement is shown in Fig. 425.

208. Battery Rooms

Batteries should be placed in clean, dry rooms located so that there will be no sharp fluctuations in temperature or exposure to jarring or vibration. At electric stations and substations it is usual to place batteries in the basement or on the ground floor of the station building.

The entrance to the battery room should have an antechamber. The doors should bear the caution signs: "Battery Room", "Fire Hazard", "Keep Off Fire", "No Smoking".

The temperature in the battery room should not be below 10°C at the battery level.

The walls, ceiling, doors, window frames, ventilation hoods, metal parts and other equipment in the room should be given a coat of acid-proof paint (where lead-acid batteries are used) at regular intervals.

The lighting wiring in battery rooms should be of special grades of cable. Lighting fixtures should be of flame-proof construction. All switches, socket outlets and fuses must be installed outside of the battery room.

The battery room should be equipped with a plenum-exhaust ventilation system. In winter, the air admitted into the battery room should be heated.

The floors in battery-rooms are usually asphalt or ceramic tile.

Cell jars should be placed on wooden stands or racks furnished by the battery manufacturer. Here, the cells are best arranged in tiers (one or two). Glass insulators must be placed between the stands and the floor.

Glass cell-jars must be set up on porcelain insulators placed on the stands.

Power connections within the battery room should be made with bare copper wire supported by porcelain insulators. Connection to the end-cell switch mounted on the control board must be through a partition panel fastened to a wall of the battery room.

A separate place must be provided in the battery room for keeping bottles of sulphuric acid and distilled water.

Review Questions

1. What are the principal characteristics of a cell?
2. What is the purpose and construction of end-cell switches?
3. What services can a storage battery perform at an electric station or substation?
4. How is a storage battery charged?
5. Name the basic types of operation of storage batteries. What is the difference between them?
6. Describe a type battery room.

Chapter Seventeen

ACCIDENT PREVENTION

209. General

There is no danger in operating or servicing electrical equipment, provided attending personnel observe all rules and regulations of operation and safety. Therefore, only duly authorized and qualified persons may be allowed to operate or service electrical equipment.

In the Soviet Union, workers handling electrical equipment undergo periodic examinations in the safety rules. All work on high-voltage equipment is done by at least two persons (according to permission cards), using the requisite protective facilities.

In Soviet practice, protective facilities are classed into basic and auxiliary.

Basic facilities are those whose insulation will withstand the working voltage of the plant being serviced and permit contact with live parts. Those for use at any voltage include insulating rods for routine switchings, measurements, application of grounds, etc., and also fuse pullers. Those for low voltages additionally include rubber gloves and gauntlets, and tools with insulated handles.

Auxiliary facilities are those which by themselves cannot protect from an electric shock and only serve as adjuncts to basic facilities, although they are capable of protecting an attendant from touch voltage, step voltage and burns due to an electric arc. Auxiliary devices for work at high voltages are rubber gloves and gauntlets, rubber boots, rubber mats, and insulating supports. In all work on high-voltage equipment, use must be made of the basic facilities in conjunction with auxiliary facilities. All facilities, both in use and in storage, must be registered and regularly checked for their condition.

Electrical equipment is classed as low-voltage if the voltage between any line and earth does not exceed 250 V, and as high-voltage if the voltage between any line and earth is upwards of 250 V.

In low-voltage equipment all current-carrying parts must be protected to exclude accidental contact. Thus, the blades of knife switches, the contacts of rheostats, etc., must be enclosed in protective coverings.

In damp locations and also when working on solidly earthed structures (such as boilers, bridges, etc.), step-down transformers (for 12 V and 24 V) should be used.

All metal parts which are located close to live parts and may come in contact with the latter must be earthed.

No work on rotating machinery and associated apparatus is allowed, except the polishing of slip-rings and commutators.

Installation and maintenance work is done only on de-energized equipment. If the equipment cannot be de-energized for one reason or another, work on live equipment must be performed in conformity with the relevant safety rules, using protective facilities (insulating supports, rubber boots, "live" tools with insulated handles, temporary grounds and short-circuits, insulating sticks, and protective goggles).

Where necessary, caution signs must be posted (skull and crossbones) and the inscription "Death on Contact", or "High Voltage—Dangerous to Life", or other similar inscription. Caution signs must be sufficiently large, with inscriptions in large clear letters made with indelible paint. The smallest size for caution signs is 20 cm by 10 cm.

In work on high-voltage equipment the following precautions must be taken:

(a) work must be done by a crew of not less than two, so that one of the maintainers can help in case of an accident;

(b) the attendants must be well insulated from earth;

(c) while working, the men must not touch persons not standing on insulating supports and must not come in contact with metal parts such as machines, piping, etc.;

(d) the workers must inspect all protective facilities before beginning any job.

Maintenance work on high-voltage equipment (substations, transformer rooms and parts, cables, etc.) is begun only after verbal or written permission has been given and the maintainers have informed that the equipment to be serviced has been de-energized.

Before commencing maintenance work, a check should be made with appropriate testing equipment to see that the equipment to be serviced is dead. Then collecting buses and transformer cables must be discharged, tested for a short-circuit, short-circuited and reliably earthed.

Also prior to work, ladders, scaffoldings, cradles, and ropes must be checked for possible defects.

210. Electric Shock

Every operator or maintainer in the electrical industry must be fully aware of the dangers associated with electric shock.

Electric shock may be due to (1) contact with parts normally alive and (2) contact with parts which are normally dead but have proved live inadvertently (this applies to the outer casings of electric machines and apparatus).

Victims of electric shock lose consciousness and often show no signs of life (there is no breathing, the heart ceases to pump blood).

If the shock has not been severe, the victim will remain unconscious for a few seconds and recover without any aid. This will not happen, however, in severe cases, when the victim must be given immediate, energetic and skilled aid. Any delay may result in the loss of human life. In many cases it is difficult to determine the actual state of a person who has experienced electric shock. Only a physician can say whether further efforts are useless and that the victim is dead. Consequently, the rule is: give resuscitation treatment (artificial respiration) at once and continue until recovery or until the onset of rigor mortis. In some cases up to several hours may be required for recovery.

In most cases the electric current in electric shock passes through the breathing centre and causes this centre to stop sending out impulses to the breathing muscles. As a consequence, breathing stops abruptly. This is why artificial respiration must be applied to the patient as a substitute for natural respiration.

211. First Aid to Victims of Electric Shock

The victim must be immediately freed from contact with current and given first aid.

Breaking the contact. Switch off the part of equipment with which the victim is in contact. If this should cause the victim to fall from a height, prevent the fall or cushion the impact.

If the equipment cannot be switched off at once, break the victim's contact with the live conductor or part.

In the case of low-voltage, use a dry stick, dry board, dry rope, dry coat, or other nonconductor. Never use metal or wet things.

The victim may also be broken away from the live parts by pulling at his clothing, if it is dry and does not fit tightly to his body. Do not touch metal objects and any exposed parts of the victim's body. Pull by his legs only if your own hands are adequately protected.

For insulation from earth or from the victim's body, the rescuer should put on rubber boots or stand on a dry board or any other dry and nonconducting material; put on rubber gloves or wrap the hands in rubberized or dry fabric.

When the current flows to earth through the victim's body, the contact may be broken by lifting the victim off ground, while taking the precuations listed above.

Where necessary, the live conductor must be cut (where several conductors are involved, they must be cut one by one) with an axe having a dry wooden handle or with an insulated cutting tool, with the rescuer reliably insulated from earth.

Where *high voltage* is involved, short-circuit the line wires if contact with the live conductors cannot be broken promptly by any other method.

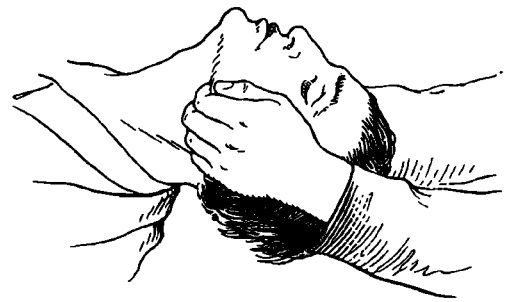


Fig. 426. Forcing a victim's mouth open

First care of a shock victim. (a) If the victim is conscious but has been unconscious or in contact with a live part for a long time, he must be allowed to have complete rest and must be watched closely for two or three hours while waiting for the doctor to arrive. Where a physician cannot be called in promptly, the victim must be taken to hospital.

(b) If the victim is unconscious but is breathing, lay him level on a soft padding, unbuckle his belt and unbutton his clothes, and allow fresh air to reach him. Put a cotton wad soaked in ammonia water to his nose, sprinkle his face with water (but not from your mouth), rub his body with cloth and cover him with a blanket for warmth. Send for a doctor at once.

(c) If the victim is breathing spasmodically, as if dying, apply artificial respiration.

(d) If the mouth is tight shut, force it open by thrusting at the lower jaw with your thumbs until the lower teeth are moved out in front of the upper teeth (Fig. 426). If this method does not work, carefully (so as not to break the teeth) insert a wooden chip, a metal plate, a spoon handle, etc., between the rear teeth at the corner of the mouth and force the mouth open. It must again be stressed that the victim may only appear to be dead (even though there is no breathing, no heart beat, etc.). Only a physician can say definitely whether the victim is dead or not.

In order to be effective, artificial respiration must be applied as soon as possible. The first few minutes are valuable. It should be continued until normal breathing has been restored or until the onset of rigor mortis. Resuscitation should be carried on at the nearest possible point to where the patient received his injuries. He should not be moved from this point unless there is some sort of danger either to the victim or to the rescuer.

With the return of normal respiration, resuscitation may be stopped. Not infrequently, however, the victim, after a temporary recovery of respiration, stops breathing again. The victim must therefore be closely

watched, and, if natural breathing stops, artificial respiration should be resumed at once.

After the recovery, the victim should be kept lying down, covered to keep him warm, and given a warm drink. It is advisable to give the victim some stimulant, such as 15 to 20 drops of valerian tincture.

Even if the shock has been very light, the victim must be taken to hospital for medical assistance within a few hours.

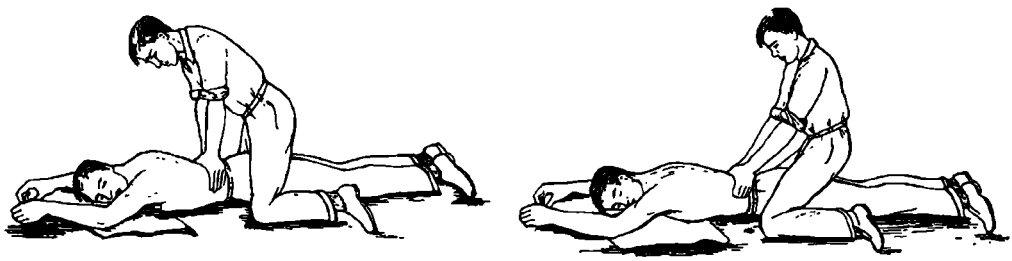


Fig. 427. The Schaefer method of resuscitation (administered by one person)

First care of burns. 1. If a burn is not severe, dress it as a cut wound with sterile dressing material.

2. In the case of serious burns covering a large area, do not attempt to undress the victim. If clothing sticks, do not peel it off; cut around it. A raw or blistered surface should be protected by a curtain of tarpaulin or overalls suspended above the stretcher.

3. Keep the victim warm, give him a hot drink, and send for a doctor.

Methods of resuscitation. There are two main methods of resuscitation: (1) the Schaefer, or prone pressure, method and (2) the Silvester, or back pressure, method.

The Schaefer Method (Fig. 427). One operator is required to apply this method. The standard technique is as follows:

(1) Lay the victim on his belly, one arm extended directly overhead, the other arm bent at elbow and with the face turned outward and resting on hand or forearm, so that the nose and mouth are free for breathing. Pull the tongue forward, but do not hold it.

(2) Kneel, straddling the victim's thighs, with your knees placed at such a distance from the hip bones as will allow you to assume the position shown in Fig. 427.

(3) Place the palms of the hands on the small of the back with fingers resting on the ribs, the little finger just touching the lowest rib, with the thumb and fingers in a natural position, and the tips of the fingers just out of sight.

(4) With arms held straight, swing forward slowly so that the weight of your body is gradually brought to bear upon the victim. The shoulder should be directly over the heel of the hand at the end of the forward

swing (exhalation). This operation should take about two seconds (count "one, two, three").

(5) Now immediately swing backward so as to completely remove the pressure, without removing your hands from the victim's back (inhalation).

(6) After two seconds (count "four, five, six") swing forward again as before. Thus, repeat deliberately twelve to fifteen times a minute the double

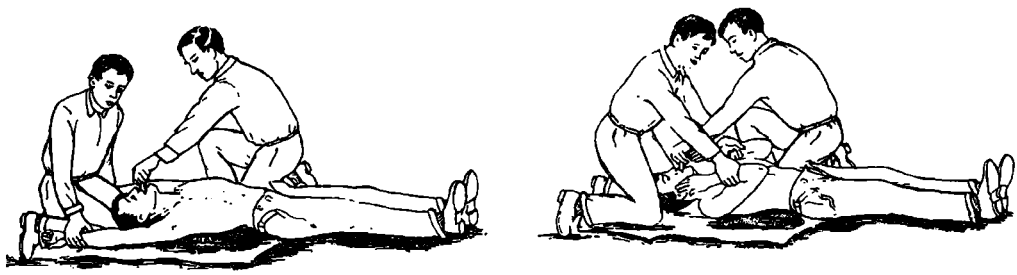


Fig. 428. The Silvester method of resuscitation (administered by two persons)

movement of exhalation and inhalation (a complete respiration in five or four seconds).

This method is, however, unsuitable where the victim has been burned or some of the ribs have been broken.

The Silvester method (Fig. 428). This method takes two operators to apply. The standard technique for this method is as follows:

(1) Lay the victim on his back on a warm padding and place a knoll of clothing under his shoulder-blades so that the head is swung back.

(2) Feel with your fingers in the victim's mouth and throat and remove any foreign body (tobacco, false teeth, etc.).

(3) Pull the tongue forward and keep it in this position by pulling it towards the chin.

(4) Kneel at the patient's head, grasp his arms at the elbows, raise the arms and throw them overhead during two seconds (count "one, two, three"). This is inhalation.

(5) After two seconds (count "four, five, six"), press the arms to the sides.

When resuscitation is carried on correctly, there will be a throbbing sound due to the air passing through the windpipe when the chest is pressed. If there is no sound, this is an indication that the tongue has dropped in.

The Silvester method cannot be used if a collar-bone or arm is broken.

Where two assistants are on hand, artificial respiration can be applied by two, each holding the patient by one hand; the third person holding the tongue pulled out.

In any method, take care not to put too much pressure in the movements so as not to break the ribs, squeeze food out of the stomach and block the respiratory path, or dislocate an arm.

APPENDICES

1. Basic Physical Units and Their Symbols

Unit	Symbol	Unit	Symbol
Gram (mass)	g	Milliwatt	mW
Kilogram (mass)	kg	Microwatt	μW
Milligram (mass)	mg	Electron (charge)	e
Gram (force)	gf	Coulomb	C
Kilogram (force)	kgf	Ampere-second	Asec
Metre	m	Ampere-hour	Ah
Kilometre	km	Wattsecond	Wsec
Centimetre	cm	Joule	J
Decimetre	dm	Watt-hour	Wh
Millimetre	mm	Hectowatt-hour	hWh
Square metre	m^2	Kilowatt-hour	kWh
Square centimetre	cm^2	Volt-ampere	VA
Square millimetre	mm^2	Kilovolt-ampere	kVA
Ohm	Ω or spelled out	Volt-ampere, reactive	VAr
		Kilovolt-ampere, reactive	kVAr
Kiloohm	kΩ or Kohm	Farad	F
Megohm	MΩ or Mohm	Microfarad	μF
Ampere	A	Picofarad	pF
Kiloampere	kA	Henry	H
Milliampere	mA	Cycles per second	c/s
Microampere	μA	Kilocycles per second	kc/s
Volt	V	Megacycles per second	Mc/s
Kilovolt	kV	Weber	Wb
Millivolt	mV	Maxwell	spelled out
Microvolt	μV	Gauss	spelled out
Watt	W	Oersted	spelled out
Hectowatt	hW	Calorie	cal
Kilowatt	kW	Kilogram-calorie	kcal
Megawatt	MW		

2. Symbols of Basic Electrical Quantities

Quantity	Symbol
Capacitance	C
Inductance, self-inductance	L
Magnetic induction (flux density)	B
Quantity of electricity (charge)	Q, q
Resistance-temperature coefficient	α
Electro-chemical equivalent	α
Power	P
Reactive power	Q
Apparent power	S
Instantaneous power	p
Voltage	V
Magnetizing force	H
Electric field strength (force)	E
Current density	δ
Magnetic flux	Φ
Linkage	ψ
Conductance	G, g
Absolute permittivity	ε
Relative permittivity	ε_r
Absolute permeability	μ
Relative permeability	μ_r
Phase angle or phase displacement	φ
Electromotive force	E
Resistance	R
Reactance	X
Impedance	Z
Resistivity	ϱ
Current	I
Initial phase angle	ψ
Frequency	f
Angular velocity	ω
Number of turns	T
Proportionality factor	c

3. Answers to Exercises

(some of the answers are rounded off)

Chapter One

1. 1.35 N = 137.7 gf
2. 18.15 dynes
3. 9 cm
4. 4.5×10^3 V/cm
5. 4×10^{-8} C
6. 10^5 V
7. 150 J
8. 4 μF
9. 1/300th μF

Chapter Two

1. 16.55 ohms
2. 3.6 ohms
3. 0.1 mm^2
4. 10.9 m
5. Iron
6. 0.25 mm^2
7. 1550 ohms
8. 2.2 mm^2
9. 50°C
10. 13.06 m
11. 28 V
12. 5.8 ohms
13. 1600 ohms
14. 6 ohms
15. 22.4 V; 9.6 V; 6.4 V
16. 270.4 ohms
17. 0.8 ohm
18. 0.88 ohm
19. 24.5 ohms
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